PC Trigonometry

Name: KEY

Trigonometric Identity Review

Simplify the following expressions:

1. $\cos(x)+\tan(x)\bullet\sin(x)$

$\frac{\cos x}{\cos x}\cdot\frac{\cos x}{1}+\frac{\sin x}{\cos x}\cdot\frac{\sin x}{1}$

$\frac{\cos^2 x+\sin^2 x}{\cos x}=\frac{1}{\cos x}$

$\boxed{=\sec x}$

2. $\cos^2(x)\left(1+\tan^2(x)\right)$

$\cos^2 x(\sec^2 x)$

$\boxed{=1}$

3. $\frac{\cot(\theta)\bullet\sec(\theta)}{\csc(\theta)}$

$\frac{\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}}=\frac{\frac{1}{\sin\theta}}{\frac{1}{\sin\theta}}=\boxed{1}$

4. $\frac{\tan(x)+\cot(x)}{\csc(x)}$

$\frac{\frac{\sin x}{\sin x}\cdot\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\cdot\frac{\cos x}{\cos x}}{\frac{1}{\sin x}}$

$\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}\cdot\frac{\sin x}{1}=\frac{1}{\cos x}=\boxed{\sec x}$

5. Verify the following identities:

a. $\frac{\cos^2 x-\tan^2 x}{\sin^2 x}=\cot^2 x-\sec^2 x$

$\frac{\frac{\cos^2 x}{\cos^2 x}\cdot\frac{\cos^2 x}{1}-\frac{\sin^2 x}{\cos^2 x}}{\sin^2 x}$ $\quad \frac{\cos^2 x}{\cos^2 x}\cdot\frac{\cos^2 x}{\sin^2 x}-\frac{1}{\cos^2 x}\cdot\frac{\sin^2 x}{\sin^2 x}$

$\frac{\cos^4 x-\sin^2 x}{\cos^2 x}\cdot\frac{1}{\sin^2 x}=$

$\boxed{\frac{\cos^4 x-\sin^2 x}{\cos^2 x\sin^2 x}=\frac{\cos^4 x-\sin^2 x}{\cos^2 x\sin^2 x}}$

b. $\frac{\cot x\sec x}{\csc x}=1$

$\frac{\frac{\cos x}{\sin x}\cdot\frac{1}{\cos x}}{\frac{1}{\sin x}}=\frac{\frac{1}{\sin x}}{\frac{1}{\sin x}}=\boxed{1}$

c. $9\sec^2\theta - 5\tan^2\theta = 5 + 4\sec^2\theta$

$$\frac{9}{\cos^2\theta} - \frac{5\sin^2\theta}{\cos^2\theta} = \frac{5\cos^2 x}{\cos^2 x} + \frac{4}{\cos^2\theta}$$

$$\frac{9-5\sin^2\theta}{\cos^2\theta} = \frac{5\cos^2\theta + 4}{\cos^2\theta}$$

$$\frac{9-5(1-\cos^2\theta)}{\cos^2\theta} = \frac{9-5+5\cos^2\theta}{\cos^2\theta} = \boxed{\frac{4+5\cos^2\theta}{\cos^2\theta}}$$

d. $1 - \dfrac{\sin^2\theta}{1-\cos\theta} = -\cos\theta$

$$1 - \frac{1-\cos^2\theta}{1-\cos\theta} = -\cos\theta$$

$$1 - \frac{(1+\cos\theta)\cancel{(1-\cos\theta)}}{\cancel{1-\cos\theta}} = -\cos\theta$$

$$1 - (1+\cos\theta) = -\cos\theta$$

$$\boxed{-\cos\theta = -\cos\theta}$$

e. $\tan\left(\dfrac{\pi}{4} + \Omega\right) = \dfrac{1+\tan\Omega}{1-\tan\Omega}$

$$\frac{\tan\frac{\pi}{4} + \tan\Omega}{1 - \tan\frac{\pi}{4}\cdot\tan\Omega} = \frac{1+\tan\Omega}{1-\tan\Omega}$$

$$\boxed{\frac{1+\tan\Omega}{1-\tan\Omega} = \frac{1+\tan\Omega}{1-\tan\Omega}}$$

