Pg. 491 #'s 1-49(odd)

1. a) $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ or 30°

b) $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ or 60°

c) $\cos^{-1}(2) =$ Not defined

3. a) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ or 45°

b) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ or 45°

c) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$ or -45°

5. a) $\sin^{-1}(1) = \frac{\pi}{2}$ or 90°

b) $\cos^{-1}(1) = 0$

c) $\cos^{-1}(-1) = \pi$

7. a) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$ or 30° Remember $\frac{\frac{1}{2}\,y}{\frac{\sqrt{3}}{2}\,x} = \frac{\sqrt{3}}{3}$

b) $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$ or -30°

c) $\sin^{-1}(-2) =$ Not defined.

9. a) $\sin^{-1}(0.7688) = 50.24625^\circ$ or 0.87696 rad

b) $\cos^{-1}(-0.5014) = 120.09267^\circ$ or 2.0960 rad

11. $\sin(\sin^{-1}(\frac{1}{3})) = \boxed{\frac{1}{3}}$

13. $\tan(\tan^{-1}(10)) = \boxed{10}$

15. $\cos^{-1}(\cos(\frac{\pi}{3})) = \boxed{\frac{\pi}{3}}$

17. $\sin^{-1}(\sin(-\frac{\pi}{6})) = \boxed{-\frac{\pi}{6}}$

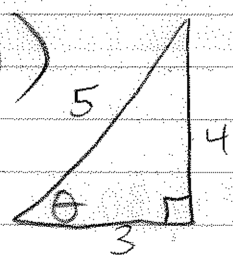
19. $\tan^{-1}(\tan(\frac{2\pi}{3})) = \boxed{-\frac{\pi}{3}}$

21. $\tan(\sin^{-1}(\frac{1}{2})) = \tan\frac{\pi}{6} = \boxed{\frac{\sqrt{3}}{3}}$

23. $\cos(\sin^{-1}(\frac{\sqrt{3}}{2})) = \cos\frac{\pi}{3} = \boxed{\frac{1}{2}}$

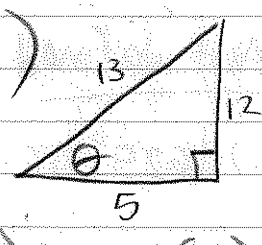
25. $\tan^{-1}(2\sin\frac{\pi}{3}) = \tan^{-1}(2\cdot\frac{\sqrt{3}}{2}) = \tan^{-1}(\sqrt{3}) = \boxed{\frac{\pi}{3}}$

27. $\sin(\cos^{-1}(\frac{3}{5}))$ $\quad \theta = \cos^{-1}\frac{3}{5}$



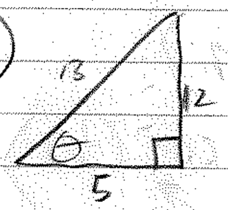
$\sin(\cos^{-1}(\frac{3}{5})) = \sin(\theta) = \boxed{\frac{4}{5}}$

29. $\sin(\tan^{-1}(\frac{12}{5}))$ $\quad \theta = \tan^{-1}(\frac{12}{5})$



$\sin(\tan^{-1}(\frac{12}{5})) = \sin(\theta) = \boxed{\frac{12}{13}}$

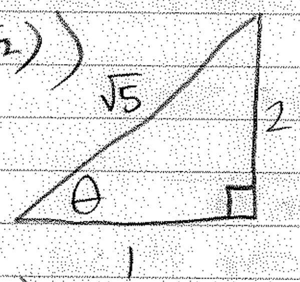
31. $\sec\left(\sin^{-1}\left(\frac{12}{13}\right)\right)$



$\theta = \sin^{-1}\left(\frac{12}{13}\right)$

$\sec\left(\sin^{-1}\left(\frac{12}{13}\right)\right) = \sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{5}{13}} = \boxed{\frac{13}{5}}$

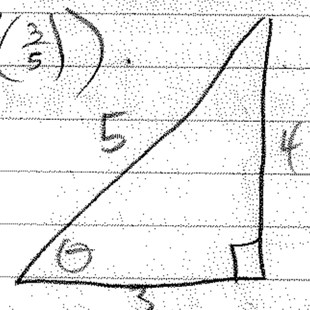
33. $\cos\left(\tan^{-1}(2)\right)$



$\theta = \tan^{-1}(2)$

$\cos\left(\tan^{-1}(2)\right) = \cos\theta = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{\sqrt{5}}{5}}$

35. $\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right)$.



