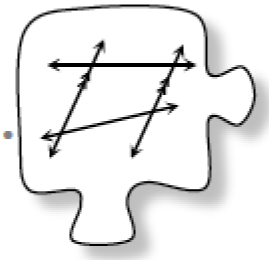
2.1.1 What is the relationship?



Complementary, Supplementary, and Vertical Angles

In Chapter 1, you compared shapes by looking at similarities between their parts. For example, two shapes might have sides of the same length or equal angles. In this chapter you will examine relationships between parts within a *single* figure or diagram. Today you will start by looking at angles to identify relationships in a diagram that make angle measures equal. As you examine angle relationships today, keep the following questions in mind to guide your discussion:

How can I name the angle?

What is the relationship?

How do I know?

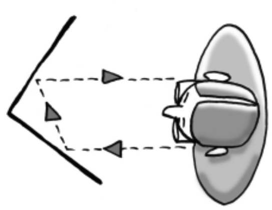
2-1. SOMEBODY'S WATCHING ME

In order to see yourself in a small mirror, you usually have to be looking directly into it—if you move off to the side, you cannot see your image any more. But Mr. Douglas knows a neat trick. He claims that if he makes a right angle with a hinged mirror, he can see himself in the mirror no matter from which direction he looks into it.

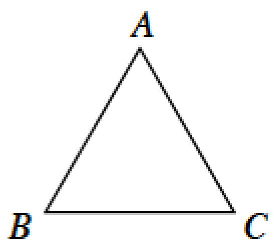
a. By forming a right angle with a hinged mirror, test Mr. Douglas's trick for yourself. Look into the place where the sides of the mirror meet. Can you see yourself? What if you look in the mirror from a different angle?

b. Does the trick work for *any* angle between the sides of the mirror? Change the angle between the sides of the mirror until you can no longer see your reflection where the sides meet.

c. Below is a diagram of a student trying out the mirror trick. What appears to be true about the lines of sight? Can you explain why Mr. Douglas's trick works? Talk about this with your team and be ready to share your ideas with the class.

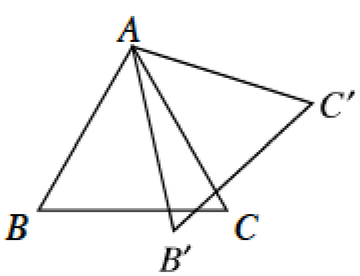


2-2. To completely understand how Mr. Douglas's reflection trick works, you need to learn more about the relationships between angles. But in order to clearly describe relationships between angles, you will need a convenient way to refer to and name them. Examine the diagram of equilateral ΔABC below.



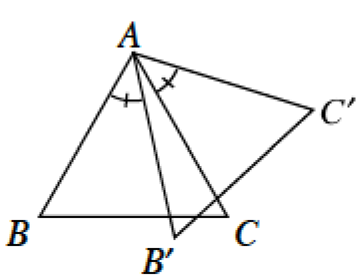
a. The "top" of this triangle is usually referred to as "angle A," written $\angle A$. Point A is called the **vertex** of this angle. The measure of $\angle A$ (the number of degrees in angle A) is written $m\angle A$. Since ΔABC is equilateral, write an equation showing the relationship between its angles.

b. Audrey rotated ΔABC around point A to form $\Delta AB'C'$. She told her teammate Maria, *"I think the two angles at A are equal."* Maria did not know which angles she was referring to. How many angles can you find at A? Are there more than three?



c. Maria asked Audrey to be more specific. She explained, "*One of my angles is* $\angle BAB'$." At the same time, she marked her two angles with the same marking below to indicate that they have the same measure.

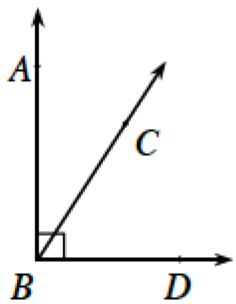
Name her other angle. Be sure to use three letters so there is no confusion about which angle you mean.



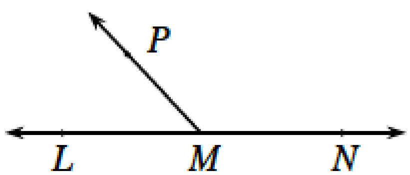
2-3. ANGLE RELATIONSHIPS

When you know two angles have a certain relationship, learning something about one of them tells you something about the other. Certain angle relationships come up often enough in geometry that they are given special names.

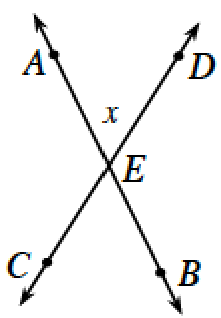
a. Two angles whose measures have a sum of 90° are called **complementary angles**. Since $\angle ABD$ is a right angle in the diagram below, angles $\angle ABC$ and $\angle CBD$ are complementary. If $m\angle CBD = 76°$, what is $m\angle ABC$? Show how you got your answer.



b. Another special angle is 180°. If the sum of the measures of two angles is 180°, they are called **supplementary angles**. In the diagram below, $\angle LMN$ is a straight angle. If $m\angle LMP = 62°$, what is $m\angle PMN$?



c. Now consider the diagram below, which shows $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersecting at *E*. If $x = 23°$, find $m\angle AEC$, $m\angle DEB$, and $m\angle CEB$. Show all work.



d. Based on your work in part (c), which angle has the same measure as $\angle AED$?

e. When two lines intersect, the angles that lie on opposite sides of the intersection point are called **vertical angles**. For example, in the diagram above, $\angle AED$ and $\angle CEB$ are vertical angles. Find another pair of vertical angles in the diagram.

