PC – Trig

Name KEY

Review – Graphing and all that jazz

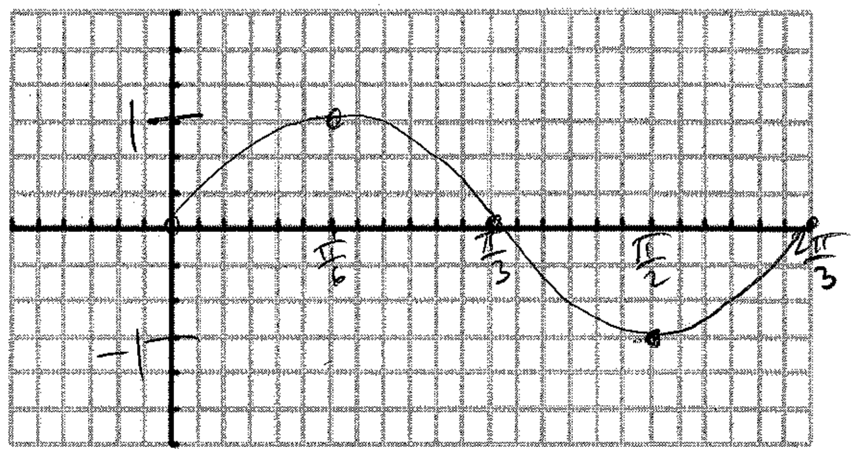
For #'s 1-10, use the attached graphing paper to graph at least five points accurately. Show any work necessary.

1. Equation: $y = \sin 3\theta$

Period: $2\pi/3$ Amplitude: 1

Phase shift: 0 Vertical shift: 0

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\frac{1}{3}\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$\sin\theta$	0	1	0	-1	0



$\csc \frac{1}{2}\theta$

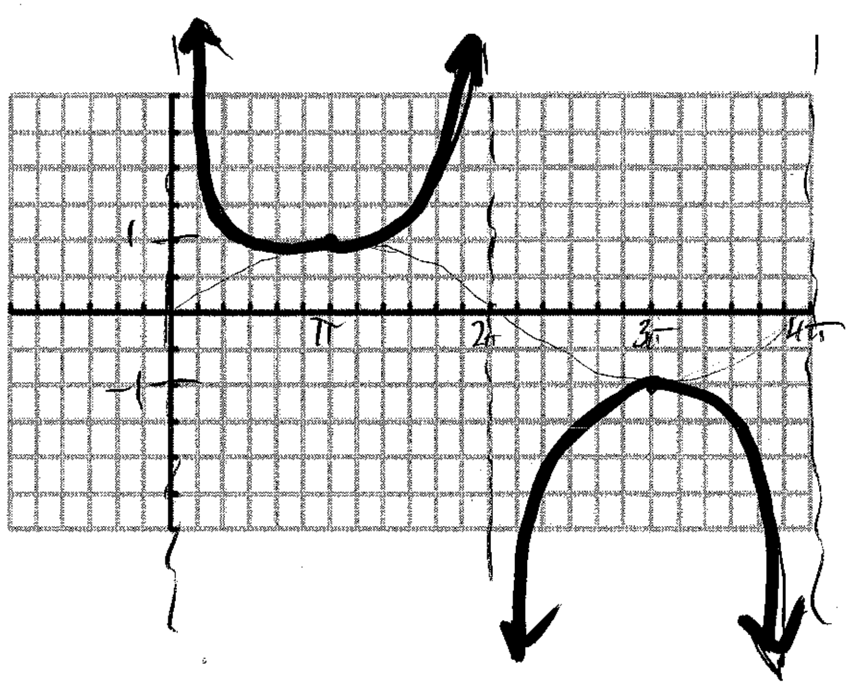
2. Equation: $y = \csc\frac{\theta}{2} \longrightarrow \frac{1}{\sin\frac{\theta}{2}}$

Period: 4π Amplitude: 1

$\frac{2\pi}{\frac{1}{2}} =$

Phase shift: 0 Vertical shift: 0

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
2θ	0	π	2π	3π	4π
$\sin\theta$	0	1	0	-1	0
$\frac{1}{\sin\theta} = \csc\theta$	Und.	1	Und	-1	Und.

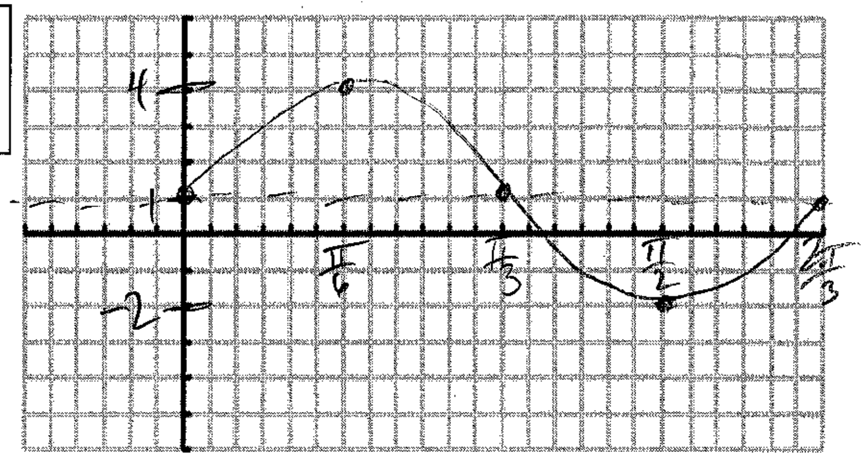


3. Equation: $y = 1 + 3\sin 3\theta$

Period: $2\pi/3$ Amplitude: 3

Phase shift: 0 Vertical shift: 1

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\frac{1}{3}\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$\sin\theta$	0	1	0	-1	0
$1+3\sin\theta$	1	4	1	-2	1



