SOLUTION We apply the second sum-to-product formula to the numerator and the third formula to the denominator.

$$\text{LHS} = \frac{\sin 3x - \sin x}{\cos 3x + \cos x} = \frac{2\cos\frac{3x+x}{2}\sin\frac{3x-x}{2}}{2\cos\frac{3x+x}{2}\cos\frac{3x-x}{2}} \quad \text{Sum-to-product formulas}$$

$$= \frac{2\cos 2x\sin x}{2\cos 2x\cos x} \quad \text{Simplify}$$

$$= \frac{\sin x}{\cos x} = \tan x = \text{RHS} \quad \text{Cancel}$$

■

7.3 EXERCISES

1–6 ■ Find $\sin 2x$, $\cos 2x$, and $\tan 2x$ from the given information.

1. $\sin x = \frac{5}{13}$, x in quadrant I

2. $\cos x = \frac{4}{5}$, $\csc x < 0$

3. $\tan x = -\frac{4}{3}$, x in quadrant II

4. $\csc x = 4$, $\tan x < 0$

5. $\sin x = -\frac{3}{5}$, x in quadrant III

6. $\cot x = \frac{2}{3}$, $\sin x > 0$

7–12 ■ Use the formulas for lowering powers to rewrite the expression in terms of the first power of cosine, as in Example 4.

7. $\sin^4 x$ **8.** $\cos^4 x$

9. $\cos^4 x \sin^4 x$ **10.** $\cos^4 x \sin^2 x$

11. $\cos^2 x \sin^4 x$ **12.** $\cos^6 x$

13–18 ■ Use an appropriate half-angle formula to find the exact value of the expression.

13. $\sin 15°$ **14.** $\tan 15°$

15. $\cos 22.5°$ **16.** $\tan\frac{\pi}{8}$

17. $\sin\frac{\pi}{12}$ **18.** $\cos\frac{5\pi}{12}$

19–24 ■ Simplify the expression by using a double-angle formula or a half-angle formula.

19. (a) $2\sin 18°\cos 18°$ (b) $2\sin 3\theta\cos 3\theta$

20. (a) $\dfrac{2\tan 7°}{1-\tan^2 7°}$ (b) $\dfrac{2\tan 7\theta}{1-\tan^2 7\theta}$

21. (a) $\cos^2 34° - \sin^2 34°$ (b) $\cos^2 5\theta - \sin^2 5\theta$

22. (a) $\cos^2\dfrac{\theta}{2} - \sin^2\dfrac{\theta}{2}$ (b) $2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$

23. (a) $\dfrac{\sin 8°}{1+\cos 8°}$ (b) $\dfrac{1-\cos 4\theta}{\sin 4\theta}$

24. (a) $\sqrt{\dfrac{1-\cos 30°}{2}}$ (b) $\sqrt{\dfrac{1-\cos 8\theta}{2}}$

25–30 ■ Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$, and $\tan\frac{x}{2}$ from the given information.

25. $\sin x = \frac{3}{5}$, $0° < x < 90°$

26. $\cos x = -\frac{4}{5}$, $180° < x < 270°$

27. $\csc x = 3$, $90° < x < 180°$

28. $\tan x = 1$, $0° < x < 90°$

29. $\sec x = \frac{3}{2}$, $270° < x < 360°$

30. $\cot x = 5$, $180° < x < 270°$

31–34 ■ Write the product as a sum.

31. $\sin 2x\cos 3x$ **32.** $\sin x\sin 5x$

33. $3\cos 4x\cos 7x$ **34.** $11\sin\dfrac{x}{2}\cos\dfrac{x}{4}$

35–40 ■ Write the sum as a product.

35. $\sin 5x + \sin 3x$

36. $\sin x - \sin 4x$

37. $\cos 4x - \cos 6x$

38. $\cos 9x + \cos 2x$

39. $\sin 2x - \sin 7x$

40. $\sin 3x + \sin 4x$

41–46 ■ Find the value of the product or sum.

41. $2 \sin 52.5° \sin 97.5°$

42. $3 \cos 37.5° \cos 7.5°$

43. $\cos 37.5° \sin 7.5°$

44. $\sin 75° + \sin 15°$

45. $\cos 255° - \cos 195°$

46. $\cos \dfrac{\pi}{12} + \cos \dfrac{5\pi}{12}$

47–64 ■ Prove the identity.

