

7.1 EXERCISES

1–6 ■ Write the trigonometric expression in terms of sine and cosine, and then simplify.

1. $\cos x \tan x$

2. $\sin \theta \cos \theta \csc \theta$

3. $\sec^2 x - \tan^2 x$

4. $\frac{\tan x + \cot x}{\sec x \csc x}$

5. $\cos u + \tan u \sin u$

6. $\cos^2 x (1 + \tan^2 x)$

7–20 ■ Simplify the trigonometric expression.

7. $\frac{\cos x \sec x}{\cot x}$

8. $\cos^3 x + \sin^2 x \cos x$

9. $\frac{1 + \sin y}{1 + \csc y}$

10. $\frac{\tan x}{\sec(-x)}$

11. $\frac{\sec^2 x - 1}{\sec^2 x}$

12. $\frac{\sec x - \cos x}{\tan x}$

13. $\frac{1 + \csc x}{\cos x + \cot x}$

14. $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x}$

15. $\frac{1 + \sin u}{\cos u} + \frac{\cos u}{1 + \sin u}$

16. $\tan x \cos x \csc x$

17. $\frac{2 + \tan^2 x}{\sec^2 x} - 1$

18. $\frac{1 + \cot A}{\csc A}$

19. $\tan \theta + \cos(-\theta) + \tan(-\theta)$

20. $\frac{\cos x}{\sec x + \tan x}$

21–82 ■ Verify the identity.

21. $\sin \theta \cot \theta = \cos \theta$

22. $\frac{\tan x}{\sec x} = \sin x$

23. $\frac{\cos u \sec u}{\tan u} = \cot u$

24. $\frac{\cot x \sec x}{\csc x} = 1$

25. $\frac{\tan y}{\csc y} = \sec y - \cos y$

26. $\frac{\cos v}{\sec v \sin v} = \csc v - \sin v$

27. $\sin B + \cos B \cot B = \csc B$

28. $\cos(-x) - \sin(-x) = \cos x + \sin x$

29. $\cot(-\alpha) \cos(-\alpha) + \sin(-\alpha) = -\csc \alpha$

30. $\csc x [\csc x + \sin(-x)] = \cot^2 x$

31. $(1 - \sin x)(1 + \sin x) = \cos^2 x$

32. $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

33. $(1 - \cos \beta)(1 + \cos \beta) = \frac{1}{\csc^2 \beta}$

34. $\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = 1$

35. $\frac{(\sin x + \cos x)^2}{\sin^2 x - \cos^2 x} = \frac{\sin^2 x - \cos^2 x}{(\sin x - \cos x)^2}$