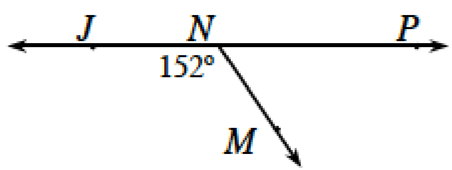
2-4. Travis noticed that the vertical angles in parts (c) and (d) of problem 2-3 have equal measure and wondered if other pairs of vertical angles also have equal measure.

a. Return to the diagram above and find *m∠CEB* if $x = 54°$. Show all work.

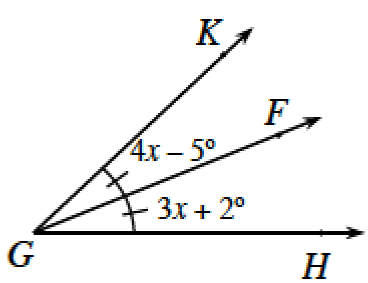
b. Based on your observations, write a **conjecture** (a statement based on an educated guess that is unproven). Start with, "*Vertical angles …*"

2-5. In the problems below, you will use geometric relationships to find angle measures. Start by finding a special relationship between some of the sides or angles, and use that relationship to write an equation. Solve the equation for the variable, then use that variable value to answer the original question.

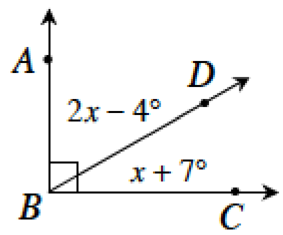
a. Find *m∠MNP*.



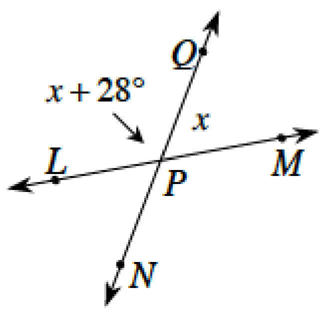
b. Find *m∠FGH*.



c. Find $m\angle DBC$.



d. Find $m\angle LPQ$ and $m\angle LPN$.

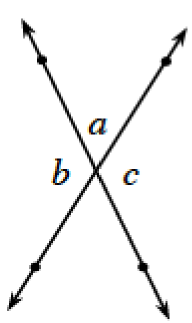


2-6. PROOF OF VERTICAL ANGLE RELATIONSHIP

When Jacob answered part (b) of problem 2-4, he wrote the conjecture: "*Vertical angles have equal measure*." (Remember that a conjecture is an educated guess that has not yet been proven.)

a. Do you think Jacob's vertical angle conjecture holds for *any* pair of vertical angles? Be prepared to convince the rest of the class.

b. Jacob's explanation included the diagram showing intersecting lines. He then wrote that $a + b = 180º$ and $a + c = 180º$. Are these statements true? Why?



c. How can you use Jacob's statements in part (b) to prove that vertical angles always have equal measure?

d. Once a conjecture is proven to be true, it is referred to as a **theorem**. Proving that vertical angles are always congruent in part (c) changed this conjecture into a theorem that can now be used in later problems without needing to reprove it again. Discuss with your team the difference between a conjecture and a theorem and write down your ideas about the difference.

2-7. LEARNING LOG

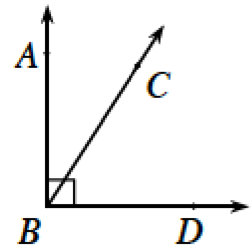
Describe each of the angle relationships you learned about today in an entry in your Learning Log. Include a diagram, a description of the angles, and what you know about the relationship. For example, are the angles always equal? Do they have a special sum? Title this entry "Angle Relationships" and include today's date.

Methods and Meanings

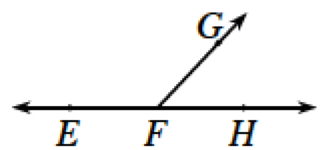
Math Notes

Angle Relationships

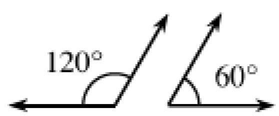
If two angles have measures that add up to 90°, they are called **complementary angles**. For example, in the diagram at right, $\angle ABC$ and $\angle CBD$ are complementary because together they form a right angle.



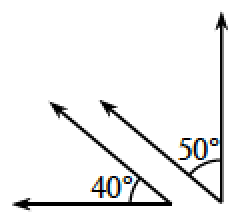
If two angles have measures that add up to 180°, they are called **supplementary angles**. For example, in the diagram at right, $\angle EFG$ and $\angle GFH$ are supplementary because together they form a straight angle.



Two angles do not have to share a vertex to be complementary or supplementary. The first pair of angles at right are supplementary; the second pair of angles are complementary.



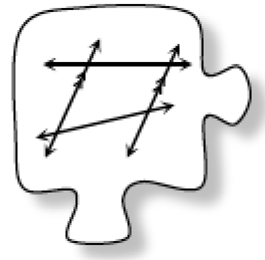
Supplementary



Complementary

2.1.2 What is the relationship?

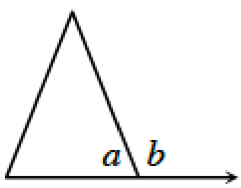
Angles Formed by Transversals



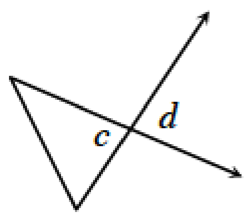
In Lesson 2.1.1, you examined vertical angles and found that vertical angles are always equal. Today you will look at another special relationship that guarantees angles have equal measure.