47. $\cos^2 5x - \sin^2 5x = \cos 10x$

48. $\sin 8x = 2 \sin 4x \cos 4x$

49. $(\sin x + \cos x)^2 = 1 + \sin 2x$

50. $\dfrac{2 \tan x}{1 + \tan^2 x} = \sin 2x$

51. $\dfrac{\sin 4x}{\sin x} = 4 \cos x \cos 2x$

52. $\dfrac{1 + \sin 2x}{\sin 2x} = 1 + \frac{1}{2} \sec x \csc x$

53. $\dfrac{2(\tan x - \cot x)}{\tan^2 x - \cot^2 x} = \sin 2x$

54. $\cot 2x = \dfrac{1 - \tan^2 x}{2 \tan x}$

55. $\tan 3x = \dfrac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$

56. $4(\sin^6 x + \cos^6 x) = 4 - 3 \sin^2 2x$

57. $\cos^4 x - \sin^4 x = \cos 2x$

58. $\tan^2\left(\dfrac{x}{2} + \dfrac{\pi}{4}\right) = \dfrac{1 + \sin x}{1 - \sin x}$

59. $\dfrac{\sin x + \sin 5x}{\cos x + \cos 5x} = \tan 3x$

60. $\dfrac{\sin 3x + \sin 7x}{\cos 3x - \cos 7x} = \cot 2x$

61. $\dfrac{\sin 10x}{\sin 9x + \sin x} = \dfrac{\cos 5x}{\cos 4x}$

62. $\dfrac{\sin x + \sin 3x + \sin 5x}{\cos x + \cos 3x + \cos 5x} = \tan 3x$

63. $\dfrac{\sin x + \sin y}{\cos x + \cos y} = \tan\left(\dfrac{x + y}{2}\right)$

64. $\tan y = \dfrac{\sin(x + y) - \sin(x - y)}{\cos(x + y) + \cos(x - y)}$

65. Show that $\sin 45° + \sin 15° = \sin 75°$.

66. Show that $\cos 87° + \cos 33° = \sin 63°$.

67. Prove the identity

$$\frac{\sin x + \sin 2x + \sin 3x + \sin 4x + \sin 5x}{\cos x + \cos 2x + \cos 3x + \cos 4x + \cos 5x} = \tan 3x$$

68. Use the identity

$$\sin 2x = 2 \sin x \cos x$$

n times to show that

$$\sin(2^n x) = 2^n \sin x \cos x \cos 2x \cos 4x \cdots \cos 2^{n-1} x$$

69. (a) Draw the graph of $f(x) = \dfrac{\sin 3x}{\sin x} - \dfrac{\cos 3x}{\cos x}$ and make a conjecture.
(b) Prove the conjecture you made in part (a).

70. (a) Draw the graph of $f(x) = \cos 2x + 2 \sin^2 x$ and make a conjecture.
(b) Prove the conjecture you made in part (a).

71. Let $f(x) = \sin 6x + \sin 7x$.
(a) Graph $y = f(x)$.
(b) Verify that $f(x) = 2 \cos \frac{1}{2}x \sin \frac{13}{2}x$.
(c) Graph $y = 2 \cos \frac{1}{2}x$ and $y = -2 \cos \frac{1}{2}x$, together with the graph in part (a), in the same viewing rectangle. How are these graphs related to the graph of f?

72. When two pure notes that are close in frequency are played together, their sounds interfere to produce *beats*; that is, the loudness (or amplitude) of the sound alternately increases and decreases. If the two notes are given by

$$f_1(t) = \cos 11t \qquad \text{and} \qquad f_2(t) = \cos 13t$$

the resulting sound is $f(t) = f_1(t) + f_2(t)$.
(a) Graph the function $y = f(t)$.
(b) Verify that $f(t) = 2 \cos t \cos 12t$.
(c) Graph $y = 2 \cos t$ and $y = -2 \cos t$, together with the graph in part (a), in the same viewing rectangle. How do these graphs describe the variation in the loudness of the sound?