7.1 pg. 466 1-7(odd) 21-55(odd)

1. $\cos x \tan x = \cos x \cdot \dfrac{\sin x}{\cos x} = \boxed{\sin x}$

2. $\sin\theta\cos\theta\csc\theta = \sin\theta\cos\theta\dfrac{1}{\sin\theta} = \boxed{\cos\theta}$

3. $\sec^2 x - \tan^2 x = \boxed{1}$

4. $\dfrac{\tan x + \cot x}{\sec x \csc x} = \dfrac{\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}}{\dfrac{1}{\cos x} \cdot \dfrac{1}{\sin x}}$

   $= \left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right) \cdot \cos x \cdot \sin x$

   $= \sin^2 x + \cos^2 x = \boxed{1}$

5. $\cos u + \tan u \sin u = \dfrac{\cos u}{1} + \dfrac{\sin^2 u}{\cos u}$

   $= \dfrac{\cos^2 u + \sin^2 u}{\cos u} = \boxed{\dfrac{1}{\cos u} = \sec u}$

6. $\cos^2 x (1 + \tan^2 x) = \cos^2 x (\sec^2 x) = \boxed{1}$

7. $\dfrac{\cos x \sec x}{\cot x} = \dfrac{\cos x \cdot \dfrac{1}{\cos x}}{\dfrac{\cos x}{\sin x}} = 1 \cdot \dfrac{\sin x}{\cos x} = \boxed{\tan x}$

8. $\cos^3 x + \sin^2 x \cos x = \cos x(\cos^2 x + \sin^2 x)$

$$= \cos x$$

9. $\dfrac{1 + \sin y}{1 + \csc y} \cdot \dfrac{\sin y}{\sin y} = \dfrac{\sin y + \sin^2 y}{\sin y + 1}$

$$= \dfrac{\sin y(1 + \sin y)}{\sin y + 1} \qquad = \sin y$$

10. $\dfrac{\tan x}{\sec(-x)} = \dfrac{\sin x}{\cos x} \cdot \dfrac{-\cos x}{1} = -\sin x$

11. $\dfrac{\sec^2 x - 1}{\sec^2 x} = \dfrac{\tan^2 x}{\sec^2 x} = \dfrac{\sin^2 x}{\cos^2 x} \cdot \dfrac{\cos^2 x}{1}$

$$= \sin^2 x$$

12. $\dfrac{\sec x - \cos x}{\tan x} = \dfrac{\frac{1}{\cos x} - \cos x}{\frac{\sin x}{\cos x}} \cdot \dfrac{\cos x}{\cos x}$

$$= \dfrac{1 - \cos^2 x}{\sin x} = \dfrac{\sin^2 x}{\sin x} \qquad = \sin x$$

13. $\dfrac{1 + \csc x}{\cos x + \cot x} = \dfrac{1 + \frac{1}{\sin x}}{\cos x + \frac{\cos x}{\sin x}} \cdot \dfrac{\sin x}{\sin x} = \dfrac{\sin x + 1}{\cos x \cdot \sin x + \cos x}$

$$= \dfrac{\sin x + 1}{\cos x(\sin x + 1)} = \dfrac{1}{\cos x} = \sec x$$

14. $$\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = \sin x \cdot \sin x + \cos x \cos x$$

$$\boxed{= 1}$$

15. $$\frac{1+\sin u}{\cos u} + \frac{\cos u}{1+\sin u} \cdot \frac{1-\sin u}{1-\sin u}$$

$$\frac{1+\sin u}{\cos u} + \frac{\cos u(1-\sin u)}{1-\sin^2 u}$$

$$\frac{1+\sin u}{\cos u} + \frac{\cos u(1-\sin u)}{\cos^2 u} = \frac{1+\sin u}{\cos u} + \frac{1-\sin u}{\cos u}$$

$$= \frac{2}{\cos u} = 2\sec u$$

16. $$\tan x \cos x \csc x = \sin x \cdot \cos x \cdot \frac{1}{\sin x} = \boxed{1}$$

$$\qquad \qquad \qquad \frac{\sin x}{\cos x}$$

17. $$\frac{2+\tan^2 x}{\sec^2 x} - 1 = \frac{2+\tan^2 x}{\sec^2 x} - \frac{\sec^2 x}{\sec^2 x} = \frac{2-1}{\sec^2 x}$$

$$= \frac{1}{\sec^2 x} = \boxed{\cos^2 x}$$

18. $$\frac{1+\cot A}{\csc A} = \left(1 + \frac{\cos A}{\sin A}\right) \cdot \sin A = \boxed{\sin A + \cos A}$$

