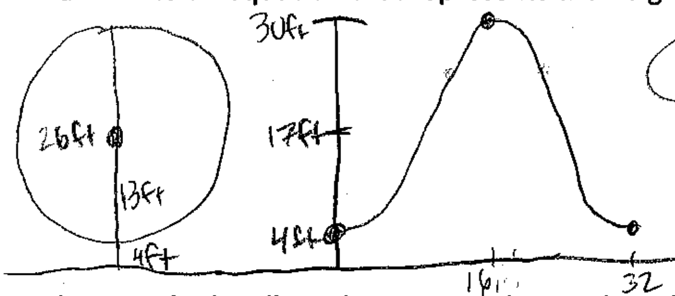
PC – Trigonometry  Name KEY

# GOING FOR A WILD RIDE AROUND AND AROUND

1. A Ferris wheel is 4 feet off the ground. It has a diameter of 26 feet, and rotates once every 32 seconds. You begin the ride sitting in a chair that is 4 feet above the ground.

   a. Write an equation that represents the height $h(t)$ of the rider over time $t$ in seconds.




$h(t) = -13\cos\left(\frac{\pi}{16}t\right) + 17$

$\frac{2\pi}{b} = 32$

   b. How high will you be 10 seconds into the ride?

$h(10) = -13\cos\left(\frac{\pi}{16}\cdot 10\right) + 17 \approx 21.97$ ft

   c. During the first minute, when will you be 20 feet high? * Use graph above for visual

$20 = -13\cos\left(\frac{\pi}{16}\cdot t\right) + 17$

$-\frac{3}{13} = \cos\left(\frac{\pi}{16}t\right)$

$\cos^{-1}\left(-\frac{3}{13}\right) = \frac{\pi}{16}t$

$1.8037 = \frac{\pi}{16}t$

$t \approx 9.19$ sec

Symmetrical, so

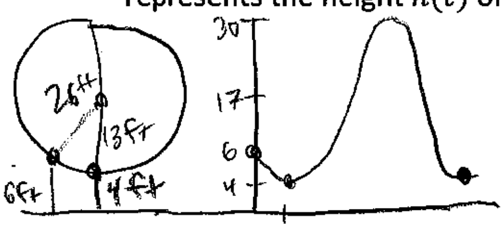
$16 - 9.19 = 6.814$

$16 + 6.814 = 22.81$ sec

$9.19 + 32$ (period) $= 41.19$ sec

$22.81 + 32 = 54.81$ sec

   d. If a similar Ferris wheel (4 feet off the ground, diameter of 26 feet, and rotates once every 32 seconds) had you begin at 6 feet above the ground (rotating counterclockwise), write an equation that represents the height $h(t)$ of the rider over time $t$ in seconds (phase shift needs to be in seconds).




$6 = -13\cos\left(\frac{\pi}{16}t\right) + 17$

$\cos^{-1}\left(\frac{11}{13}\right) = \frac{\pi}{16}t$

$0.5621 = \frac{\pi}{16}t$

$t \approx 2.863$ sec to travel 2ft higher.

$h(t) = -13\cos\frac{\pi}{16}(t - 2.863) + 17$

2. A car's tire has a diameter of 32 inches. It runs over a nail at 88 $m.p.h.$, but it is able to continue moving. Write a function that describes the height of the nail $h(t)$ above ground as a function of time $t$.

dist = rate · time (period)

$\frac{88\text{ mi}}{\text{hr}} \cdot \frac{1\text{ hr}}{3600\text{ sec}} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{12\text{ in}}{1\text{ ft}} = \frac{5575680}{3600} = \frac{7744\text{ in}}{5\text{ sec}}$

$= 32\pi$

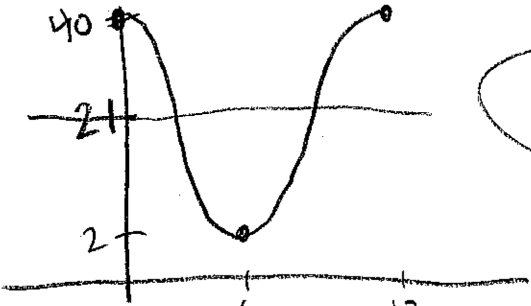
time $= \frac{32\pi}{\frac{7744}{5}} = \frac{5\pi}{242}$ sec (period) $\rightarrow \frac{2\pi}{b} = \frac{5\pi}{242}$

$b = \frac{484}{5}$

$h(t) = -16\cos\left(\frac{484}{5}t\right) + 16$

3. Each day, the tide continuously goes in and out, raising and lowering a boat (sinusoidally) in the harbor. Low tide at 6 a.m. has the boat only 2 feet above the ocean floor. Six hours later, at peak high tide, the boat is 40 feet above the ocean floor.

a. Write an equation that represents the height $h(t)$ of the boat over time $t$ in hours from midnight.




$h(t) = 19\cos\frac{\pi}{6}t + 21$

$\frac{2\pi}{b} = 12$

b. For safety, the boat needs 14 feet of depth to sail. If high tide occurs at noon, between what times can the boat go out to sea?

$14 = 19\cos\left(\frac{\pi}{6}t\right) + 21$

$\cos^{-1}\left(-\frac{7}{19}\right) = \frac{\pi}{6}t$

$t \approx 3.72$ hours past midnight ⇒ 3:43 a.m. + 12 hr. . .

→ Symmetrical

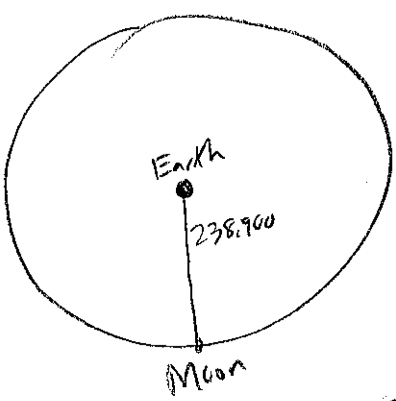
$6 - 3.72 = 2.28$

$6 + 2.28 = 8.28$ hrs after midnight ⇒ 8:16 am

+12 hr

Period

4. The mean distance of the Moon from Earth is 238,900 *miles*. Assuming that the orbit of the Moon around Earth is circular and that 1 revolution takes 27.3 days, write a sine function that represents the path of the Moon around Earth as a function of time in *days*.




$\frac{2\pi}{b} = 27.3$

$b = \frac{20\pi}{273}$

$d(t) = 238{,}900 \sin\left(\frac{20\pi}{273}t\right)$