36. $(\sin x + \cos x)^4 = (1 + 2 \sin x \cos x)^2$

37. $\frac{\sec t - \cos t}{\sec t} = \sin^2 t$

38. $\frac{1 - \sin x}{1 + \sin x} = (\sec x - \tan x)^2$

39. $\frac{1}{1 - \sin^2 y} = 1 + \tan^2 y$

40. $\csc x - \sin x = \cos x \cot x$

41. $(\cot x - \csc x)(\cos x + 1) = -\sin x$

42. $\sin^4 \theta - \cos^4 \theta = \sin^2 \theta - \cos^2 \theta$

43. $(1 - \cos^2 x)(1 + \cot^2 x) = 1$

44. $\cos^2 x - \sin^2 x = 2 \cos^2 x - 1$

45. $2 \cos^2 x - 1 = 1 - 2 \sin^2 x$

46. $\tan y + \cot y = \sec y \csc y$

47. $\frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha}$

48. $\sin^2 \alpha + \cos^2 \alpha + \tan^2 \alpha = \sec^2 \alpha$

49. $\frac{\sin x - 1}{\sin x + 1} = \frac{-\cos^2 x}{(\sin x + 1)^2}$

50. $\frac{\sin w}{\sin w + \cos w} = \frac{\tan w}{1 + \tan w}$

51. $\frac{(\sin t + \cos t)^2}{\sin t \cos t} = 2 + \sec t \csc t$

52. $\sec t \csc t (\tan t + \cot t) = \sec^2 t + \csc^2 t$

53. $\frac{1 + \tan^2 u}{1 - \tan^2 u} = \frac{1}{\cos^2 u - \sin^2 u}$

$$54. \frac{1 + \sec^2 x}{1 + \tan^2 x} = 1 + \cos^2 x$$

$$55. \frac{\sec x}{\sec x - \tan x} = \sec x(\sec x + \tan x)$$

$$56. \frac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$$

$$57. \sec v - \tan v = \frac{1}{\sec v + \tan v}$$

$$58. \frac{\sin A}{1 - \cos A} - \cot A = \csc A$$

$$59. \frac{\sin x + \cos x}{\sec x + \csc x} = \sin x \cos x$$

$$60. \frac{1 - \cos x}{\sin x} + \frac{\sin x}{1 - \cos x} = 2 \csc x$$

$$61. \frac{\csc x - \cot x}{\sec x - 1} = \cot x \quad 62. \frac{\csc^2 x - \cot^2 x}{\sec^2 x} = \cos^2 x$$

$$63. \tan^2 u - \sin^2 u = \tan^2 u \sin^2 u$$

$$64. \frac{\tan v \sin v}{\tan v + \sin v} = \frac{\tan v - \sin v}{\tan v \sin v}$$

$$65. \sec^4 x - \tan^4 x = \sec^2 x + \tan^2 x$$

$$66. \frac{\cos \theta}{1 - \sin \theta} = \sec \theta + \tan \theta$$

$$67. \frac{\cos \theta}{1 - \sin \theta} = \frac{\sin \theta - \csc \theta}{\cos \theta - \cot \theta}$$

$$68. \frac{1 + \tan x}{1 - \tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$69. \frac{\cos^2 t + \tan^2 t - 1}{\sin^2 t} = \tan^2 t$$

$$70. \frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} = 2 \sec x \tan x$$

$$71. \frac{1}{\sec x + \tan x} + \frac{1}{\sec x - \tan x} = 2 \sec x$$

$$72. \frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = 4 \tan x \sec x$$

$$73. (\tan x + \cot x)^2 = \sec^2 x + \csc^2 x$$

$$74. \tan^2 x - \cot^2 x = \sec^2 x - \csc^2 x$$

$$75. \frac{\sec u - 1}{\sec u + 1} = \frac{1 - \cos u}{1 + \cos u}$$

$$76. \frac{\cot x + 1}{\cot x - 1} = \frac{1 + \tan x}{1 - \tan x}$$

$$77. \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cos x$$

$$78. \frac{\tan v - \cot v}{\tan^2 v - \cot^2 v} = \sin v \cos v$$

$$79. \frac{1 + \sin x}{1 - \sin x} = (\tan x + \sec x)^2$$

$$80. \frac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$$

$$81. (\tan x + \cot x)^4 = \csc^4 x \sec^4 x$$

$$82. (\sin \alpha - \tan \alpha)(\cos \alpha - \cot \alpha) = (\cos \alpha - 1)(\sin \alpha - 1)$$

83–88 ■ Make the indicated trigonometric substitution in the given algebraic expression and simplify (see Example 7). Assume $0 \leq \theta < \pi/2$.

$$83. \frac{x}{\sqrt{1-x^2}}, \quad x = \sin \theta$$

$$84. \sqrt{1+x^2}, \quad x = \tan \theta$$

$$85. \sqrt{x^2-1}, \quad x = \sec \theta$$

$$86. \frac{1}{x^2\sqrt{4+x^2}}, \quad x = 2 \tan \theta$$

$$87. \sqrt{9-x^2}, \quad x = 3 \sin \theta$$

$$88. \frac{\sqrt{x^2-25}}{x}, \quad x = 5 \sec \theta$$

89–92 ■ Show that the equation is not an identity.

$$89. \sin 2x = 2 \sin x$$

$$90. \sin(x+y) = \sin x + \sin y$$

$$91. \sec^2 x + \csc^2 x = 1$$

$$92. \frac{1}{\sin x + \cos x} = \csc x + \sec x$$

93–96 ■ Graph f and g in the same viewing rectangle. Do the graphs suggest that the equation $f(x) = g(x)$ is an identity? Prove your answer.

$$93. f(x) = \cos^2 x - \sin^2 x, \quad g(x) = 1 - 2 \sin^2 x$$