2-13. Examine the diagrams below. For each pair of angles marked on the diagram, quickly decide what relationship their measures have. Your responses should be limited to one of three relationships: same (equal measures), complementary (have a sum of 90°), and supplementary (have a sum of 180°).

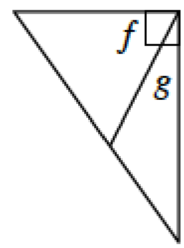
a.



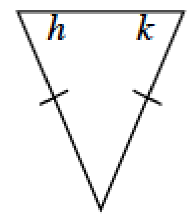
b.



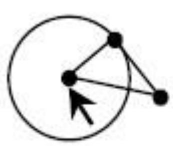
c.



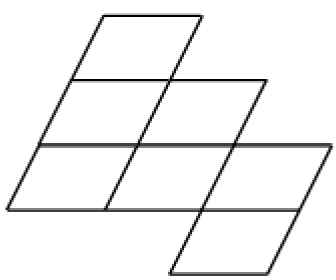
d.



2-14. Marcos was walking home after school thinking about special angle relationships when he happened to notice a pattern of parallelogram tiles on the wall of a building. Marcos saw lots of special angle relationships in this pattern, so he decided to copy the pattern into his notebook.

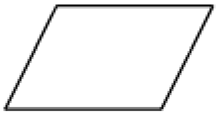


The beginning of Marcos's diagram is shown below and provided on the [Lesson 2.1.2 Resource Page](). This type of pattern is sometimes called a **tiling**. In this tiling, a parallelogram is copied and translated to fill an entire page without gaps or overlaps. View this interactive eTool about *[Marcos' Tile Pattern]()* (CPM).



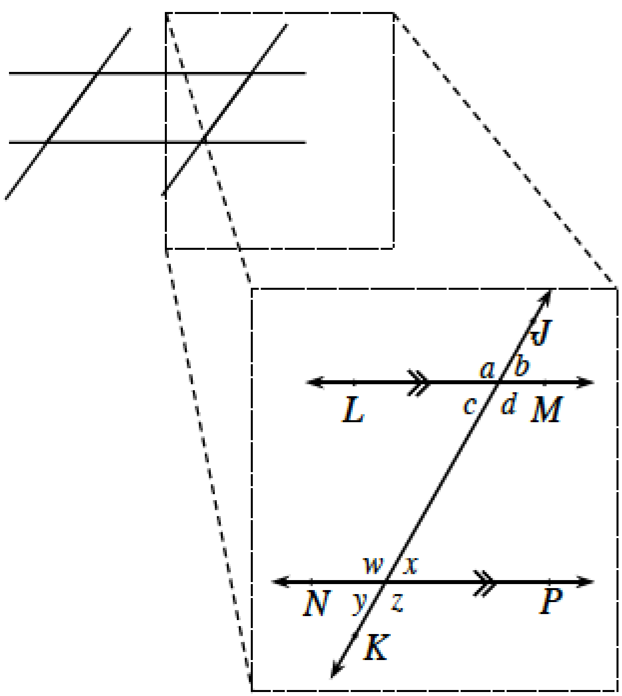
a. Since each parallelogram is a translation of another, what can be stated about the angles in the rest of Marcos' tiling? Use a technology tool or tracing paper to determine which angles must have the same measure. Color all angles that must be equal the same color.

b. Consider the angles inside a single parallelogram. Which angles must have equal measure? How can you justify your claim?



c. What about the relationship between lines? Can you identify any lines that must be parallel? Mark all of the lines on your diagram with the same number of arrows to show which lines are parallel.

2-15. Julia wants to learn more about the angles in Marcos's diagram and has decided to focus on just a part of his tiling. An enlarged view of that section is shown in the image below, with some points and angles labeled.



a. A line that crosses two or more other lines is called a **transversal**. In Julia's diagram, which line is the transversal? Which lines are parallel?

b. Trace $\angle x$ on tracing paper and shade its interior. Then translate $\angle x$ by sliding the tracing paper along the transversal until it lies on top of another angle and matches it exactly. Which angle in the diagram corresponds with *x*?

c. In this diagram, $\angle x$ and $\angle b$ are called **corresponding angles** because they are in the same position at two different intersections of the transversal. What is the relationship between the measures of angles *x* and*b*? Must one be greater than the other, or must they be equal? Explain how you know.

2-16. CORRESPONDING ANGLES FORMED BY PARALLEL LINES

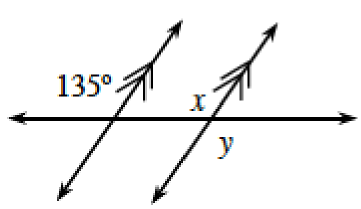
The corresponding angles in Julia's diagram in problem 2-15 have equal measure because they were formed by translating a parallelogram.

a. Name all the other pairs of corresponding angles you can find in Julia's diagram from problem 2-15.

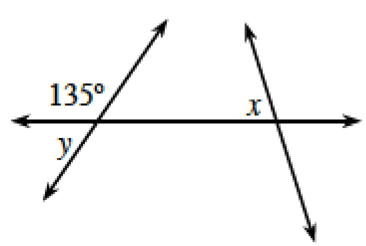
b. Suppose *b* = 60º. Use what you know about vertical, supplementary, and corresponding angle relationships to find the measures of all the other angles in Julia's diagram.

2-17. Frank wonders whether corresponding angles *always* have equal measure. For parts (a) through (d) below, use tracing paper to decide if corresponding angles have the same measure. Then determine if you have enough information to find the measures of *x* and *y*. If you do, find the angle measures and state the relationship.

