

### Sometimes Trig happens...

Simplify the following expressions.

1.  $\cos \theta \tan^2 \theta + \cos \theta$

$$\cancel{\cos \theta} \cdot \frac{\sin^2 \theta}{\cos^2 \theta} + \cos \theta$$

$$\frac{\sin^2 \theta}{\cos \theta} + \frac{\cos \theta \cdot \cos \theta}{1 \cdot \cos \theta}$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\cos \theta} = \frac{1}{\cos \theta}$$

$$= \boxed{\sec \theta}$$

2.  $\frac{\cos \theta}{\sec \theta + \tan \theta}$

$$\frac{\cos \theta}{\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}} = \frac{\cos \theta}{\frac{1 + \sin \theta}{\cos \theta}} = \cos \theta \cdot \frac{\cos \theta}{1 + \sin \theta} = \frac{\cos^2 \theta}{1 + \sin \theta} = \frac{1 - \sin^2 \theta}{1 + \sin \theta}$$

$$\frac{\sqrt{(1 - \sin \theta)(1 + \sin \theta)}}{1 + \sin \theta} = \boxed{1 - \sin \theta}$$

3.  $(\csc \theta - \cot \theta)^2$

$$\left( \frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 \rightarrow \frac{(1 - \cos \theta)^2}{1 - \cos^2 \theta}$$

$$\frac{\left( \frac{1 - \cos \theta}{\sin \theta} \right)^2}{\frac{(1 - \cos \theta)^2}{(1 + \cos \theta)(1 - \cos \theta)}} = \boxed{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

4.  $\tan \theta + \cot \theta - \sec \theta \csc \theta$

$$\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} - \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta}$$

$$\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} - \frac{1}{\cos \theta \sin \theta}$$

$$\frac{1}{\sin \theta \cos \theta} - \frac{1}{\cos \theta \sin \theta} = \boxed{0}$$

$$\frac{\cos \theta}{\cos \theta} \cdot \frac{(\sin \theta + \cos \theta)}{\sin \theta} - \frac{(\cos \theta - \sin \theta)}{\cos \theta} \cdot \frac{\sin \theta}{\sin \theta}$$

$$= \frac{\cos \theta \sin \theta + \cos^2 \theta - \cos \theta \sin \theta + \sin^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{1}{\sin \theta \cos \theta}$$

$$= \csc \theta \sec \theta$$

$$\frac{(1+\sin \theta)(1+\sin \theta)}{(1+\sin \theta)(1-\sin \theta)} - \frac{1-\sin \theta}{(1+\sin \theta)} \cdot \frac{(1-\sin \theta)}{(1-\sin \theta)} \rightarrow (1-\sin^2 \theta)$$

$$\frac{1+2\sin \theta + \sin^2 \theta - (1-2\sin \theta + \sin^2 \theta)}{1-\sin^2 \theta}$$

$$= \frac{4\sin \theta}{1-\sin^2 \theta} = \frac{4\sin \theta}{\cos^2 \theta} = \boxed{4\sin \theta \sec^2 \theta}$$

7.  $(\tan \theta + \cot \theta)^2$

$$= \left( \frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta} \right)^2$$

$$= \left( \frac{1}{\sin^2 \theta + \cos^2 \theta} \right)^2$$

$$= \left( \frac{1}{\sin \theta \cos \theta} \right)^2$$

$$= \frac{1}{\sin^2 \theta \cos^2 \theta} = \boxed{\csc^2 \theta \sec^2 \theta}$$