f. $(\cot\alpha + \tan\alpha)^2 = \csc^2\alpha + \sec^2\alpha$

$$\cot^2 a + 2\underbrace{\cot a\tan a}_{=1} + \tan^2 a =$$

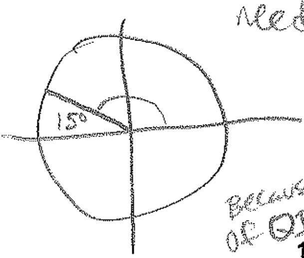
$$\cot^2 a + 2 + \tan^2 a = \csc^2 a + \sec^2 a$$

$$\csc^2 a - 1 + 2 + \sec^2 a - 1 = \csc^2 a + \sec^2 a$$

$$\boxed{\csc^2 a + \sec^2 a = \csc^2 a + \sec^2 a}$$

6. Find the exact value for each after sketching a reference triangle:

a) $\cos 165°$ so essentially we need to find $\cos 15°$ in QII.

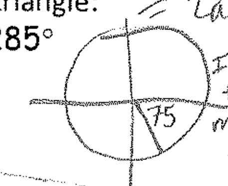


$$\cos 15 = \cos\frac{30}{2} =$$

Because of QII

$$-\sqrt{\frac{1+\cos 30}{2}} = -\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = -\sqrt{\frac{2}{2^2} + \frac{\sqrt{3}}{4}} = \boxed{-\frac{\sqrt{2+\sqrt{3}}}{2}}$$

b) $\tan 285°$



In QIV $\tan\theta < 0$ make negative

$$= \tan 75 = \tan\frac{150}{2} = -\sqrt{\frac{1-\cos 150}{1+\cos 150}}$$

$$= -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}}$$

used calc

$$\boxed{= -\sqrt{-4\sqrt{3}+7}}$$

7. If $\cos x = -\dfrac{12}{13}$ in quadrant III, find the y-value in the unit circle. Then determine the value of the $\sin 2\theta$.

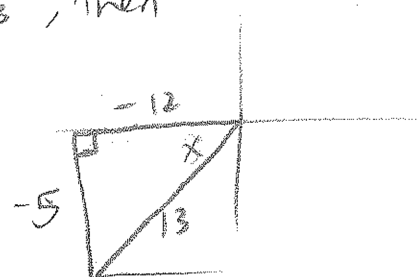
$$\sin 2\theta = 2\cos\theta\sin\theta$$

$$= 2\left(\frac{-12}{13}\right)\left(\frac{-5}{13}\right)$$

$$\boxed{= \frac{120}{169}}$$

$\cos x = \dfrac{-12}{13}$, then

$\sin x = \dfrac{-5}{13}$



$$\sqrt{13^2 - (-12)^2} = 5$$

For #'s 8-13, find the exact value of the expression. Do not use a calculator unless you are checking your answer.

8. $\sin 5^\circ \cos 55^\circ + \cos 5^\circ \sin 55^\circ$

$= \sin(5+55) = \sin 60^\circ$

$\boxed{= \frac{\sqrt{3}}{2}}$

9. $2\cos^2 22.5 - 1$

$2\left(\cos\frac{45}{2}\right)^2 - 1$

$2\left(\sqrt{\frac{1+\cos 45}{2}}\right)^2 - 1$

$2\cdot\frac{1+\frac{\sqrt{2}}{2}}{2} - 1 = 1 + \frac{\sqrt{2}}{2} - 1$

$\boxed{= \frac{\sqrt{2}}{2}}$

10. $\frac{\tan 53^\circ - \tan 23^\circ}{1 + \tan 53^\circ \tan 23^\circ}$

$= \tan(53-23) = \tan 30$

$= \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$

$\boxed{= \frac{\sqrt{3}}{3}}$

11. $\sin(105^\circ)$ → positive in Q II

$\sin\frac{210}{2} = \sqrt{\frac{1+\cos 210}{2}}$

$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \boxed{\frac{\sqrt{2+\sqrt{3}}}{2}}$

12. $\cos(\frac{7\pi}{12})$

$\cos\frac{\frac{7\pi}{6}}{2} = \sqrt{\frac{1+\cos\frac{7\pi}{6}}{2}}$

$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$\boxed{= \frac{\sqrt{2-\sqrt{3}}}{2}}$

13. $\tan(\frac{7\pi}{16})$

See attached paper.