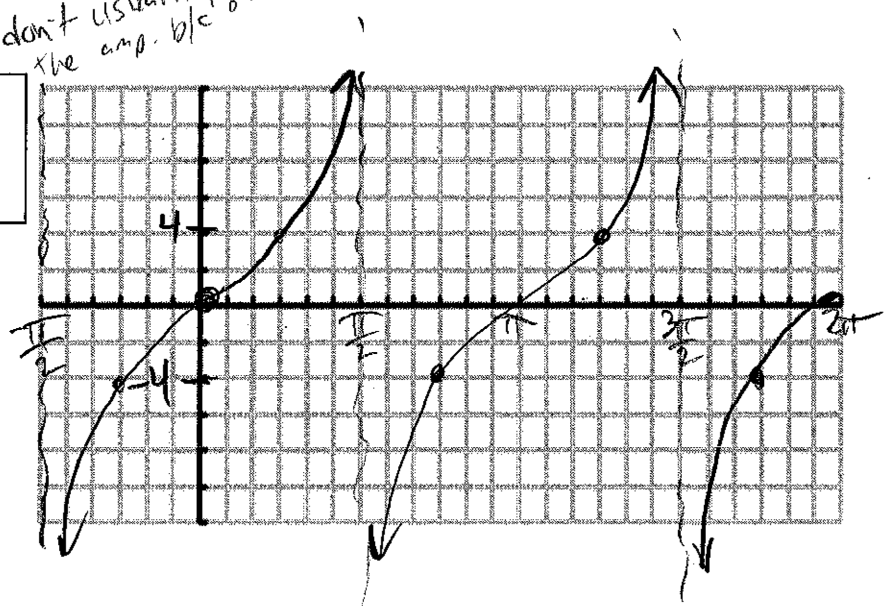
4. Equation $y = 4\tan\theta$

Remember, we don't usually plot the amp. b/c of scale issues

Period: π Amplitude: 4

Phase shift: 0 Vertical shift: 0

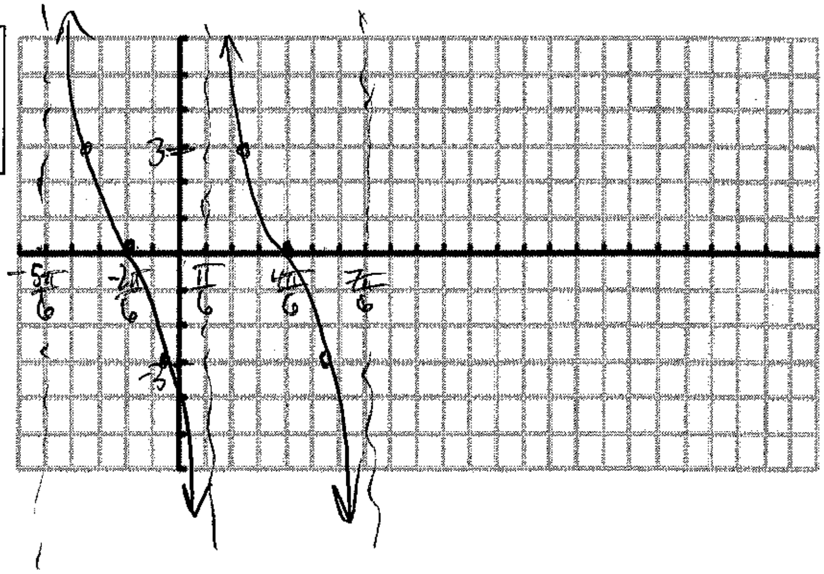
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
terminal points	(1,0)	(0,1)	(-1,0)	(0,-1)	(1,0)
$\tan\theta$	$\frac{0}{1}$	$\frac{1}{0}$	$\frac{0}{-1}$	$\frac{-1}{0}$	$\frac{0}{1}$
$\tan\theta$	0	und.	0	und	0



5. Equation: $y = 3\cot\left(\theta + \frac{5\pi}{6}\right)$

Period: π	Amplitude: 3
Phase shift: $-\frac{5\pi}{6}$	Vertical shift: 0

θ	0	$\frac{\pi}{2}\frac{3}{3}$	$\pi\frac{6}{6}$	$\frac{3\pi}{2}\frac{3}{3}$	$2\pi\frac{6}{6}$
θ	0	$\frac{3\pi}{6}$	$\frac{6\pi}{6}$	$\frac{9\pi}{6}$	$\frac{12\pi}{6}$
$\theta - \frac{5\pi}{6}$	$-\frac{5\pi}{6}$	$-\frac{2\pi}{6}$	$\frac{\pi}{6}$	$\frac{4\pi}{6}$	$\frac{7\pi}{6}$
$\cot\theta$	und	0	und	0	und

