Pg. 482 #'s 7, 8, 13-24 (all), 31-34 (all), 47-57 (odd)

7. $\sin^4 x = \sin^2 x \cdot \sin^2 x = \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{2}\right)$

$= \frac{(1-\cos 2x)^2}{4}$ Not technically 1st power, so we distribute!

Change into lower power

$= \frac{1-2\cos 2x+\cos^2 2x}{4} = \frac{1-2\cos 2x+\left(\frac{1+\cos 4x}{2}\right)}{4}$

$= \frac{1}{4}\left(1-2\cos 2x+\frac{1+\cos 4x}{2}\right)$

$= \frac{1}{4}-\frac{1}{2}\cos 2x+\frac{1+\cos 4x}{8}$

$= \frac{2 \cdot 1}{2 \cdot 4}-\frac{4\cos 2x}{4 \cdot 2}+\frac{1+\cos 4x}{8}$

$= \frac{2-4\cos 2x+1+\cos 4x}{8}$

$\boxed{= \frac{3-4\cos 2x+\cos 4x}{8}}$

8. $\cos^4 x = \cos^2 x \cos^2 x = \left(\frac{1+\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right)$

Do this again!

$= \frac{1 + 2\cos 2x + \cos^2 2x}{4}$

$= \frac{1 + 2\cos 2x + \frac{1+\cos 4x}{2}}{4}$

$= \frac{2}{2}\frac{1}{4} + \frac{2 \cdot 2\cos 2x}{2 \cdot 4} + \frac{1+\cos 4x}{8}$

$= \frac{2 + 4\cos 2x + 1 + \cos 4x}{8}$

$\boxed{= \frac{3 + 4\cos 2x + \cos 4x}{8}}$

13. $\sin 15 = \sin \frac{30}{2} = +\sqrt{\frac{1-\cos 30}{2}} = +\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$= +\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}} = +\sqrt{\frac{2-\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$

Also, you could use addition or subtraction. See next page.

13. $\sin 15° = \sin(45° - 30°) = \sin 45 \cos 30 - \cos 45 \sin 30$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

14. $\tan(15°) = \tan(45 - 30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$

$= \frac{1 - \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}}{1 + 1 \cdot \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}} = \frac{\frac{3}{3} 1 - \frac{\sqrt{3}}{3}}{\frac{3}{3} 1 + \frac{\sqrt{3}}{3}} = \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3} \cdot \frac{3}{3+\sqrt{3}} = \boxed{\frac{3-\sqrt{3}}{3+\sqrt{3}}}$

$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$= \frac{3-\sqrt{3}}{3+\sqrt{3}} \cdot \frac{(3-\sqrt{3})}{(3-\sqrt{3})}$

$= \frac{(3-\sqrt{3})(3-\sqrt{3})}{6}$

$= \frac{9 - 6\sqrt{3} + 3}{6}$

$= \frac{12 - 6\sqrt{3}}{6}$

$\boxed{= 2 - \sqrt{3}}$

OR

$\tan \frac{30}{2} = \frac{1 - \cos 30}{\sin 30}$

$= \frac{1 - \frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= \left(1 - \frac{\sqrt{3}}{2}\right) \cdot \frac{2}{1}$

$\boxed{= 2 - \sqrt{3}}$

15. $\cos 22.5 = \cos\frac{45}{2} = +\sqrt{\frac{1+\cos 45}{2}}$

$= \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \sqrt{\left(1+\frac{\sqrt{2}}{2}\right)\cdot\frac{1}{2}} = \sqrt{\frac{2}{2}\cdot\frac{1}{2}+\frac{\sqrt{2}}{4}}$

$= \sqrt{\frac{2+\sqrt{2}}{4}} = \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$

16. $\tan\frac{\pi}{8} = \tan\frac{\frac{\pi}{4}}{2} = \frac{1-\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} = \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

$= \left(1-\frac{\sqrt{2}}{2}\right)\left(\frac{2}{\sqrt{2}}\right) = \frac{\sqrt{2}}{\sqrt{2}}\frac{2}{\sqrt{2}} - 1 = \boxed{\sqrt{2}-1}$

17. $\sin\frac{\pi}{12} = \sin\frac{\frac{\pi}{6}}{2} = +\sqrt{\frac{1-\cos\frac{\pi}{6}}{2}}$

$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\left(1-\frac{\sqrt{3}}{2}\right)\cdot\frac{1}{2}} = \sqrt{\frac{2}{2}\cdot\frac{1}{2}-\frac{\sqrt{3}}{4}}$

$= \sqrt{\frac{2-\sqrt{3}}{4}} = \boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$

18. $\cos\frac{5\pi}{12} = \cos\frac{\frac{5\pi}{6}}{2} = +\sqrt{\frac{1+\cos 5\pi/6}{2}}$

$= \sqrt{\frac{1+-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\left(1-\frac{\sqrt{3}}{2}\right)\cdot\frac{1}{2}} = \boxed{\frac{\sqrt{2-\sqrt{3}}}{4}}$

19. (a) $2\sin 18\cos 18 = \sin 2\cdot 18 = \boxed{\sin 36^\circ}$

(b) $2\sin 3\theta\cos 3\theta = \sin 2\cdot 3\theta = \boxed{\sin 6\theta}$

20 a) $\frac{2\tan 7}{1-\tan^2 7} = \tan 2\cdot 7 = \boxed{\tan 14°}$

b) $\frac{2\tan 7\theta}{1-\tan^2 7\theta} = \tan 2\cdot 7\theta = \boxed{\tan 14\theta}$