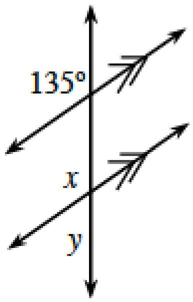
a.



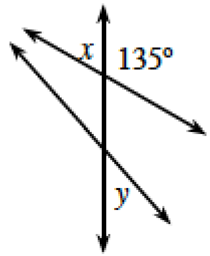
b.



c.



d.



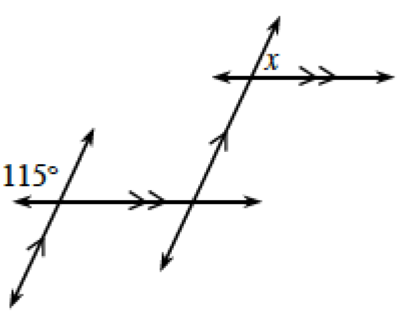
e. Answer Frank's question: Do corresponding angles always have equal measure? If not, when are their measures equal?

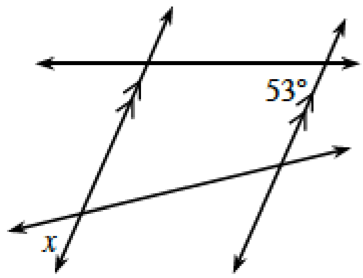
f. Conjectures are often written in the form, "*If…, then…*". A statement in if-then form is called a **conditional statement**. Make a conjecture about corresponding angles by completing this conditional statement: "*If …, then corresponding angles have equal measure.*"

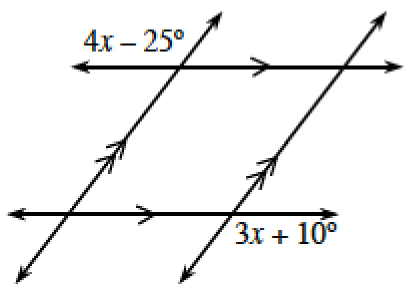
g. Prove that your conjecture in part (f) is always true. That is, explain why this conjecture is a theorem.

2-18. For each diagram below, find the value of *x* , *if possible*. If it is not possible, explain how you know. State the relationships you use. Be prepared to justify every measurement you find to other members of your team.

a.



b. 

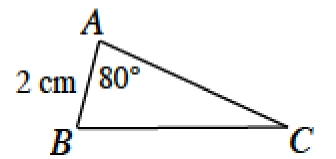
c. 

Methods and Meanings

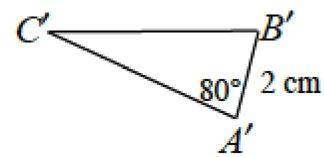
Math Notes

Naming Parts of Shapes

Part of geometry is the study of parts of shapes, such as points, line segments, and angles. To avoid confusion, standard notation is used to name these parts.
A **point** is named using a single capital letter. For example, the vertices (corners) of the triangle at right are named *A*, *B*, and *C*.



If a shape is transformed, the image shape is often named using **prime notation**. The image of point *A* is labeled *A*′ (read as "A prime"), the image of *B* is labeled *B*′ (read as "B prime"), etc. At right, *A*′*B*′*C*′ is the image of Δ*ABC*.

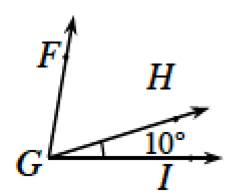


The side of a polygon is a line segment. A **line segment** is a portion of a line between two points and is named by naming its endpoints and placing a bar above them.

For example, one side of the first triangle above is named $\overline{AB}$. When referring to the length of a segment, the bar is omitted. In Δ*ABC* above, *AB* = 2 cm.
A **line**, which differs from a segment in that it extends infinitely in either direction, is named by naming two points on the line and placing a bar with arrows above them.

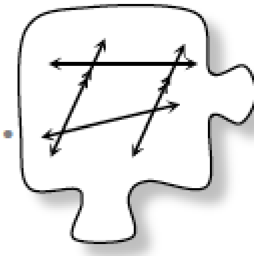
For example, the line below is named $\overleftrightarrow{DE}$. When naming a segment or line, the order of the letters is unimportant. The line below could also be named $\overleftrightarrow{ED}$.

An **angle** can be named by putting an angle symbol in front of the name of the angle's vertex. For example, the angle measuring 80° in Δ*ABC* above is named ∠*A*. Sometimes using a single letter makes it unclear which angle is being referenced. For example, in the diagram at right, it is unclear which angle is referred to by ∠*G*. When this happens, the angle is named with three letters. For example, the angle measuring 10° is called ∠*HGI* or ∠*IGH*. Note that the name of the vertex must be the second letter in the name; the order of the other two letters is unimportant.



To refer to an angle's measure, an *m* is placed in front of the angle's name. For example, $m\angle HGI = 10°$ means "the measure of ∠*HGI* is 10°."

2.1.3 What is the relationship?



More Angles Formed by Transversals

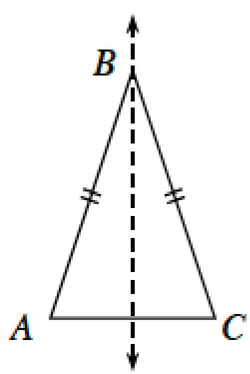
In Lesson 2.1.2, you looked at corresponding angles formed when a transversal intersects two parallel lines. Today you will investigate other special angle relationships formed in this situation.

2-24. Whenever one geometric figure can be translated, rotated, or reflected (or a combination of these) so that it lies on top of another, the figures must have the same shape and size. When this is possible, the figures are said to be **congruent** and the symbol ≅ is used to represent the relationship.