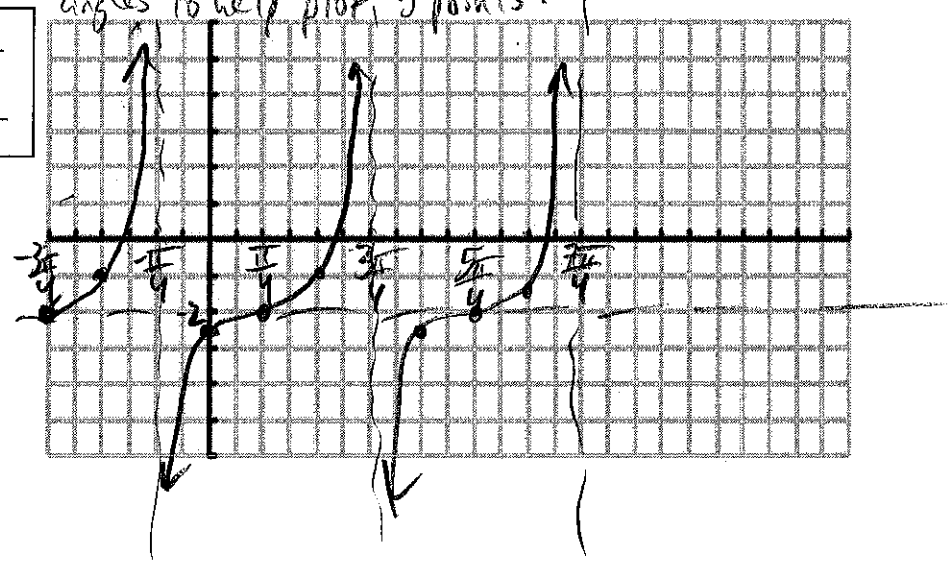


6. Equation: $y = \frac{1}{2}\tan\left(\theta + \frac{7\pi}{4}\right) - 2$

So, the θ values are not "great." Use coterminal angles to help plot 5 points.

Period: π	Amplitude: $\frac{1}{2}$
Phase shift: $-\frac{7\pi}{4}$	Vertical shift: -2

θ	0	$\frac{\pi \cdot 2}{2 \cdot 2}$	$\pi\frac{4}{4}$	$\frac{3\pi}{2 \cdot 2}\,2$	$\frac{2\pi}{4}\,4$
$\theta - \frac{7\pi}{4}$	$-\frac{7\pi}{4}$	$-\frac{5\pi}{4}$	$-\frac{3\pi}{4}$	$-\frac{\pi}{4}$	$\frac{\pi}{4}$
$\tan\theta$ $\frac{y}{x}$	0	und	0	und	0
$\frac{1}{2}\tan\theta - 2$	-2	und	-2	und	-2



@ θ values in between -2 and vertical asymptotes, outputs could be graphed (for more accuracy, but NOT required) where 1 and -1 are transformed by $\frac{1}{2}\tan\theta - 2$.

$-1 \rightarrow -2.5$

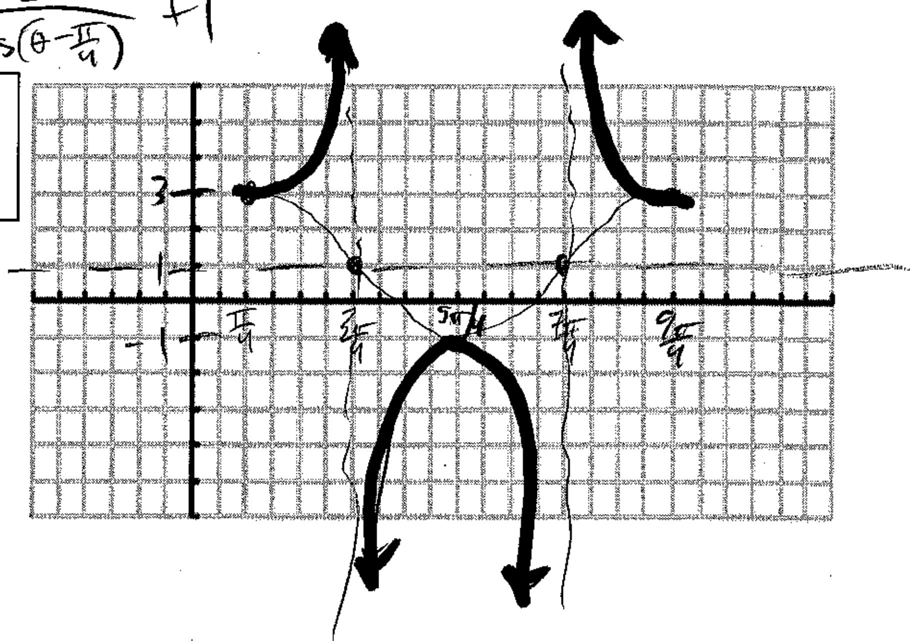
$+1 \rightarrow -1.5$

7. Equation: $y = 2\sec\left(\theta - \frac{\pi}{4}\right) + 1 = \frac{2}{\cos(\theta - \frac{\pi}{4})} + 1$

Period: 2π Amplitude: 2

Phase shift: $+\frac{\pi}{4}$ Vertical shift: +1

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\theta + \frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{3\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$	$\frac{9\pi}{4}$
$\cos\theta$	1	0	-1	0	1
$2\cos\theta + 1$	3	1	-1	1	3
$2\sec\theta + 1$	3	und	-1	und	3



Asymptotes still occur where $\cos\theta = 0$ because when those specific θ values ($\frac{3\pi}{4}$ + $\frac{7\pi}{4}$) are input, $\cos\theta$ will output 0. → $\frac{2}{\cos(\theta - \frac{\pi}{4})} + 1$

