PC – Trig

Name KEY

It's all about Sine and Cosine

For #'s 1 – 6, write the trigonometric expression in terms of sine and cosine, and then simplify.

1. $\cos x \tan x$

$\cos x \cdot \frac{\sin x}{\cos x} = \sin x$

2. $\sin x \cos x \csc x$

$\sin x \cdot \cos x \cdot \frac{1}{\sin x} = \cos x$

3. $\sec^2 x - \tan^2 x$

$\frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} = \frac{1-\sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x}$

$= 1$

4. $\frac{\tan x + \cot x}{\sec x \csc x}$

$\frac{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}{\frac{1}{\cos x} \cdot \frac{1}{\sin x}} = \frac{\frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x}}{\frac{1}{\cos x \sin x}} = 1$

5. $\cos x + \tan x \sin x$

$\cos x + \frac{\sin x}{\cos x} \sin x$

$\frac{\cos x}{\cos x}\cos x + \frac{\sin^2 x}{\cos x}$

$\frac{\cos^2 x + \sin^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$

6. $\cos^2 x(1 + \tan^2 x)$

$\cos^2 x \left(1 + \frac{\sin^2 x}{\cos^2 x}\right)$

$\cos^2 x \left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x}\right) = 1$

For #'s 7 – 19, simplify the trigonometric expression.

7. $\frac{\cos x \sec x}{\cot x}$

$= \frac{\cos x \cdot \frac{1}{\cos x}}{\frac{\cos x}{\sin x}} = \frac{\sin x}{\cos x} = \tan x$

8. $\cos^3 x + \sin^2 x \cos x$

$\cos x(\cos^2 x + \sin^2 x) = \cos x$

9. $\frac{1+\sin x}{1+\csc x} = \frac{1+\sin x}{1+\frac{1}{\sin x}} = \frac{1+\sin x}{\frac{\sin x+1}{\sin x}} = 1+\sin x \cdot \frac{\sin x}{\sin x+1} = \sin x$

10. $\frac{\tan x}{\sec x} = \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \frac{\sin x}{\cos x} \cdot \cos x = \sin x$

11. $\frac{\sec^2 x - 1}{\sec^2 x} = \frac{\tan^2 x}{\sec^2 x} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} = \sin^2 x$

12. $\frac{\sec x - \cos x}{\tan x} = \frac{\frac{1}{\cos x} - \cos x}{\frac{\sin x}{\cos x}} = \frac{\frac{1-\cos^2 x}{\cos x}}{\frac{\sin x}{\cos x}} = \frac{\sin^2 x}{\cos x} \cdot \frac{\cos x}{\sin x} = \sin x$

13. $\frac{1+\csc x}{\cos x+\cot x} = \frac{1+\frac{1}{\sin x}}{\cos x+\frac{\cos x}{\sin x}} = \frac{\frac{\sin x+1}{\sin x}}{\frac{\cos x \cdot \sin x+\cos x}{\sin x}} = \frac{\sin x+1}{\sin x} \cdot \frac{\sin x}{\cos x(\sin x+1)} = \frac{1}{\cos x} = \sec x$

14. $\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} \cdot \frac{1-\sin x}{1-\sin x}$

$\frac{1+\sin x}{\cos x} + \frac{\cos x(1-\sin x)}{1-\sin^2 x}$

$\frac{1+\sin x}{\cos x} + \frac{\cos x(1-\sin x)}{\cos^2 x}$

$\frac{1+\sin x}{\cos x} + \frac{1-\sin x}{\cos x} = \frac{2}{\cos x} = 2\sec x$

See back for alternate way

15. $\tan x \cos x \csc x$

$$\frac{\sin x}{\cos x} \cdot \cos x \cdot \frac{1}{\sin x} = \boxed{1}$$

16. $\frac{2+\tan^2 x}{\sec^2 x} - 1$

$$\frac{2 + \frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} - 1 = 2\cos^2 x + \sin^2 x - 1$$

($\sin^2 x + \cos^2 x$)

$$2\cos^2 x + \sin^2 x - (\sin^2 x + \cos^2 x)$$

$$2\cos^2 x + \sin^2 x - \sin^2 x - \cos^2 x = \boxed{\cos^2 x}$$

See back for another method

17. $\frac{1+\cot x}{\csc x}$

$$\frac{1 + \frac{\cos x}{\sin x}}{\frac{1}{\sin x}} = \frac{\frac{\sin x + \cos x}{\sin x}}{\frac{1}{\sin x}} = \boxed{\sin x + \cos x}$$

18. $\tan x + \cos x - \tan x$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{1} \cdot \frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}$$

$$\frac{\sin x + \cos^2 x - \sin x}{\cos x} = \frac{\cos^2 x}{\cos x} = \boxed{\cos x}$$

19. $\frac{\cos x}{\sec x + \tan x}$

$$= \frac{\cos x}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} = \frac{\cos x}{\frac{1+\sin x}{\cos x}} = \frac{\cos x \cdot \cos x}{1+\sin x} = \frac{\cos^2 x}{1+\sin x}$$

$$= \frac{1-\sin^2 x}{1+\sin x} = \frac{(1-\sin x)(1+\sin x)}{(1+\sin x)} = \boxed{1-\sin x}$$

14 $\frac{1+\sin x}{1+\sin x}\cdot\frac{1+\sin x}{\cos x}+\frac{\cos x}{1+\sin x}\cdot\frac{\cos x}{\cos x}$

$$\frac{(1+\sin x)^2+\cos^2 x}{(1+\sin x)\cos x}=\frac{1+2\sin x+\sin^2 x+\cos^2 x}{(1+\sin x)(\cos x)}$$

$$=\frac{2+2\sin x}{(1+\sin x)\cos x}=\frac{2(1+\sin x)}{(1+\sin x)\cos x}=\frac{2}{\cos x}=2\sec x$$

16 $\frac{2+\tan^2 x}{\sec^2 x}-1\cdot\frac{\sec^2 x}{\sec^2 x}$

$$\frac{2+\tan^2 x-\sec^2 x}{\sec^2 x}=\frac{2+-1}{\sec^2 x}=\frac{1}{\sec^2 x}$$

$$=\cos^2 x$$