 Name ________________________

Solving Exponentials and Logarithms

Exponentials and Logarithms are inverse functions. We use one to solve for the other; therefore, we need to be able to change from exponential form to logarithmic form and visa versa. Here is one way this change can be written.

> The logarithmic function where $b > 0$ and $b \neq 1$ (read as "y is the logarithm with base b of x") can be written as an exponential function.
>
> $$log_b x = y \overset{if\ \&\ only\ if}{\Longleftrightarrow} x = b^y$$

Examples:

Logarithm Form		Exponential Form
1. $\mathbf{\log_3 81 = 4}$	means	
2.	means	$\mathbf{10^{-3} = .001}$
3. $\mathbf{\log_5 125 = y}$	means	
4.	means	$\mathbf{6^w = 7776}$

Do we remember how to enter these in the calculator?

What is the common log?

What is the natural log?

$$\text{if \& only if} \; \Longleftrightarrow$$

e is about growth

The number e is about continuous growth. Assume 100% rate.

$$e^x = e^{rate \cdot time} = e^{1.0 \cdot time} = e^{time}$$

Intuitively, e^x means:

- How much growth do I get after after x units of time (and 100% continuous growth)
- For example: after 3 time periods I have $e^3 = 20.08$ times the amount of "stuff".

e^x is a scaling factor, showing us how much growth we'd get after x units of time.

Natural Log is about time

The natural log is the inverse of e, a fancy term for opposite. Speaking of fancy, the Latin name is *logarithmus naturali*, giving the abbreviation *ln*.

Now what does this inverse mean?

- e^x lets us plug in **time** and get **growth**.
- ln(x) lets us plug in **growth** and get the **time it would take.**

For example:

- e^3 is 20.08. After 3 units of time, we end up with 20.08 times what we started with.
- ln(20.08) is about 3. If we want growth of 20.08, we'd wait 3 units of time (again, assuming a 100% continuous growth rate).

Are you with me? The natural log gives us the time needed to hit our desired growth.

We're Talking about Practice!

First, rewrite each given equation in its inverse form and then try to find the answer *without* a calculator then check.

1) $\log_2 x = 4$

5) $3^y = 81$

2) $\log_4 w = 3$

6) $5^p = 125$

3) $\log_8 a = 3$

7) $6^f = 1296$

4) $\log_3 z = 5$

8) $2^g = 32$

Natural Practice – So naturally you get to use your calculator…HA! (Please be accurate to four decimal places.)

9) $\ln x = 5.1$

12) $e^y = 8103.084$

10) $\ln d = 8$

13) $e^w = 2.117$

11) $\ln m = .7$

14) $e^g = 45$

Challenge Problems – Solve each without using a calculator

1) $\log_2 1$

2) $\log_8 8$

3) $\log_5 625$

4) $\log_3 \left(\frac{1}{9}\right)$

5) $\log_{\frac{1}{2}} 16$

6) $\log_{\frac{1}{3}} 9$

7) $\log_{10} \sqrt{10}$

8) $\log_5 \sqrt[3]{25}$

9) $\log_{\sqrt{2}} 4$

10) $\log_{\sqrt{3}} 9$

11) $\ln \sqrt{e}$

Domain and Range:

Logarithmic functions:

Exponential functions: