PC – Trig Name ________________________

Other Trig Formulas

The addition and subtraction formulas are a useful set of identities that are helpful for finding exact value solutions and verifying other identities.

$\sin(a+b) = \sin a \cos b + \cos a \sin b$	$\sin(a-b) = \sin a \cos b - \cos a \sin b$
$\cos(a+b) = \cos a \cos b - \sin a \sin b$	$\cos(a-b) = \cos a \cos b + \sin a \sin b$
$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$	$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$

Example:

Find the exact value of each expression.

a) $\cos 75°$

b) $\cos \frac{\pi}{12}$

Solution:

a) Find two familiar angles that sum up to 75º.

$30°$ & $45°$ $work!$ $So, \cos 75° = \cos(30° + 45°) = \cos 30° \cdot \cos 45° - \sin 30° \cdot sin45°$ by making $a = 30°$ & $b = 45°$

Find exact values (from the unit circle) and substitute into the equation!

$$\cos 30° \cdot \cos 45° - \sin 30° \cdot sin45° = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

This would have worked the same if we used $a = 45°$ & $b = 30°$ because addition is commutative.

b) Find two familiar angles that have a difference of $\frac{\pi}{12}$.

This one is a little more difficult because it's in radians and we're looking for the *difference*. So, convert all of the unit circles radian measures into twelfths. We find that many different measure will work once we do this. Here is one example: $\frac{\pi}{4} - \frac{\pi}{6} = \frac{3\pi}{12} - \frac{2\pi}{12}, so \cos \frac{\pi}{12} = \cos \frac{\pi}{4} - \frac{\pi}{6} = \cos \frac{\pi}{4} \cdot \cos \frac{\pi}{6} + \sin \frac{\pi}{4} \cdot \sin \frac{\pi}{6}$ by making $a = \frac{\pi}{4}$ & $b = \frac{\pi}{6}$

Find exact values (from the unit circle) and substitute into the equation!

$$\cos \frac{\pi}{4} \cdot \cos \frac{\pi}{6} + \sin \frac{\pi}{4} \cdot \sin \frac{\pi}{6} = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

For #'s 1 - 6, use an addition or subtraction formula to find the exact value of the expression, as demonstrated above.

1. $\sin 15°$

2. $\cos 165°$

3. $\tan 105°$

4. $\sin\frac{\pi}{12}$

5. $\sin\frac{11\pi}{12}$

6. $\sin 75°$

For #'s 7 – 10, use an addition or subtraction formula to write the expression as a trigonometric function of one number and find its exact value.

7. $\sin 18° \cos 27° + \cos 18° \sin 27°$

8. $\cos\frac{3\pi}{7}\cos\frac{2\pi}{21} + \sin\frac{3\pi}{7}\sin\frac{2\pi}{21}$

9. $\frac{\tan 73° - \tan 13°}{1+\tan 73° \tan 13°}$

10. $\cos\frac{13\pi}{15}\cos\left(-\frac{\pi}{5}\right) - \sin\frac{13\pi}{15}\sin\left(-\frac{\pi}{5}\right)$

Use the addition formula above to prove:

a) $\sin 2\theta = 2\sin\theta\cos\theta$

b) $\cos 2\theta = \cos^2\theta - \sin^2\theta$

The two above formulas are the double – angle formulas, used to find the values of the trigonometric functions that are 2θ from their values at θ. There also exists one for tangent, as seen in the following table.

$$\sin 2\theta = 2\sin\theta\cos\theta$$

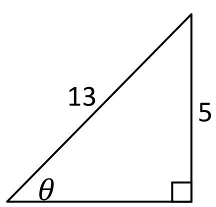
$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

For #'s 11 – 14, find $\sin 2\theta, \cos 2\theta, and \tan 2\theta$ from the given information.

11. $\sin\theta = \frac{5}{13}$ (x in quadrant I)

Example: So this means that $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{5}{13}\cdot\cos\theta$. To finish this and calculate the value of $\sin 2\theta$, we need to find the ratio for $\cos\theta$. We can do this by looking at a right triangle and finding the missing side. Using the Pythagorean Theorem on the right triangle below gives us an adjacent side of 12.



Therefore, $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\frac{5}{13}\cdot\frac{12}{13} = \mathbf{\frac{120}{169}}$. The quadrant means that our values will be positive for our two leg lengths.

Now find $\cos 2\theta$ & $\tan 2\theta$ using the same right triangle above.

12. $\cos\theta = \frac{4}{5}$ $(\csc\theta < 0)$

13. $\tan\theta = \frac{-4}{3}$ (θ in quadrant II)

14. $\sin\theta = \frac{-3}{5}$ (θ in quadrant III)

The following formulas allow us to write any trigonometric expression involving even powers of sine and cosine in terms of the first power of cosine only. This technique is important in calculus. The half – angle formulas are immediate consequences of these formulas. Formulas for lowering powers are in the following table.

$$\sin^2\theta = \frac{1-\cos 2\theta}{2} \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

The half angle formulas are in the following table.

$$sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

The choice of + or – sign depends on the quadrant in which $\frac{\theta}{2}$ lies.