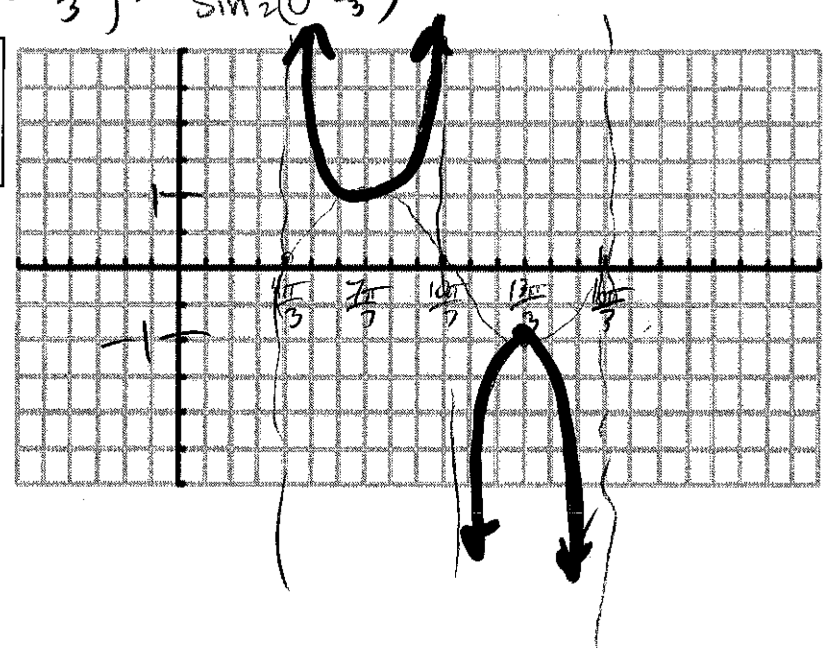
8. Equation $y = \csc\left(\frac{\theta}{2} - \frac{2\pi}{3}\right) = \csc\frac{1}{2}\left(\theta - \frac{4\pi}{3}\right) = \frac{1}{\sin\frac{1}{2}(\theta - \frac{4\pi}{3})}$

Period: 4π Amplitude: 1

$\frac{2\pi}{1/2}$

Phase shift: $+\frac{4\pi}{3}$ Vertical shift: 0

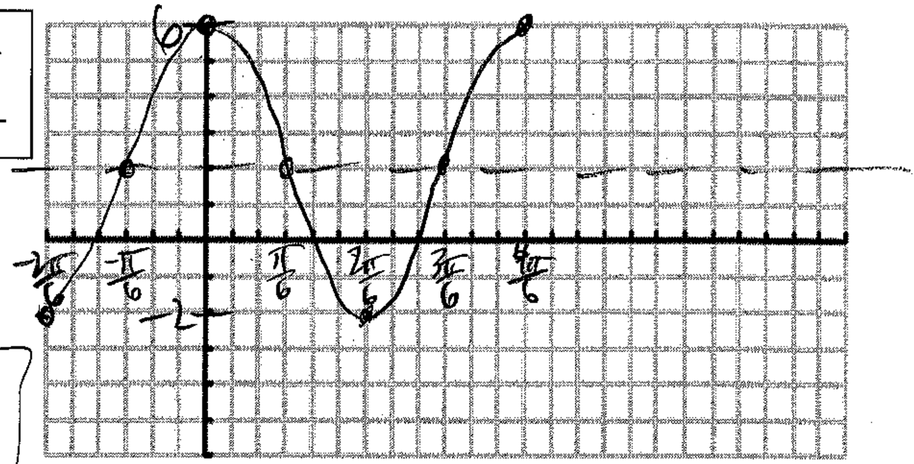
θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
2θ	0	$\pi\frac{3}{3}$	$2\pi\frac{3}{3}$	$3\pi\frac{3}{3}$	$4\pi\frac{3}{3}$
$2\theta + \frac{4\pi}{3}$	$\frac{4\pi}{3}$	$\frac{7\pi}{3}$	$\frac{10\pi}{3}$	$\frac{13\pi}{3}$	$\frac{16\pi}{3}$
$\sin\theta$	0	1	0	-1	0
$\csc\theta$	und	1	und	-1	und



9. Equation: $y = 4\sin\left(3\theta + \frac{5\pi}{3}\right) + 2 = 4\sin 3\left(\theta + \frac{5\pi}{6}\right) + 2$

Period: $2\pi/3$	Amplitude: 4
Phase shift: $-\frac{5\pi}{6}$	Vertical shift: $+2$

θ	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\frac{1}{3}\theta$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}\frac{2}{2}$	$\frac{\pi}{2}\frac{3}{3}$	$\frac{2\pi}{3}\frac{2}{2}$
$\frac{1}{3}\theta - \frac{5\pi}{6}$	$-\frac{5\pi}{6}$	$-\frac{4\pi}{6}$	$-\frac{3\pi}{6}$	$-\frac{2\pi}{6}$	$-\frac{\pi}{6}$
$\sin\theta$	0	1	0	-1	0
$4\sin\theta + 2$	2	6	2	-2	2

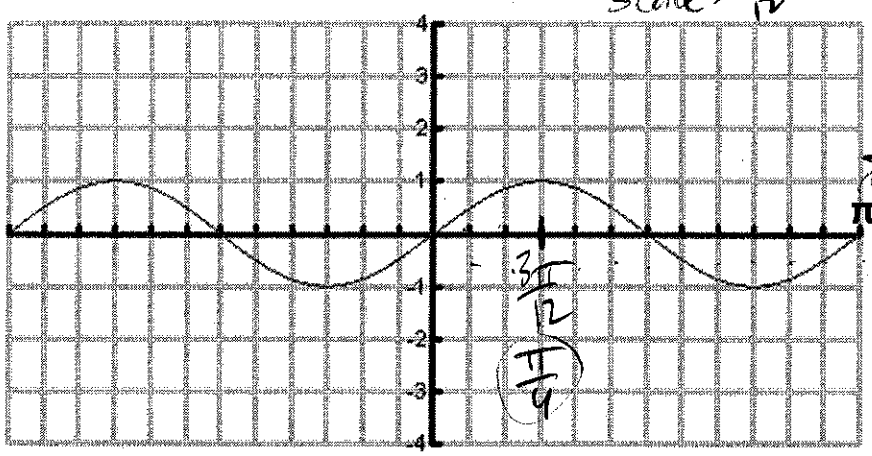


Our x-axis is not pretty, so we can continue our pattern and expand our scale: $\frac{2\pi}{6}, \frac{-\pi}{6}, 0, \frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}, \frac{4\pi}{6}, \frac{5\pi}{6}$ - - -

For #'s 10-15, find the equation that best fits the trigonometric graph.