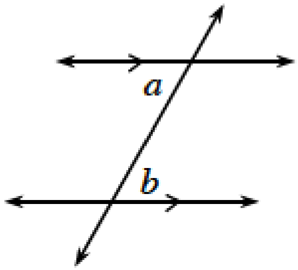
a. Review the angle relationships you have studied so far. Which types of angles must be congruent?

b. Angles are not the only type of figure that can be congruent. For example, sides of a figure can be congruent to another side. Also, a complex shape (such as a trapezoid) can be congruent to another if there is a sequence of rigid transformations that carry it onto the other.

Consider an isosceles triangle, like the one shown below. Because of its reflection symmetry, which parts must be congruent? State each relationship using symbols.



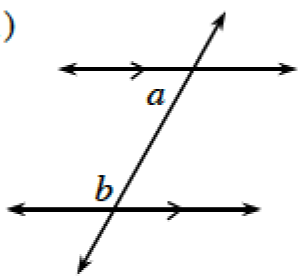
2-25. Suppose $\angle a$ in the diagram below measures 48°.

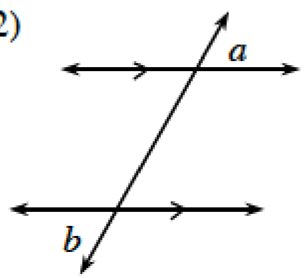


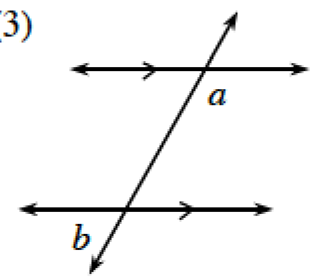
a. Use what you know about vertical, corresponding, and supplementary angle relationships to find the measure of $\angle b$.

b. Julia is still having trouble seeing the angle relationships clearly in this diagram. Her teammate, Althea explains, "*When I translate one of the angles along the transversal, I notice its image and the other given angle are a pair of vertical angles. That way, I know that angles a and b must be congruent.*"

Use Althea's method and tracing paper to determine if the following angle pairs are congruent or supplementary. Be sure to state whether the pair of angles created after the translation is a vertical pair or forms a straight angle. Be ready to justify your answer for the class.

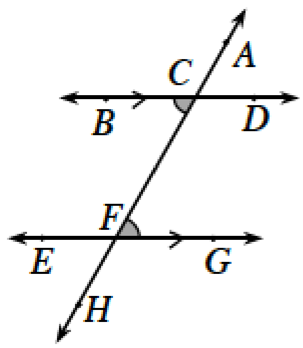
(1)


(2)


(3)


2-26. ALTERNATE INTERIOR ANGLE RELATIONSHIP

In problem 2-25, Althea showed that the shaded angles in the diagram are congruent. However, these angles also have a name for their geometric relationship (their relative positions on the diagram). These angles are called **alternate interior angles**. They are called "alternate" because they are on opposite sides of the transversal, and "interior" because they are both inside (that is, between) the parallel lines.



a. Find another pair of alternate interior angles in this diagram.

b. Think about the relationship between the measures of alternate interior angles. If the lines are parallel, are they always congruent? Are they always supplementary? Complete the conjecture, "*If lines are parallel, then alternate interior angles are…*"

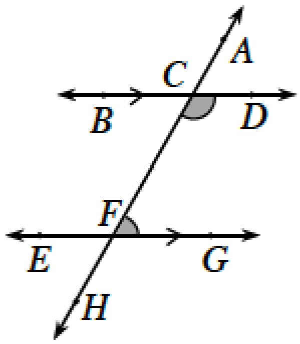
c. Instead of writing conditional statements, Roxie likes to write **arrow diagrams** to express her conjectures. She expresses the conjecture from part (b) as:

Lines are parallel → *alternate interior angles are congruent.*

This arrow diagram says the same thing as the conditional statement you wrote in part (b). How is it different from your conditional statement? What does the arrow mean?

d. Prove that alternate interior angles are congruent. That is, how can you use rigid transformations to move ∠*CFG* so that it lands on ∠*BCF*? Explain. Be sure your team agrees.

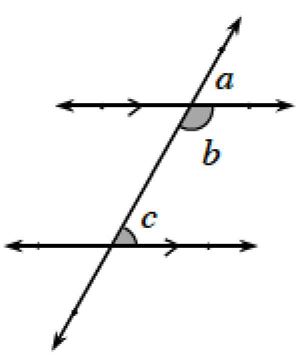
2-27. The shaded angles in the diagram below have another special angle relationship. They are called **same-side interior angles**.



a. Why do you think they have this name?

b. What is the relationship between the angle measures of same-side interior angles? Are they always congruent? Supplementary? Talk about this with your team. Then write a conjecture about the relationship of the angle measures. Your conjecture can be in the form of a conditional statement or an arrow diagram. If you write a conditional statement, it should begin, "*If lines are parallel, then same-side interior angles are*…"

c. Claudio decided to prove this theorem this way. He used letters in his diagram to represent the measures of the angles. Then, he wrote $a + b = 180º$ and $a = c$. Is he correct? Explain why or why not.



d. Finish Claudio's proof to explain why same-side interior angles are always supplementary whenever lines are parallel.

2-28. THE REFLECTION OF LIGHT

You know enough about angle relationships now to start analyzing how light bounces off mirrors. Examine the two diagrams below. Diagram A shows a beam of light emitted from a light source at A. In Diagram B, someone has placed a mirror across the light beam. The light beam hits the mirror and is reflected from its original path.