19. $\tan\theta + \cos(\theta) + \tan(-\theta)$

$$\frac{\sin\theta}{\cos\theta} + \cos\theta - \frac{\sin}{\cos\theta} = \boxed{\cos\theta}$$

20. $$\frac{\cos x}{\sec x + \tan x} = \frac{\cos x}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} = \frac{\cos x}{\frac{1+\sin x}{\cos x}} = \cos x \cdot \frac{\cos x}{1+\sin x}$$

$$= \frac{\cos^2 x}{1+\sin x} = \frac{1-\sin^2 x}{1+\sin x} = \frac{(1+\sin x)(1-\sin x)}{1+\sin x}$$

$$= \boxed{1 - \sin x}$$

21. $\sin\theta \cot\theta = \cos\theta$

$\sin\theta \cdot \frac{\cos\theta}{\sin\theta} \quad \boxed{= \cos\theta}$

22. $$\frac{\tan x}{\sec x} = \sin x$$

$$\frac{\sin x}{\cos x} \cdot \cos x = \boxed{\sin x}$$

23. $$\frac{\cos u \cdot \sec u}{\tan u} = \cot u$$

$$\cos u \cdot \frac{1}{\cos u} \cdot \frac{\cos u}{\sin u} = \boxed{\cot u}$$

24. $$\frac{\cot x \cdot \sec x}{\csc x} = 1$$

$$\frac{\cos x}{\sin x} \cdot \frac{1}{\cos x} \cdot \sin x \boxed{= 1}$$

25. $\dfrac{\tan y}{\csc y} = \sec y - \cos y$

$$\dfrac{\frac{\sin y}{\cos y} \cdot \sin y}{} = \dfrac{1}{\cos y} - \cos y \cdot \dfrac{\cos y}{\cos y}$$

$$\dfrac{\sin^2 y}{\cos y} = \dfrac{1 - \cos^2 y}{\cos y}$$

26. $\dfrac{\cos v}{\sec v \cdot \sin v} = \csc v - \sin v$

$$\dfrac{\cos v}{\frac{1}{\cos v} \cdot \sin v} = \dfrac{1}{\sin v} - \sin v \cdot \dfrac{\sin v}{\sin v}$$

$$\dfrac{\cos^2 v}{\sin v} = \dfrac{1 - \sin^2 v}{\sin v}$$

27. $\sin B + \cos B \cot B = \csc B$

$$\dfrac{\sin B}{\sin B} \quad \sin B + \cos B \cdot \dfrac{\cos B}{\sin B} = \dfrac{1}{\sin B}$$

$$\dfrac{\sin^2 B + \cos^2 B}{\sin B} = \dfrac{1}{\sin B}$$

28. $\cos(-x) - \sin(-x) = \cos x + \sin x$

$\cos x + \sin x = \cos x + \sin x$

29. $\cot(-a)\cos(-a) + \sin(-a) = -\csc a$

$\dfrac{\cos a \cdot \cos a}{-\sin a} - \sin a = -\csc a$

$\dfrac{\cos^2 a}{-\sin a} - \dfrac{\sin^2 a}{\sin a} = \dfrac{1}{-\sin a} = -\csc a$

30. $\csc x \left[\csc x + \sin(-x) \right] = \cot^2 x$

$\csc^2 x - 1 = \cot^2 x$

$\cot^2 x = \cot^2 x$

31. $(1 - \sin x)(1 + \sin x) = \cos^2 x$

$1 - \sin^2 x = \cos^2 x$

$\cos^2 x = \cos^2 x$

32. $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

$\sin^2 x + 2\cos x \sin x + \cos^2 x = 1 + 2\sin x \cos x$

33. $(1 - \cos B)(1 + \cos B) = \dfrac{1}{\csc^2 B} = \sin^2 B$

$1 - \cos^2 B = \sin^2 B$

34. $\dfrac{\cos x}{\sec x} + \dfrac{\sin x}{\csc x} = 1$

$\cos x \cdot \cos x + \sin x \cdot \sin x = 1$

$\cos^2 x + \sin^2 x = 1$

35. $\dfrac{(\sin x + \cos x)^2}{\sin^2 x - \cos^2 x} = \dfrac{\sin^2 x - \cos^2 x}{(\sin x - \cos x)^2}$