21. a) $\cos^2 34 - \sin^2 34 = \cos 2\cdot 34 = \boxed{\cos 68'}$

b) $\cos^2 5\theta - \sin^2 5\theta = \cos 2\cdot 5\theta = \boxed{\cos 10\theta}$

22. a) $\cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} = \cos 2\cdot \frac{\theta}{2} = \boxed{\cos\theta}$

b) $2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \sin 2\cdot\frac{\theta}{2} = \boxed{\sin\theta}$

23. a) $\frac{\sin 8°}{1+\cos 8°} = \tan\frac{8}{2} = \boxed{\tan 4°}$

b) $\frac{1-\cos 4\theta}{\sin 4\theta} = \tan\frac{4\theta}{2} = \boxed{\tan 2\theta}$

24. a) $\sqrt{\frac{1-\cos 30}{2}} = = \sin\frac{30}{2} = \sin 15°$

b) $\sqrt{\frac{1-\cos 8\theta}{2}} = \sin\frac{8\theta}{2} = \boxed{\sin 4\theta}$

31. $\sin 2x \cdot \cos 3x$

$= \frac{1}{2}\left[\sin(2x+3x) + \sin(2x-3x)\right]$

$= \frac{1}{2}\left[\sin 5x + \sin -x\right]$

$= \frac{1}{2}\left[\sin 5x - \sin x\right]$

$\boxed{= \frac{1}{2}\sin 5x - \frac{1}{2}\sin x}$

32. $\sin x \cdot \sin 5x = \frac{1}{2}\left[\cos(x-5x) - \cos(x+5x)\right]$

$= \frac{1}{2}\left[\cos(-4x) - \cos(6x)\right]$

$\boxed{= \frac{1}{2}\cos 4x - \frac{1}{2}\cos 6x}$

33. $3\cos 4x \cdot \cos 7x = 3 \cdot \frac{1}{2}\left[\cos(4x+7x) + \cos(4x-7x)\right]$

$= \frac{3}{2}\left[\cos 11x + \cos(-3x)\right]$

$\boxed{= \frac{3}{2}\cos 11x + \frac{3}{2}\cos 3x}$

34. $11 \sin\frac{x}{2}\cos\frac{x}{4} = 11 \cdot \frac{1}{2}\left[\sin\left(\frac{x}{2}+\frac{x}{4}\right) + \sin\left(\frac{x}{2}-\frac{x}{4}\right)\right]$

$= \frac{11}{2}\left[\sin\frac{3x}{4} + \sin\frac{x}{4}\right]$

$\boxed{= \frac{11}{2}\sin\frac{3x}{4} + \frac{11}{2}\sin\frac{x}{4}}$

47. $\cos^2 5x - \sin^2 5x = \cos 10x$

$\cos^2 5x - \sin^2 5x = \cos 2(5x)$ Double angle identity, so done!

$\cos^2\theta - \sin^2\theta = \cos 2\theta$

$\boxed{\theta = 5x}$

49. $(\sin x + \cos x)^2 = 1 + \sin 2x$

$\sin^2 x + 2\sin x\cos x + \cos^2 x = 1 + 2\sin x\cos x$

$\boxed{1 + 2\sin x\cos x = 1 + 2\sin x\cos x}$

51. $\dfrac{\sin 4x}{\sin x} = 4\cos x\cos 2x$

$\dfrac{2\sin 2x\cos 2x}{\sin x} = 4\cos x \cdot \cos 2x$

$\dfrac{2 \cdot 2\cancel{\sin x}\cos x\cos 2x}{\cancel{\sin x}} = 4\cos x\cos 2x$

$\boxed{4\cos x\cos 2x = 4\cos x\cos 2x}$

53. $\dfrac{2(\tan x - \cot x)}{\tan^2 x - \cot^2 x} = \sin 2x$

$\dfrac{2\cancel{(\tan x - \cot x)}}{\cancel{(\tan x - \cot x)}(\tan x + \cot x)} = \sin 2x$

$\dfrac{2}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \sin 2x$

$\dfrac{2}{\frac{1}{\sin x\cos x}} = \sin 2x$

$\rightarrow 2\sin x\cos x = \sin 2x$

$\boxed{2\sin x\cos x = 2\sin x\cos x}$

55. $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

$$\tan(2x + x) = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x} =$$

$$= \frac{\dfrac{2\tan x}{1-\tan^2 x} + \tan x}{1 - \dfrac{2\tan x}{1-\tan^2 x} \cdot \tan x} =$$

$$= \frac{\dfrac{2\tan x}{1-\tan^2 x} + \dfrac{\tan x}{1} \cdot \dfrac{(1-\tan^2 x)}{1-\tan^2 x}}{\dfrac{1-\tan^2 x}{1-\tan^2 x} \cdot \dfrac{1}{1} - \dfrac{2\tan^2 x}{1-\tan^2 x}} =$$

$$= \frac{\dfrac{3\tan x - \tan^3 x}{1-\tan^2 x}}{\dfrac{1-3\tan^2 x}{1-\tan^2 x}} =$$

$$= \boxed{\frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x} = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}}$$

57. $\cos^4 x - \sin^4 x = \cos 2x$

$(\cos^2 x - \sin^2 x)\underbrace{(\cos^2 x + \sin^2 x)}_{1} = \cos 2x$

$\cos^2 x - \sin^2 x = \cos 2x$ This is the double angle identity!

Done