10.

Scale $= \frac{\pi}{12}$



Period $= \pi$

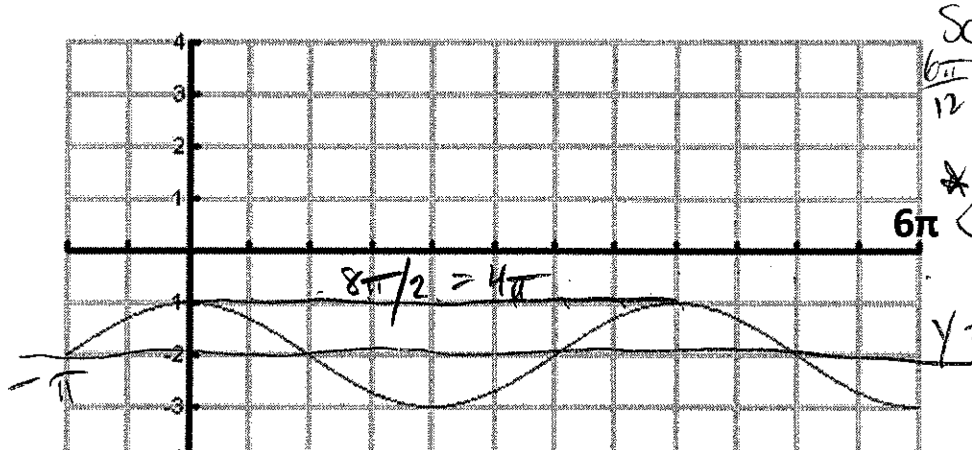
$\frac{2\pi}{b} = \pi \Rightarrow b = 2$

* $y = \sin 2\theta$ *

or

$y = \cos 2\left(\theta - \frac{\pi}{4}\right)$

11.



Scale $\frac{6\pi}{12} = \frac{\pi}{2}$

Period $= 4\pi$

$b = 1/2$

* $y = \cos\left(\frac{1}{2}\theta\right) - 2$ *

or

$y = \sin\frac{1}{2}(\theta + \pi) - 2$

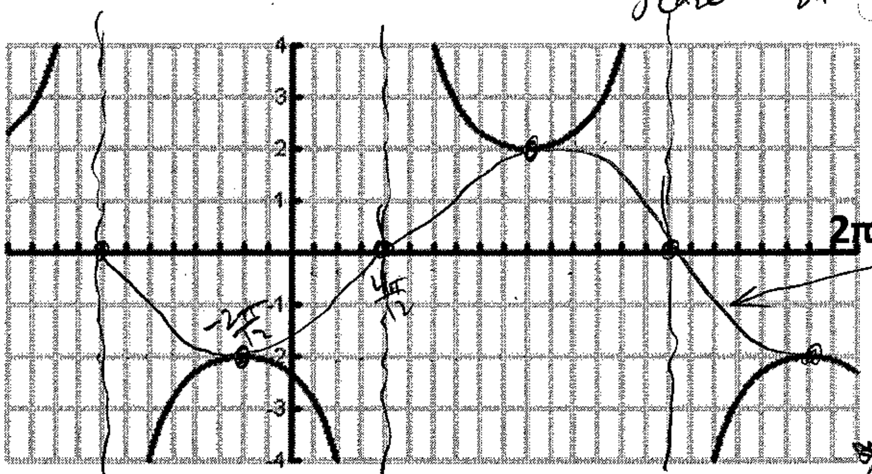
or

$y = -\sin\frac{1}{2}(\theta - \pi) - 2$

12.

Scale: $\frac{2\pi}{24} = \frac{\pi}{2}$

Plot sine/cosine wave to assist!



Asymptote is half-way between min/max.

Period $= 2\pi$

$b = 1$

$a = 2$

Maybe $y = 2\sin(\theta - \frac{\pi}{3})$

So ...

$y = 2\csc(\theta - \frac{\pi}{3})$

or

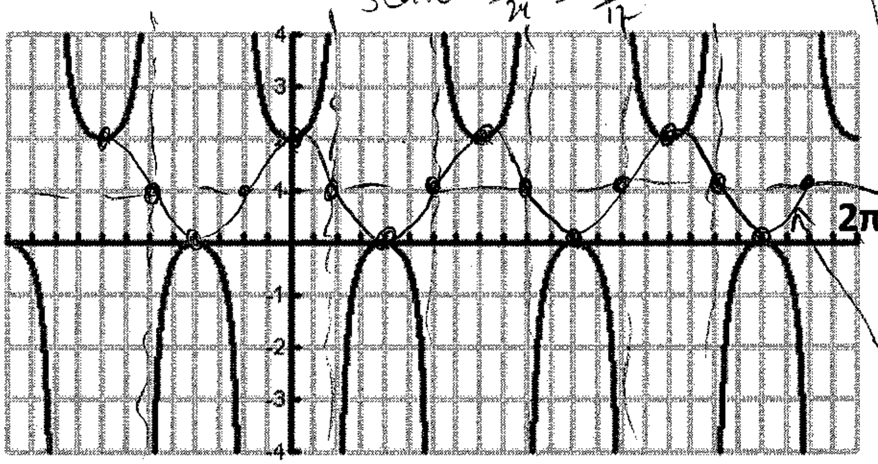
$y = -2\cos(\theta + \frac{\pi}{6})$

So ...