This one is crazy!

Verify the following identities.

14. $(\cos(\theta) - \sin(\theta))^2 = 1 - \sin(2\theta)$

$\cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta =$

$1 - 2\cos\theta\sin\theta = 1 - \sin 2\theta$

$\boxed{1 - \sin 2\theta = 1 - \sin 2\theta}$

15. $\csc(2\theta) = \frac{\cot(\theta) + \tan(\theta)}{2}$

$\frac{1}{\sin 2\theta} =$

$\frac{1}{2\cos\theta\sin\theta} = \frac{\frac{\cos\theta\cos\theta}{\cos\theta\sin\theta} + \frac{\sin\theta\sin\theta}{\cos\theta\sin\theta}}{2}$

$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}\cdot\frac{1}{2}$

$\boxed{\frac{1}{2\cos\theta\sin\theta} = \frac{1}{2\sin\theta\cos\theta}}$

16. $\sin(4\theta) = 4\sin(\theta)\cos^3(\theta) - 4\sin^3(\theta)\cos(\theta)$

$\sin 4\theta =$

$\sin(2\theta + 2\theta) =$

$\sin 2\theta\cos 2\theta + \cos 2\theta\sin 2\theta =$

$2\sin\theta\cos\theta(\cos^2\theta - \sin^2\theta) + (\cos^2\theta - \sin^2\theta)2\sin\theta\cos\theta =$

$2\sin\theta\cos^3\theta - 2\sin^3\theta\cos\theta + 2\sin\theta\cos^3\theta - 2\sin^3\theta\cos\theta =$

$4\sin\theta\cos^3\theta - 4\sin^3\theta\cos\theta =$

Flash back

17. What is the period of the tangent and cotangent functions? If a tangent function has a period of $\frac{2\pi}{7}$, what is the k-value? Show work necessary to find this value.

Period $= \frac{\pi}{k}$

$\frac{\pi}{k} = \frac{2\pi}{7}$

$\frac{2\pi k}{2\pi} = \frac{7\pi}{2\pi}$

$\boxed{k = \frac{7}{2}}$

18.

Covert the given measurement into radians, degrees, and revolutions.

a. $\frac{35\pi}{6} = 35 \cdot 30 = \boxed{1050^\circ}$

$\frac{1050}{360} = \boxed{\frac{35}{12} \text{ of a rev.}}$

b. $\frac{645^\circ \pi}{180} = \boxed{\frac{43\pi}{12}}$

$\frac{645}{360} = \boxed{\frac{43}{24} \text{ rev.}}$

c. 1050°

Duh....

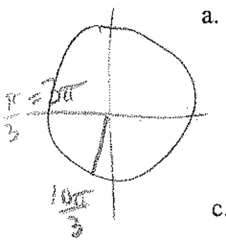
d. $-\frac{2\pi}{9} = -2 \cdot 20 = \boxed{-40^\circ}$

$-\frac{40}{360} = \boxed{-\frac{1}{9} \text{ rev.}}$

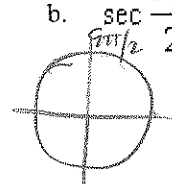
19. Sketch the reference triangle, then

Find the exact value of each trigonometric function.

a. $\cos\frac{10\pi}{3} = \boxed{-\frac{1}{2}}$



b. $\sec\frac{9\pi}{2} = \frac{1}{\cos\frac{9\pi}{2}} = \boxed{\frac{1}{0}} = \text{Und.}$

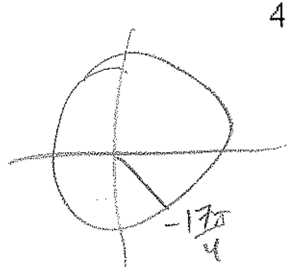


c. $\tan -3\pi = \frac{y}{x} = \frac{0}{-1} = \boxed{0}$

d. $\sec -\frac{17\pi}{4} = \frac{1}{\cos -\frac{17\pi}{4}} = \frac{1}{\frac{\sqrt{2}}{2}}$

$= \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$

$\boxed{= \sqrt{2}}$



20.