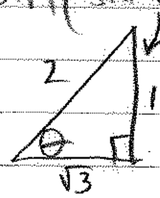
$\theta = \cos^{-1}\left(\frac{3}{5}\right)$

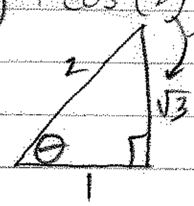
$\sin\left(2\cos^{-1}\left(\frac{3}{5}\right)\right) = \sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$= \boxed{\frac{24}{25}}$

37. $\sin\left(\underbrace{\sin^{-1}\left(\frac{1}{2}\right)}_{a} + \underbrace{\cos^{-1}\left(\frac{1}{2}\right)}_{b}\right) = \sin\left(\sin^{-1}\frac{1}{2}\right)\cos\left(\cos^{-1}\frac{1}{2}\right) + \cos\left(\sin^{-1}\frac{1}{2}\right)\sin\left(\cos^{-1}\frac{1}{2}\right)$





$= \frac{1}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}$

$= \frac{1}{4} + \frac{\sqrt{9}}{4} = \frac{1}{4} + \frac{3}{4} = \boxed{1}$

37. Another (easier way)

$$\sin\left(\sin^{-1}\left(\tfrac{1}{2}\right) + \cos^{-1}\left(\tfrac{1}{2}\right)\right)$$

we know exact angles for these two!

$$\sin\left(\tfrac{\pi}{6} + \tfrac{\pi}{3}\right) = \sin\left(\tfrac{\pi}{2}\right) = \boxed{1}$$

39. $\cos(\sin^{-1}(x))$ $\quad \theta = \sin^{-1}x$

$$\cos\theta = +\sqrt{1-\sin^2\theta}$$

Write $\cos\theta$ in terms of sin, so that we can "undo" the inverse function!

Positive because outputs for $\sin^{-1}x$ are $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, which give a positive cosine.

$$\cos\theta = \sqrt{1-\sin^2(\sin^{-1}x)} = \sqrt{1-[\sin(\sin^{-1}x)]^2}$$

$$\boxed{= \sqrt{1-x^2}}$$

41. $\tan(\sin^{-1}(x))$ Write tan as a function of sine, so we can undo the $\sin^{-1}x$.

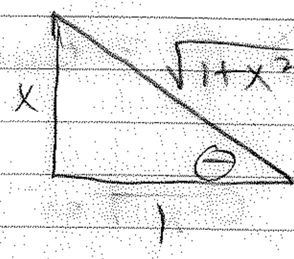
$\theta = \sin^{-1}(x)$ $\quad$ Now replace θ w/ $\sin^{-1}x$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} = \frac{\sin(\sin^{-1}x)}{\sqrt{1-(\sin(\sin^{-1}x))^2}}$$

from #39

$$\boxed{= \frac{x}{\sqrt{1-x^2}}}$$

43. $\cos(2\tan^{-1}(x))$



$\theta = \tan^{-1}(x)$ from # 41

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \left(\frac{1}{\sqrt{1+x^2}}\right)^2 - \left(\frac{x}{\sqrt{1+x^2}}\right)^2$$

$$= \frac{1}{1+x^2} - \frac{x}{1+x^2}$$

$$\boxed{= \frac{1-x^2}{1+x^2}}$$

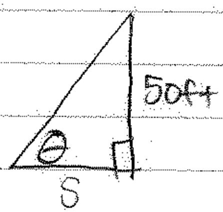
45. $\cos(\cos^{-1}(x) + \sin^{-1}(x))$

$$= \cos(\cos^{-1}x)\cos(\sin^{-1}x) + \sin(\cos^{-1}x)\sin(\sin^{-1}x)$$

$$= x \cdot \sqrt{1-x^2} + \sqrt{1-x^2} \cdot x$$

$$= \boxed{0}$$

47.



so... $\tan\theta = \frac{50}{s}$

$$\boxed{\theta = \tan^{-1}\left(\frac{50}{s}\right)}$$

θ interms of "s."

49. a) $\tan\theta = \tan(a-B) = \frac{\tan a - \tan B}{1 + \tan a \tan B}$

$$= \frac{\frac{5-h}{x} - \frac{3-h}{x}}{1 + \frac{5-h}{x} \cdot \frac{3-h}{x}} = \frac{\frac{2}{x}}{\frac{x^2}{x^2} + \frac{(5-h)(3-h)}{x^2}} = \frac{2}{x} \cdot \frac{x^2}{x^2+(5-h)(3-h)}$$

$$= \boxed{\frac{2x}{x^2+(5-h)(3-h)}}$$

49. b) $\Theta = \tan^{-1}\left(\frac{2x}{x^2+(1)(3)}\right) = \frac{2x}{x^2+3}$

$\Theta = \tan^{-1}\left(\frac{2\cdot 0.5}{0.5^2+3}\right) = \boxed{17.1°}$

$\Theta = \tan^{-1}\left(\frac{2\cdot 2}{2^2+3}\right) = \boxed{29.7°}$

$\Theta = \tan^{-1}\left(\frac{2\cdot 5}{5^2+3}\right) = \boxed{19.7°}$