$y = -2\sec(\theta + \frac{\pi}{6})$

13.

Scale: $\frac{2\pi}{24} = \frac{\pi}{12}$



Period $= \frac{8\pi}{12} = \frac{2\pi}{3}$

$b = 3$

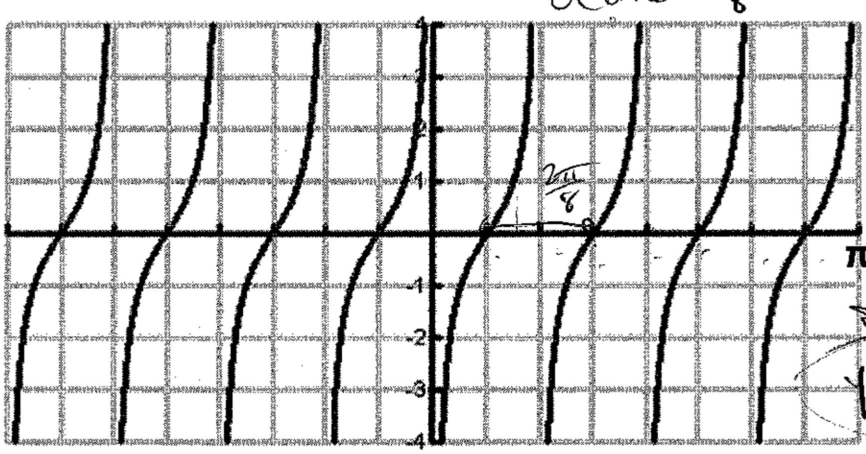
$y = \cos(3\theta) + 1$

So ...

$y = \sec(3\theta) + 1$

14.

Scale $= \frac{\pi}{8}$

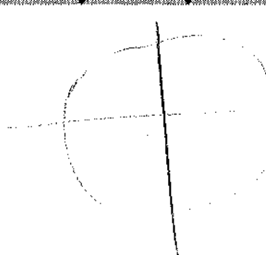


Period $= \frac{\pi}{b}$

$\frac{\pi}{4} = \frac{\pi}{b}$

$b = 4$

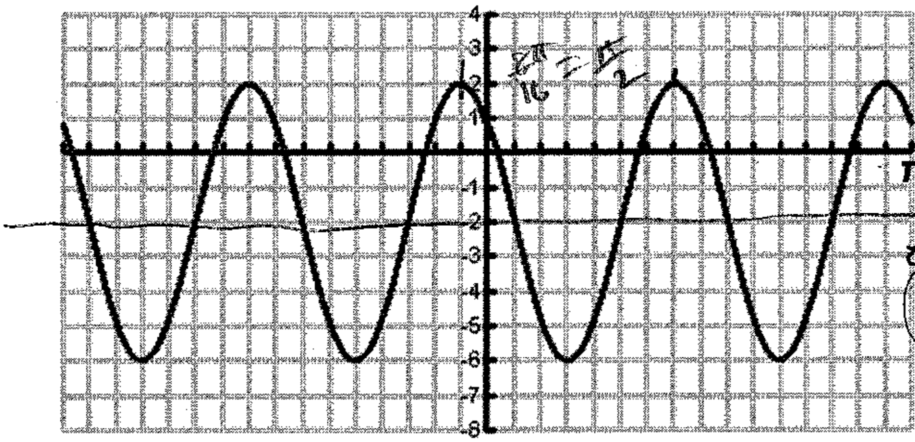
$y = \tan 4(\theta - \frac{\pi}{8})$



$(1,0)$ $\frac{y}{x} = \frac{0}{1}$

15.

Scale: $\frac{\pi}{16}$



$a = 4$

V.S. $= -2$

$\frac{\pi}{2} = \frac{2\pi}{b}$

$b = 4$

$y = 4\cos 4\left(\theta + \frac{\pi}{16}\right) - 2$

For #'s 16-19, write an equation for the given situation.

16. A Porsche 911 is traveling at a speed of 65 mph. Its tires have an outside diameter of 25.086 inches. Find an equation that would represent a nail stuck in the tire for time "t" in minutes. (Hint: The nail is picked up from the ground and your "b" value needs to be in radians/minute)

$\frac{65\text{ mi}}{1\text{ hr}} \cdot \frac{1\text{ hr}}{60\text{ min}} \cdot \frac{5280\text{ ft}}{1\text{ mi}} \cdot \frac{12\text{ in}}{1\text{ ft}} = 68640 \frac{\text{in}}{\text{min}}$ rate

$C = 25.086\pi$

$C = \frac{12543\pi}{500}$

d = rate · time

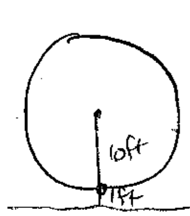
time (period) $= \frac{\frac{12543\pi}{500}}{68640} = \frac{12543\pi}{34320000}$ min

$\frac{2\pi}{b} = \frac{12543\pi}{34320000}$

$b = \frac{68640000}{12543} \approx 5472.38$

$y = -12.543\cos\left(\frac{68640000}{12543}t\right) + 12.543$

17. A water wheel has a 10 ft radius and the center of the wheel is 11 ft off the water surface. The rate of the river makes the water wheel travel at 14 revolutions per minute. Write an equation that would represent the water as it is first picked up from the river and travels around the water wheel.