$\sqrt{(\sin x + \cos x)^2 (\sin x - \cos x)^2} = \sqrt{(\sin^2 x - \cos^2 x)^2}$

$(\sin x + \cos x)(\sin x - \cos x) = \sin^2 x - \cos^2 x$

$\sin^2 x - \cos^2 x = \sin^2 x - \cos^2 x$

36. $\sqrt{(\sin x + \cos x)^4} = \sqrt{(1 + 2\sin x \cos x)^2}$

$(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

$\sin^2 x + 2 \cos x \sin x + \cos^2 x = 1 + \sin x \cos x$

37. $\dfrac{\sec t - \cos t}{\sec t} = \sin^2 t$

$\dfrac{\cos t}{\cos t}$

$\dfrac{\frac{1}{\cos t} - \cos t}{\frac{1}{\cos t}} = \sin^2 t$

$1 - \cos^2 t = \sin^2 t$

38. $\dfrac{(1-\sin x)}{(1+\sin x)} = (\sec x - \tan x)^2$

$\dfrac{(1-\sin x)}{(1-\sin x)} \cdot \dfrac{1-\sin x}{1+\sin x} = \left(\dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x}\right)^2$

$\dfrac{(1-\sin x)^2}{1-\sin^2 x} = \left(\dfrac{1-\sin x}{\cos x}\right)^2$

$\dfrac{(1-\sin x)^2}{\cos^2 x} = \dfrac{(1-\sin x)^2}{\cos^2 x}$

39. $\dfrac{1}{1-\sin^2 y} = 1 + \tan^2 y$

$\dfrac{1}{\cos^2 y} = \dfrac{\cos^2 y + \sin^2 y}{\cos^2 y \quad \cos^2 y}$

$\dfrac{1}{\cos^2 y} = \dfrac{1}{\cos^2 y}$

40. $\csc x - \sin x = \cos x \cot x$

$\dfrac{1}{\sin x} - \sin x = \cos x \cdot \dfrac{\cos x}{\sin x}$

$\dfrac{1-\sin^2 x}{\sin x} = \dfrac{\cos^2 x}{\sin x}$

41. $(\cot x - \csc x)(\cos x + 1) = -\sin x$

$\left(\dfrac{\cos x}{\sin x} - \dfrac{1}{\sin x}\right)(\cos x + 1) = -\sin x$

$\left(\dfrac{\cos x - 1}{\sin x}\right)(\cos x + 1) = -\sin x$

$\dfrac{\cos^2 x - \cos x}{\sin x} + \dfrac{\cos x - 1}{\sin x} = \dfrac{\cos^2 x - 1}{\sin x} = \dfrac{-\sin^2 x}{\sin x}$

$= -\sin x$

42. $\sin^4 \theta - \cos^4 \theta = \sin^2 \theta - \cos^2 \theta$

$(\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta) = \sin^2 \theta - \cos^2 \theta$

$\sin^2 \theta - \cos^2 \theta \cdot 1 = \sin^2 \theta - \cos^2 \theta$

43. $(1 - \cos^2 x)(1 + \cot^2 x) = 1$

$\sin^2 x \cdot \csc^2 x = 1$

$\sin^2 x \cdot \dfrac{1}{\sin^2 x} = 1$

44. $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

$\cos^2 x + (1 + \cos^2 x)$

$2\cos^2 x - 1 = 2\cos^2 x - 1$

45. $2\cos^2 x - 1 = 1 - 2\sin^2 x$

$2(1 - \sin^2 x) - 1 = 1 - 2\sin^2 x$

$2 - 2\sin^2 x - 1 = 1 - 2\sin^2 x$

$1 - 2\sin^2 x = 1 - 2\sin^2 x$

46. $\tan y + \cot y = \sec y \csc y$

$$\frac{\sin y}{\sin y} \cdot \frac{\sin y}{\cos y} + \frac{\cos y}{\sin y} \cdot \frac{\cos y}{\cos y} = \frac{1}{\cos y} \cdot \frac{1}{\sin y}$$