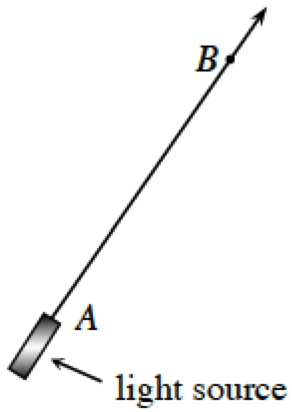


Diagram A

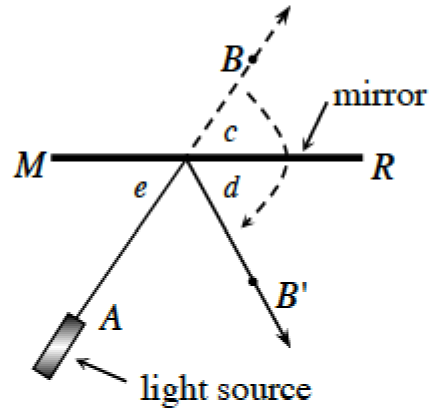


Diagram B

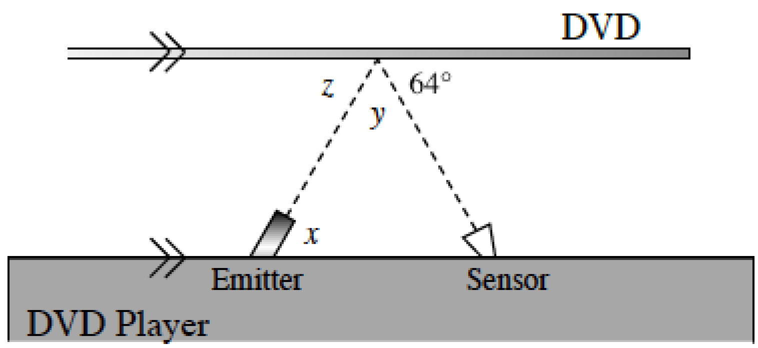
a. What is the relationship between angles *c* and *d*? Why?

b. What is the relationship between angles *c* and *e*? How do you know?

c. What is the relationship between angles *e* and *d*? How do you know?

d. Use your conclusions from parts (a) through (c) to prove that the measure of the angle at which light hits a mirror equals the measure of the angle at which it bounces off the mirror.

2-29. A DVD player (or a CD-ROM reader) works by bouncing a laser off the surface of the DVD, which acts like a mirror. An emitter sends out the light, which bounces off the DVD and then is detected by a sensor. The diagram below shows a DVD held parallel to the surface of the DVD player, on which an emitter and a sensor are mounted.



a. The laser is supposed to bounce off the DVD at a 64° angle as shown in the diagram above. For the laser to go directly into the sensor, at what angle does the emitter need to send the laser beam? In other words, what does the measure of angle *x* have to be? Justify your conclusion.

b. The diagram above shows two parts of the laser beam: the one coming out of the emitter and the one that has bounced off the DVD. What is the angle ($\angle y$) between these beams? How do you know?

2-30. ANGLE RELATIONSHIPS TOOLKIT

Obtain a Lesson 2.1.3 Resource Page (“Angle Relationships Toolkit”) from your teacher. This will be a continuation of the Geometry Toolkit you started in Chapter 1. Think about the new angle relationships you have studied so far in Chapter 2. Then, in the space provided, add a diagram and a description of the relationship for each special angle relationship you know. Be sure to specify any relationship between the measures of the angles (such as whether or not they are always congruent). In later lessons, you will continue to add relationships to this toolkit, so be sure to keep this resource page in a safe place. At this point, your Toolkit should include:

- Vertical angles
- Corresponding angles
- Same-side interior angles
- Straight angles
- Alternate interior angles

METHODS AND MEANINGS

MATH NOTES

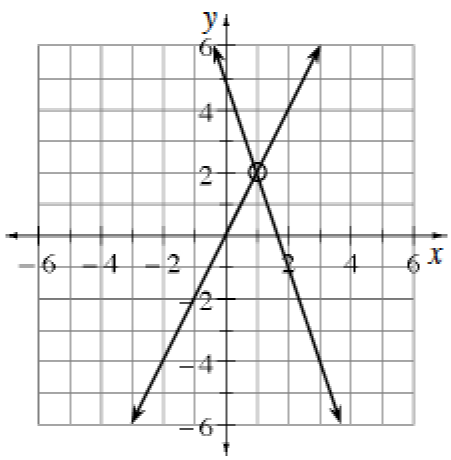
Systems of Linear Equations

In a previous course, you learned that a **system of linear equations** is a set of two or more linear equations that are given together, such as the example at right. In a system, each variable represents the same quantity in both equations. For example, y represents the same quantity in *both* equations below.

$y = 2x$

$y = -3x + 5$

To represent a system of equations graphically, you can simply graph each equation on the same set of axes. The graph may or may not have a **point of intersection**, as shown circled at right.



Sometimes two lines have *no* points of intersection. This happens when the two lines are parallel. It is also possible for two lines to have an *infinite* number of intersections. This happens when the graphs of two lines lie on top of each other. Such lines are said to **coincide**.

The **Substitution Method** is a way to change two equations with two variables into one equation with one variable. It is convenient to use when only one equation is solved for a variable. For example, to solve the system at right:

- Use substitution to rewrite the two equations as one. In other words, replace x with $(-3y + 1)$ to get $4(-3y + 1 - 3y = -11$. This equation can then be solved to find y. In this case, $y = 1$.

- To find the point of intersection, substitute to find the other value.

- Substitute $y = 1$ into $x = -3y + 1$ and write the answer for x and y as an ordered pair.