$\left(\frac{\text{Rev}}{\text{min}}\right)$ is the reciprocal of the period (in min). $\frac{\text{min}}{\text{rev.}}$ ← one time around usually

Maybe a hanging bucket... oops

14 rev = 1 min

1 rev $= \frac{1}{14}$ min Period

$\frac{2\pi}{b} = \frac{1}{14}$ min → $b = 28\pi$

* In seconds

14 rev = 60 sec.

1 rev $= \frac{30}{7}$ sec.

$\frac{2\pi}{b} = \frac{30}{7}$ $b = \frac{7\pi}{15}$

$y = -10\cos\left(\frac{7\pi}{15}t\right) + 11$

* In minutes

$y = -10\cos(28\pi t) + 11$

18. The Earth is 93,000,000 miles from the sun and traverses its orbit, which is nearly circular, every 365.25 days. Write an equation that would represent the distance the Earth travels around the sun over the course of one period.

No starting point or vert. shift given, So choose your own!

Period = 365.25 days

$365.25 = \frac{2\pi}{b}$

$b = \frac{8\pi}{1461}$

Possible equations:

$y = 93000000\cos\left(\frac{8\pi}{1461}t\right)$

$y = 93000000\sin\left(\frac{8\pi}{1461}t\right)$

19. Lance Armstrong won the 1999 Tour de France bicycle race. The wheel of his bicycle had a 58 cm diameter. His overall average speed during the race was 40.273 km/h.

a. How many revolutions per minute did his wheel travel?

$\frac{\text{Revolutions}}{\text{min}}$ is the # of times around in one min.

$$\frac{40.273\text{ Km}}{1\text{ hr}} \cdot \frac{100000\text{ cm}}{1\text{ Km}} \cdot \frac{1\text{ hr}}{60\text{ min}} = \frac{4027300}{60} = \frac{201365\text{ cm}}{3\text{ min}}$$

See # 17 for an example.

$= 2\pi \cdot 58$

$= 116\pi$

$$\text{Period} = \frac{116\pi\text{ cm}}{\frac{210365\text{ cm}}{3\text{ min}}} = \frac{348\pi\text{ min}}{210365\text{ rev}} \quad \text{So} \ldots \quad \frac{210365\text{ rev}}{348\pi\text{ min}} \approx 192.42\ \frac{\text{rev}}{\text{min}}$$

Ends up being the reciprocal of the period.

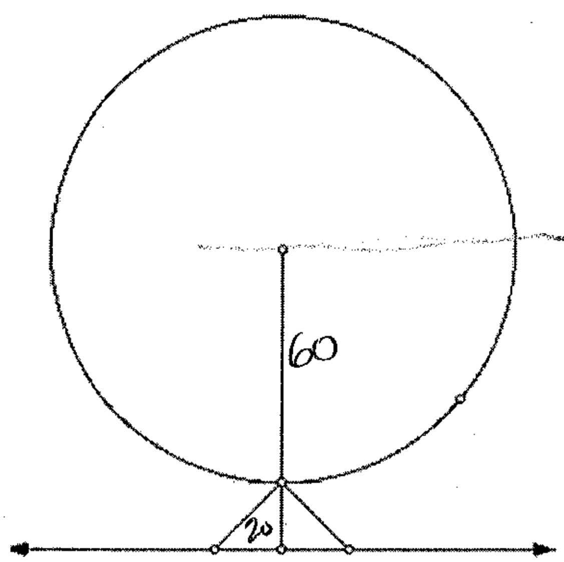
b. Write a sine equation (at time = 0, height = 0) that would represent the height of a point on the wheel at any time "t." In minutes

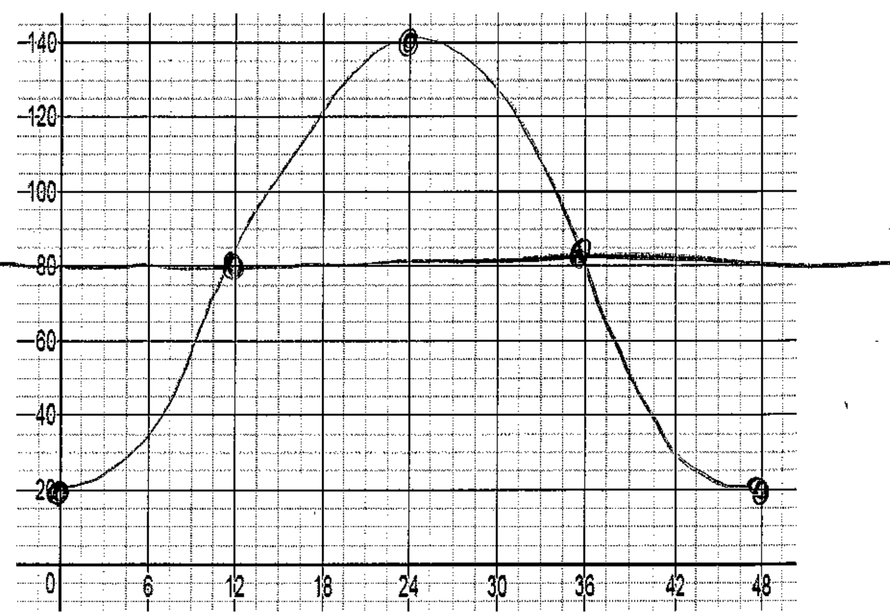
$$\frac{2\pi}{b} = \frac{348\pi}{210365}$$

$$b = \frac{210365}{174}$$

$$h(t) = 29\sin\left(\frac{210365}{174}t\right) + 29$$

20. Suppose a Ferris wheel with a diameter of 120 feet makes a complete revolution in 48 seconds. Develop a mathematical model that describes the relationship between the height h of a rider above the bottom of a Ferris wheel (20 feet above the ground) and time t.