$$\frac{\sin^2 y + \cos^2 y}{\cos y \sin y} = \boxed{\frac{1}{\cos y \sin y}}$$

47. $\dfrac{1 - \cos a}{\sin a} = \dfrac{\sin a}{1 + \cos a}$

$$\sin^2 a = (1 - \cos a)(1 + \cos a)$$
$$\sin^2 a = 1 - \cos^2 a$$

48. $\sin^2 a + \cos^2 a + \tan^2 a = \sec^2 a$

$$1 + \tan^2 a = \sec^2 a$$

49. $\dfrac{\sin x + 1}{\sin x + 1} \cdot \dfrac{\sin x - 1}{\sin x + 1} = \dfrac{-\cos^2 x}{(\sin x + 1)^2}$

$$\boxed{\frac{\sin^2 x - 1}{(\sin x + 1)^2}} = \frac{-\cos^2 x}{(\sin x + 1)^2}$$

50. $\dfrac{\sin w}{\sin w + \cos w} = \dfrac{\tan w}{1 + \tan w}$

$\dfrac{\frac{\sin w}{\cos w}}{\frac{\cos w}{\cos w} + \frac{\sin w}{\cos w}} = \dfrac{\frac{\sin w}{\cos w}}{\frac{\cos w + \sin w}{\cos w}} \cdot \dfrac{\cos w}{\cos w}$

$$= \boxed{\frac{\sin w}{\cos w + \sin w}}$$

51. $\dfrac{(\sin t + \cos t)^2}{\sin t \cos t} = 2 + \sec t \csc t$

$$= 2 + \dfrac{1}{\sin t \cos t}$$

$$\dfrac{\sin^2 t + 2\cos t \sin t + \cos^2 t}{\sin t \cos t} = \dfrac{2\sin t \cos t + 1}{\sin t \cos t}$$

$$\dfrac{1 + 2\cos t \sin t}{\sin t \cos t} = \dfrac{1 + 2\sin t \cos t}{\sin t \cos t}$$

52. $\sec t \csc t (\tan t + \cot t) = \sec^2 t + \csc^2 t$

$$\dfrac{1}{\sin t \cos t} \cdot \dfrac{\sin t}{\cos t} + \dfrac{1}{\sin t \cos t} \cdot \dfrac{\cos t}{\sin t}$$

$$\dfrac{1}{\cos^2 t} + \dfrac{1}{\sin^2 t} = \sec^2 t + \csc^2 t$$

53. $\dfrac{1 + \tan^2 u}{1 - \tan^2 u} = \dfrac{1}{\cos^2 u - \sin^2 u}$

$$\dfrac{\sec^2 x}{1 - \tan^2 x} = \dfrac{\frac{1}{\cos^2 x}}{1 - \frac{\sin^2 x}{\cos^2 x}} \cdot \dfrac{\cos^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x - \sin^2 x}$$

54. $\dfrac{1 + \sec^2 x}{1 + \tan^2 x} = 1 + \cos^2 x$

$$\dfrac{1 + \frac{1}{\cos^2 x}}{1 + \frac{\sin^2 x}{\cos^2 x}} \cdot \dfrac{\cos^2 x}{\cos^2 x} = \dfrac{\cos^2 x + 1}{\cos^2 x + \sin^2 x} = \boxed{\cos^2 x + 1}$$

55. $\dfrac{\sec x}{\sec x - \tan x} = \sec x(\sec x + \tan x)$

$\sec x = (\sec x - \tan x)(\sec^2 x + \sec x \tan x)$

$\sec x = \sec^3 x + \sec^2 x \tan x - \sec^2 x \tan x - \sec x \tan^2 x$

$\sec x = \sec x(\sec^2 x - \tan^2 x)$

$\sec x = \sec x$

56. $\dfrac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x$

$\dfrac{\dfrac{1}{\cos x} + \dfrac{1}{\sin x}}{\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}} = \dfrac{\dfrac{\sin x + \cos x}{\cos x \cdot \sin x}}{\dfrac{1 = \sin^2 x + \cos^2 x}{\cos x \cdot \sin x}} \cdot \dfrac{\cos x \sin x}{\cos x \sin x} = \boxed{\sin x + \cos x}$