Solve each equation for $0 \le \theta < 2\pi$.

a. $0 = 2 + \csc\theta$

$-2 \quad -2$

$\frac{1}{\sin\theta} = -2$

$\sin\theta = -\frac{1}{2}$ $\boxed{\theta = \frac{7\pi}{6}, \frac{11\pi}{6}}$

b. $-3 = -2 - \cot\theta$

$+2 \quad +2$

$\frac{-1}{-1} = \frac{-\cot\theta}{-1}$

$\cot\theta = 1$

$\boxed{\theta = \frac{\pi}{4}, \frac{5\pi}{4}}$

c. $2 + 3\csc\theta = -4$

$-2 \quad -2$

$\frac{3\csc\theta}{3} = \frac{-6}{3}$

$\frac{1}{\sin\theta} = -2$

$\sin\theta = -\frac{1}{2}$ $\boxed{\theta = \frac{7\pi}{6}, \frac{11\pi}{6}}$

d. $2 + 4\sin\theta = 4$

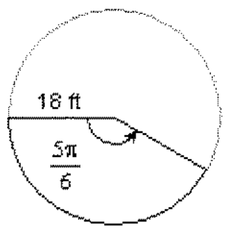
$-2 \quad \frac{\;}{4} \quad \frac{-2}{4}$

$\sin\theta = \frac{1}{2}$

$\boxed{\theta = \frac{\pi}{6}, \frac{5\pi}{6}}$

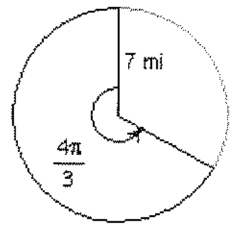
21. **Find the area of each sector. Round your answers to the nearest tenth.**

a.



$A = \frac{1}{2} \cdot \frac{5\pi}{6} \cdot 18^2 = 135\pi \approx \boxed{424.1 \text{ ft}^2}$

b.



$A = \frac{1}{2} \cdot \frac{4\pi}{3} \cdot 7^2 = \frac{98\pi}{3}$

$\boxed{\approx 102.6 \text{ mi}^2}$

22. **Show using any strategy that the two angles are coterminal.**

a. 40° and 19,120°

X needs to be whole!

$40 + 360x = 19120$

$\boxed{x = 53, \text{ so yes!}}$

b. -200° and 40120°

$-200 + 360x = 40120$

$\boxed{x = 112, \text{ so yes!}}$

c. -120° and 22,700°

$-120 + 360x = 22700$

$\boxed{x = 63.39}$

Not coterminal!

d. $\frac{\pi}{3}$ and $\frac{73\pi}{3}$

$\frac{\pi}{3} + 2\pi x = \frac{73\pi}{3}$

$\boxed{x = 12, \text{ so yes!}}$

13. $\tan\left(\frac{7\pi}{16}\right) = \tan\frac{\frac{7\pi}{8}}{2} = \sqrt{\frac{1-\cos\frac{7\pi}{8}}{1+\cos\frac{7\pi}{8}}}$

$$= \sqrt{\frac{1-\cos\frac{\frac{7\pi}{4}}{2}}{1+\cos\frac{\frac{7\pi}{4}}{2}}} = \sqrt{\frac{1-\sqrt{\frac{1-\cos\frac{7\pi}{4}}{2}}}{1+\sqrt{\frac{1-\cos\frac{7\pi}{4}}{2}}}}$$

$$= \sqrt{\frac{1-\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}}{1+\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}}} = \sqrt{\frac{1-\sqrt{\frac{2-\sqrt{2}}{4}}}{1+\sqrt{\frac{2-\sqrt{2}}{4}}}}$$

$$= \sqrt{\frac{1-\frac{\sqrt{2-\sqrt{2}}}{2}}{1+\frac{\sqrt{2-\sqrt{2}}}{2}}}$$

Probably good enough here!

Don't worry about something this wild on the test!