$a = 60 \quad \text{V.S.} = 80$

$\text{Period} = 48\text{s} \rightarrow \frac{2\pi}{b} = 48 \rightarrow b = \frac{\pi}{24}$

$$h(t) = -60\cos\left(\frac{\pi}{24}t\right) + 80$$

Review previous unit's material.

The following questions only represent a small portion of that unit.

Find a coterminal angle between 0 and 2π for each given angle.

2π again!

21) $-\frac{11\pi}{6} + \frac{2\pi \cdot 6}{6} = \frac{\pi}{6}$

22) $-\frac{37\pi}{12} + 2\pi\frac{12}{12} = -\frac{13\pi}{12} + \frac{24\pi}{12} = \frac{11\pi}{12}$

Convert each degree measure into radians and each radian measure into degrees.

23) $\frac{330°\pi}{180} = \frac{11\pi}{6}$

24) $\frac{7\pi}{3} = 7 \cdot 60 = 420°$

25) $\frac{13\pi}{3} = 13 \cdot 30 = 390°$

26) $\frac{-450°\,\pi}{180} = \frac{-5\pi}{2}$

Solve each equation for $0 \le \theta < 2\pi$.

27) $2 = 2\cot\theta$

$\cot\theta = 1$ $\quad \frac{x}{y} = 1$ in QI & QIII @ $\frac{\pi}{4}$

$\boxed{\theta = \frac{\pi}{4} \quad \theta = \frac{5\pi}{4}}$

28) $4 + \tan\theta = 4$

$\tan\theta = 0$ $\quad \frac{y}{x} = 0$ when $\frac{y}{x} = \frac{0}{1}$ or $\frac{0}{-1}$

$\boxed{\theta = 0 \quad \theta = \pi}$

29) $\frac{2-\sqrt{3}}{2} = 1 + \sin\theta$
$-1 \qquad -1$

$\frac{2-\sqrt{3}}{2} - \frac{1 \cdot 2}{2} = \sin\theta$

$\sin\theta = -\frac{\sqrt{3}}{2}$

y-value is negative in QIII, QIV
$y = -\frac{\sqrt{3}}{2}$ @ $\frac{\pi}{3}$

$\boxed{\theta = \frac{4\pi}{3} \quad \theta = \frac{7\pi}{3}}$

30) $1 + 4\csc\theta = 5$

$\csc\theta = 1$

$\frac{1}{\sin\theta} = 1$

$\sin\theta = 1$

y-value is 1 @ $\frac{\pi}{2}$

$\theta = \frac{\pi}{2}$

31) $-1 - 6\cos\theta = -4$

$\cos\theta = \frac{1}{2}$

$x = \frac{1}{2}$ @ $\frac{\pi}{3}$ Positive in QI, QIV

$\boxed{\theta = \frac{\pi}{3} \quad \theta = \frac{7\pi}{3}}$

32) $-7 = -3 + 2\sec\theta$

$\sec\theta = -2$

$\frac{1}{\cos\theta} = -2$

$\cos\theta = \frac{-1}{2}$

$x = -\frac{1}{2}$ QII, QIII

R.A. of $\frac{\pi}{3}$

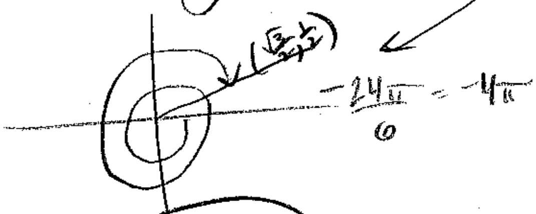
$\boxed{\theta = \frac{2\pi}{3} \quad \theta = \frac{5\pi}{3}}$

Find the exact value of each trigonometric function.

33) $\cos -\frac{23\pi}{6}$ x @ $\frac{\pi}{6}$ in

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$-\frac{24\pi}{6} = -4\pi$



$= \frac{\sqrt{3}}{2}$

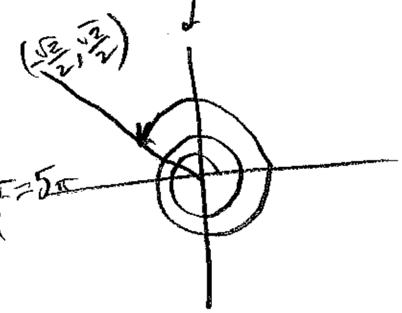
34) $\sec \frac{19\pi}{4}$ $\frac{1}{x}$ @ $\frac{\pi}{4}$ in

$\frac{1}{-\frac{\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$= -\sqrt{2}$

$\frac{20\pi}{4} = 5\pi$

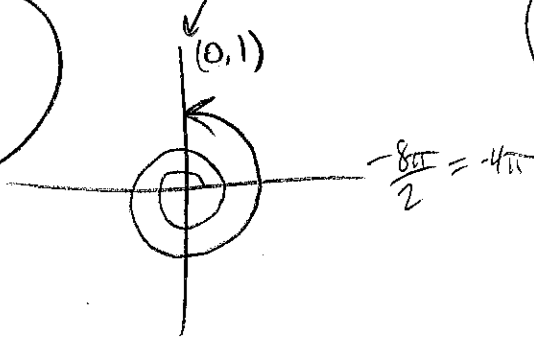


35) $\cot -\frac{9\pi}{2}$ $\frac{x}{y}$ @ $\frac{\pi}{2}$ in

$\frac{x}{y} = 0$

(0, 1)

$-\frac{8\pi}{2} = -4\pi$



36) $\sin \frac{17\pi}{4}$ y @ $\frac{\pi}{4}$ in

$= \frac{\sqrt{2}}{2}$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\frac{16\pi}{4} = 4\pi$