57. $\sec v - \tan v = \dfrac{1}{\sec v + \tan v}$

$\dfrac{1}{\cos v} - \dfrac{\sin v}{\cos v} = \dfrac{1}{\dfrac{1}{\cos v} + \dfrac{\sin}{\cos}}$

$\dfrac{1 - \sin v}{\cos v} = \dfrac{1}{\dfrac{1 + \sin v}{\cos v}}$

$\dfrac{1 - \sin v}{\cos v} = \dfrac{\cos v}{1 + \sin v}$

$\cos^2 v = (1 - \sin v)(1 + \sin v)$

$\cos^2 v = 1 - \sin^2 v$

58. $\dfrac{\sin A}{1-\cos A} - \cot A = \csc A$

$\dfrac{\sin A}{\sin A}\cdot\dfrac{\sin A}{(1-\cos A)} - \dfrac{\cos A}{\sin A}\cdot\dfrac{(1-\cos A)}{(1-\cos A)} \quad \dfrac{1}{\sin A}$

$\dfrac{\sin^2 A}{\sin A(1-\cos A)} - \dfrac{(\cos A - \cos^2 A)}{\sin A(1-\cos A)} = \dfrac{1}{\sin A}$

$= \dfrac{(1-\cos A)}{\sin(1-\cos A)} = \dfrac{1}{\sin A}$

59. $\dfrac{\sin x + \cos x}{\sec x + \csc x} = \sin x \cos x$

$\dfrac{\sin x + \cos x}{\frac{1}{\cos} + \frac{1}{\sin}} = \dfrac{\sin x + \cos x}{\frac{\sin x + \cos x}{\cos x \cdot \sin x}} = \sin x + \cos x \cdot \dfrac{\cos x \cdot \sin x}{\sin x + \cos x}$

$\boxed{= \cos x \cdot \sin x}$

60. $\dfrac{(1-\cos x)(1-\cos x}{(1-\cos x)\sin x} + \dfrac{\sin x(\sin x)}{(1-\cos x)(\sin x)} = 2\csc x$

$\dfrac{(1-\cos x)^2}{\sin x(1-\cos x)} + \dfrac{\sin^2 x}{\sin x(1-\cos x)} = \dfrac{2}{\sin x}$

$\dfrac{1-2\cos x + \cos^2 x + \sin^2 x}{\sin x(1-\cos x)} = \dfrac{1-2\cos x}{\sin x(1-\cos x)}$

$\boxed{= \dfrac{2}{\sin x}}$

61. $\dfrac{\csc x - \cot x}{\sec x - 1} = \cot x$

$$\dfrac{\dfrac{1}{\sin x} - \dfrac{\cos x}{\sin x}}{\dfrac{1}{\cos x} - 1} = \dfrac{\cos x}{\sin x}$$

$$\dfrac{\dfrac{1-\cos x}{\sin x}}{\dfrac{1-\cos x}{\cos x}} = \dfrac{1-\cos x}{\sin} \cdot \dfrac{\cos x}{1-\cos x} = \boxed{\dfrac{\cos x}{\sin x}}$$

62. $\dfrac{\csc^2 x - \cot^2 x}{\sec^2 x} = \cos^2 x$

$$\dfrac{\dfrac{1}{\sin^2 x} - \dfrac{\cos^2 x}{\sin^2 x}}{\dfrac{1}{\cos^2 x}} = \dfrac{1-\cos^2 x}{\sin^2 x} \cdot \cos^2 x$$

$$= \dfrac{\sin^2 x}{\sin^2 x} \cdot \cos^2 x$$

$$\boxed{= \cos^2 x}$$

63. $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

$$\dfrac{\sin^2 x}{\cos^2 x} - \dfrac{\sin^2 x \cdot \cos^2 x}{\cos^2 x} = \dfrac{\sin^2 x(1-\cos^2 x)}{\cos^2 x}$$

$$= \dfrac{\sin^2 x \cdot \sin^2 x}{\cos^2 x}$$

$$\boxed{= \sin^2 x \tan^2 x}$$

64. $\dfrac{\tan x \sin x}{\tan x + \sin x} = \dfrac{\tan x - \sin x}{\tan x \sin x}$

$\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

#63 proves rest.