- To test the solution, substitute $x = -2$ and $y = 1$ into $4x - 3y = -11$ to verify that it makes the equation true. Since $4(-2) - 3(1) = -11$, the solution $(-2, 1)$ must be correct.

$$x = -3y + 1$$
$$4x - 3y = -11$$

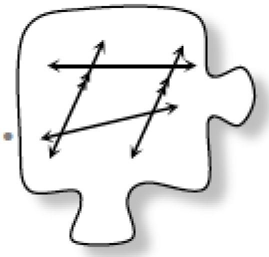
$$x = -3y + 1$$

$$4(\quad) - 3y = -11$$
$$4(-3y + 1) - 3y = -11$$
$$-12y + 4 - 3y = -11$$
$$-15y + 4 = -11$$
$$-15y = -15$$
$$y = 1$$

$$x = -3(1) + 1 = -2$$
$$(-2, 1)$$

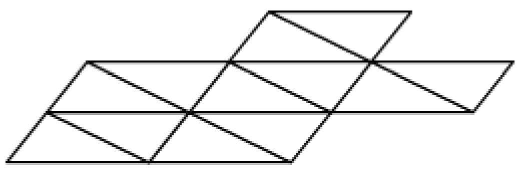
2.1.4 How can I use it?

Angles in a Triangle

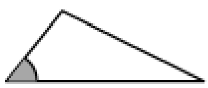


So far in this chapter, you have investigated the angle relationships created when two lines intersect, forming vertical angles. You have also investigated the relationships created when a transversal intersects two parallel lines. Today you will study the angle relationships that result when three non-parallel lines intersect, forming a triangle.

2-37. Marcos decided to change his tiling from problem 2-14 by drawing diagonals in each of the parallelograms. Find his pattern, shown below, on the Lesson 2.1.4 Resource Page.



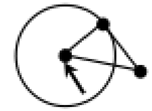
a. Copy one of Marcos's triangles onto tracing paper. Use a colored pen or pencil to shade one of the triangle's angles on the tracing paper. Then use the same color to shade every angle on the resource page that is equal to the shaded angle.

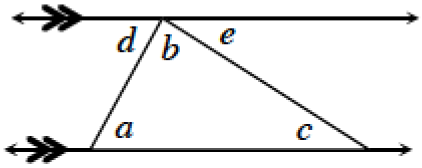


b. Repeat this process for the other two angles of the triangle, using a different color for each angle in the triangle. When you are done, every angle in your tiling should be shaded with one of the three colors.

c. Now examine your colored tiling. What relationship can you find between the three different-colored angles? You may want to focus on the angles that form a straight angle. What does this tell you about the angles in a triangle? Write a conjecture in the form of a conditional statement or an arrow diagram. If you write a conditional statement, it should begin, "*If a polygon is a triangle, then the measure of its angles*…".

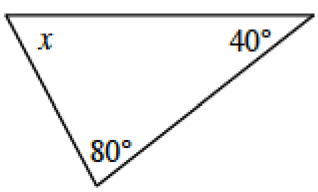
d. How can you convince yourself that your conjecture is true for all triangles? That is, given parallel lines (since the tiling was generated by translating parallelograms), why does $a = d$ and $c = e$ in the diagram at below? If technology is available, use it to test many different angle measures. Explore using the Triangle Angle Sum Theorem eTool (Desmos).

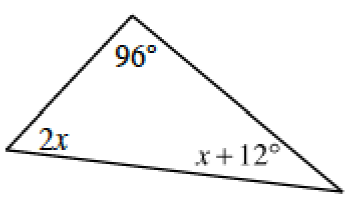




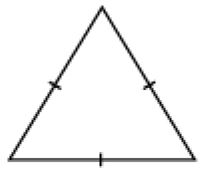
Then add this angle relationship to your Angle Relationships Toolkit from Lesson 2.1.3. This will be referred to as the **Triangle Angle Sum Theorem**. (A theorem is a statement that has been proven.)

2-38. Use your theorem from problem 2-37 about the angles in a triangle to find *x* in each diagram below. Show all work.

a. 

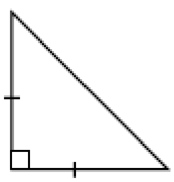
b. 

2-39. What can the Triangle Angle Sum Theorem help you learn about special triangles?

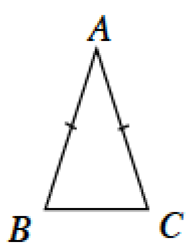


a. Find the measure of each angle in an equilateral triangle. Justify your conclusion.

b. Consider the isosceles right triangle (also sometimes referred to as a "half-square") below. Find the measures of all the angles in a half-square.

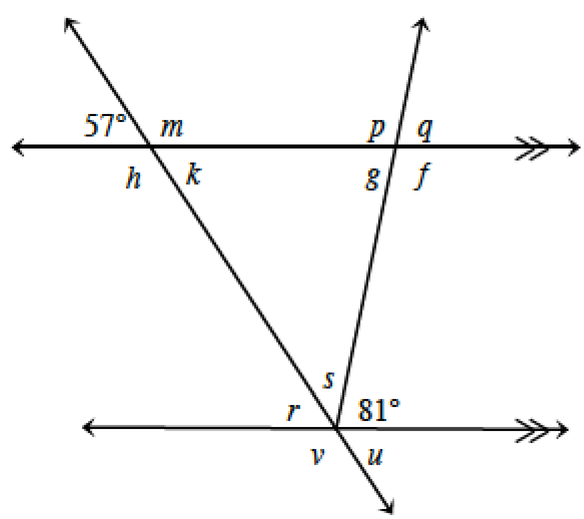


c. What if you only know one angle of an isosceles triangle? For example, if $m\angle A$ = 34°, what are the measures of the other two angles?