65. $\sec^4 x - \tan^4 x = \sec^2 x + \tan^2 x$

$\dfrac{1}{\cos^4 x} - \dfrac{\sin^4 x}{\cos^4 x} = \dfrac{1 - \sin^4 x}{\cos^4 x} = \dfrac{(1 + \sin^2 x)(1 - \sin^2 x)}{\cos^4 x}$

$= \dfrac{(1 + \sin^2 x)(\cos^2 x)}{(\cos^4 x)}$

$= \dfrac{1 + \sin^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x} + \dfrac{\sin^2 x}{\cos^2 x} = \sec^2 x + \tan^2 x$

66. $\dfrac{\cos x}{1 - \sin x} = \sec x + \tan x$

$= \dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}$

$\dfrac{\cos x}{1 - \sin x} = \dfrac{1 + \sin x}{\cos x}$

$\cos^2 x = (1 - \sin x)(1 + \sin x)$

$\cos^2 x = 1 - \sin^2 x$

67. $\dfrac{\cos x}{1-\sin x} = \dfrac{\sin x - \csc x}{\cos x - \cot x}$

$\dfrac{\cos x}{1-\sin x} = \dfrac{\sin x - \dfrac{1}{\sin x}}{\dfrac{\sin x}{\sin x}\cos x - \dfrac{\cos x}{\sin x}} = \dfrac{\dfrac{\sin^2 x - 1}{\sin x}}{\dfrac{\sin x \cos x - \cos x}{\sin x}}$

$\dfrac{\cos x}{1-\sin x} = \dfrac{\sin^2 x - 1}{\sin x} \cdot \dfrac{\sin x}{\cos x(\sin x - 1)}$

$= \dfrac{\sin^2 x - 1}{\cos x(\sin x - 1)} = \dfrac{(\sin x + 1)(\sin x - 1)}{\cos x(\sin x - 1)}$

$\dfrac{\cos x}{1-\sin x} = \dfrac{\sin x + 1}{\cos x}$

$\cos^2 x = 1 - \sin^2 x$

68. $\dfrac{1 + \tan x}{1 - \tan x} = \dfrac{\cos x + \sin x}{\cos x - \sin x}$

$\left(1 + \dfrac{\sin x}{\cos x}\right)(\cos x - \sin x) = \left(1 - \dfrac{\sin x}{\cos x}\right)(\cos x + \sin x)$

$\dfrac{\cos x - \sin x + \sin x - \sin^2 x}{\cos x} = \dfrac{\cos x + \sin x - \sin x - \sin^2 x}{\cos x}$

$\dfrac{\cos x - \sin^2 x}{\cos x} = \dfrac{\cos x - \sin^2 x}{\cos x}$

69. $\dfrac{\cos^2 x + \tan^2 x - 1}{\sin^2 t} = \tan^2 x$

$\dfrac{-\sin^2 x + \tan^2 x}{\sin^2 x}$

$\dfrac{\dfrac{\cos^2 x \cdot \sin^2 x}{\cos^2 x} + \dfrac{\sin^2 x}{\cos^2 x}}{\sin^2 x} = \dfrac{\sin^2 x (1 - \cos^2 x)}{\cos^2 x} \cdot \dfrac{1}{\sin^2 x}$

$\boxed{= \dfrac{\sin^2 x}{\cos^2 x} = \tan^2 x}$

70. $\dfrac{(1+\sin x)\; 1}{(1+\sin x)\, 1 - \sin x} - \dfrac{1(1-\sin x)}{1+\sin x (1-\sin x)} = 2 \sec x \tan x$

$\dfrac{1+\sin x}{1-\sin^2 x} + \dfrac{-1+\sin x}{1-\sin^2 x} = \dfrac{2\sin x}{1-\sin^2 x} = \dfrac{2\sin}{\cos^2 x} = \dfrac{2\sin x}{\cos x} \cdot \dfrac{1}{\cos x}$

$\boxed{= 2\tan x \cdot \sec x}$

71. $\dfrac{1}{\sec x + \tan x} + \dfrac{1}{\sec x - \tan x} = 2\sec x$

$\dfrac{\sec x - \tan x}{\sec^2 x - \tan^2 x} + \dfrac{\sec x + \tan x}{\sec^2 x - \tan^2 x} = 2\sec x$

$\boxed{\dfrac{2\sec x}{1} = 2\sec x}$

72. $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} = 4\tan x \sec x$

$\dfrac{(1+\sin x)^2}{(1-\sin^2 x)} - \dfrac{(1-\sin x)^2}{(1-\sin^2 x)} = \dfrac{(1+2\sin x+\sin^2 x)+(-1+2\sin x-\sin^2 x)}{\cos^2 x}$

$= \dfrac{4\sin x}{\cos^2 x} = \dfrac{4\sin x}{\cos x}\cdot\dfrac{1}{\cos x}$

$\boxed{= 4\tan x \cdot \sec x}$

73. $(\tan x + \cot x)^2 = \sec^2 x + \csc^2 x$

$\left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right)^2 = \dfrac{1}{\cos^2 x} + \dfrac{1}{\sin^2 x}$

$\left(\dfrac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right)^2 = \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}$

$\boxed{\dfrac{1}{\cos^2 x \sin^2 x} = \dfrac{1}{\cos^2 x \sin^2 x}}$

74. $\tan^2 x - \cot^2 x = \sec^2 x - \csc^2 x$

$\boxed{1 = 1}$

75. $\dfrac{\sec x - 1}{\sec x + 1} = \dfrac{1 - \cos x}{1 + \cos x}$

$(\sec x - 1)(1 + \cos x) = (\sec x + 1)(1 - \cos x)$

$\sec x + 1 - 1 - \cos x = \sec x - 1 + 1 - \cos x$

$\sec x - \cos x = \sec x - \cos x$

76. $\dfrac{\cot x + 1}{\cot x - 1} = \dfrac{1 + \tan x}{1 - \tan x}$

$(\cot x + 1)(1 - \tan x) = (\cot x - 1)(1 + \tan x)$

$\cot x - 1 + 1 - \tan x = \cot x + 1 - 1 - \tan x$

$\cot x - \tan x = \cot x - \tan x$

77. $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cos x$

$\dfrac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{\sin x + \cos x}$

$\sin^2 x + \cos^2 x - \sin x \cdot \cos x = \boxed{1 - \sin x \cdot \cos x}$

78. $\dfrac{\tan x - \cot x}{\tan^2 x - \cot^2 x} = \sin x \cos x$

$\dfrac{\dfrac{\sin x}{\cos x} - \dfrac{\cos x}{\sin x}}{\dfrac{\sin^2 x}{\cos^2 x} - \dfrac{\cos^2 x}{\sin^2 x}} = \dfrac{\dfrac{\sin^2 x - \cos^2 x}{\cos x \sin x}}{\dfrac{\sin^4 x - \cos^4 x}{\sin^2 x \cos^2 x}}$

$= \dfrac{\sin^2 x - \cos^2 x}{\cos^2 x \sin x} \cdot \dfrac{\sin^2 x \cos^2 x}{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)} = \boxed{\sin x \cos x}$

79. $\dfrac{1+\sin x}{1-\sin x} = (\tan x + \sec x)^2$

$\dfrac{(1+\sin x)}{(1+\sin x)} \dfrac{1+\sin x}{1-\sin x} = \left(\dfrac{\sin x}{\cos x} + \dfrac{1}{\cos x}\right)^2$

$\dfrac{(1+\sin x)^2}{1-\sin^2 x} = \left(\dfrac{\sin x + 1}{\cos x}\right)^2$

$\dfrac{(1+\sin x)^2}{\cos^2 x} = \dfrac{(1+\sin x)^2}{\cos^2 x}$

80. $\dfrac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$

$\dfrac{\dfrac{\sin x}{\cos x} + \dfrac{\sin y}{\cos y}}{\dfrac{\cos x}{\sin x} + \dfrac{\cos y}{\sin y}} = \dfrac{\dfrac{\sin x \cos y + \sin y \cos x}{\cos x \cos y}}{\dfrac{\sin y \cos x + \sin x \cos y}{\sin x \cdot \sin y}}$

$\quad = \dfrac{\sin x \cdot \sin y}{\cos x \cdot \cos y} = \boxed{\tan x \cdot \tan y}$

81. $\sqrt[4]{(\tan x + \cot x)^4} = \sqrt[4]{\csc^4 x \sec^4 x}$

$(\tan x + \cot x) = \csc x \sec x$

$\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = \dfrac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \dfrac{1}{\cos \cdot \sin x}$

$\boxed{= \sec x \cdot \csc x}$