2-40. TEAM REASONING CHALLENGE

How much can you figure out about the figure at right using your knowledge of angle relationships? Work with your team to find the measures of all the labeled angles in the diagram below. Justify your solutions with the name of the angle relationship you used. Carefully record your work as you go and be prepared to share your reasoning with the rest of the class.

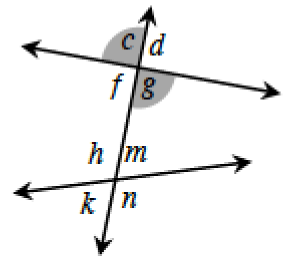


METHODS AND MEANINGS

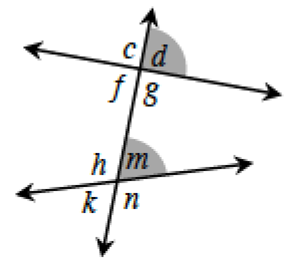
MATH NOTES

More Angle Pair Relationships

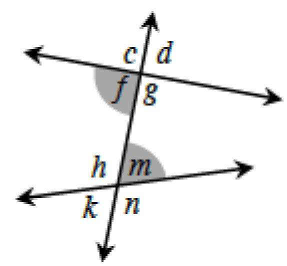
Vertical angles are the two opposite (that is, non-adjacent) angles formed by two intersecting lines, such as angles $\angle c$ and $\angle g$ in the diagram at right. $\angle c$ by itself is not a vertical angle, nor is $\angle g$, although $\angle c$ and $\angle g$ together are a pair of vertical angles. Vertical angles always have equal measure.



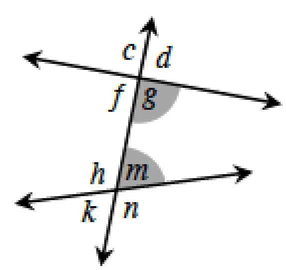
Corresponding angles lie in the same position but at different points of intersection of the transversal. For example, in the diagram at right, $\angle m$ and $\angle d$ form a pair of corresponding angles, since both of them are to the right of the transversal and above the intersecting line. Corresponding angles are congruent when the lines intersected by the transversal are parallel.



$\angle f$ and $\angle m$ are **alternate interior angles** because one is to the left of the transversal, one is to the right, and both are between (inside) the pair of lines. Alternate interior angles are congruent when the lines intersected by the transversal are parallel.

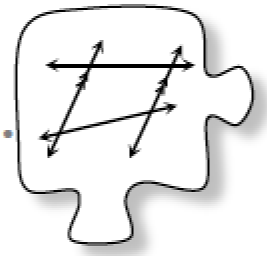


$\angle g$ and $\angle m$ are **same-side interior angles** because both are on the same side of the transversal and both are between the pair of lines. Same-side interior angles are supplementary when the lines intersected by the transversal are parallel.



2.1.5 What is the relationship?

Applying Angle Relationships

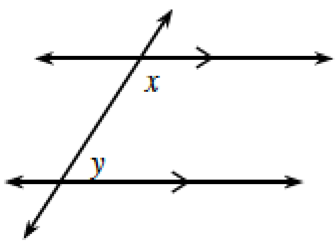


During Section 2.1, you have been learning about various special angle relationships that are created by intersecting lines. Today you will investigate those relationships a bit further, then apply what you know to explain how Mr. Douglas's hinged mirror trick (from problem 2-1) works. As you work in your teams today, keep the following questions in mind to guide your discussion:

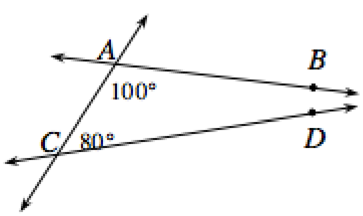
What's the relationship?
Are the angles equal? Are they supplementary?
How can I be sure?

2-46. Use your knowledge of angle relationships to answer the questions below.

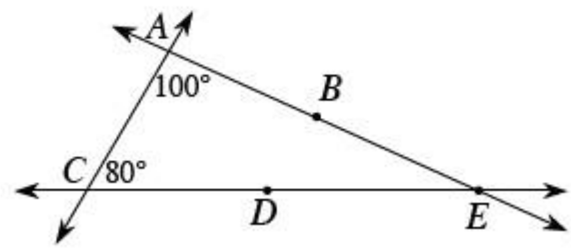
a. In the diagram below, what is the sum of angles *x* and *y*? How do you know?



b. While looking at the diagram below, Rianna exclaimed, "*I think something is wrong with this diagram.*" What do you think she is referring to? Be prepared to share your thinking with the class.



2-47. Maria is not convinced that the lines in part (b) of problem 2-46 *must* be parallel. She decides to assume that they are not parallel and draws the diagram below.

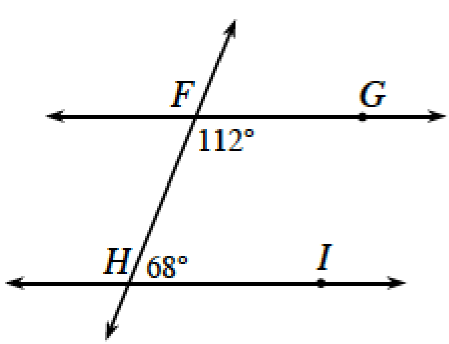


a. Why must lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect in Maria's diagram?

b. What is $m\angle BED$? Discuss this question with your team and explain what it tells you about $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$.

c. If the angle measures at points A and C are as marked, could $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at a point on the other side of $\overleftrightarrow{AC}$? Why or why not?

2-48. Examine the diagram below.

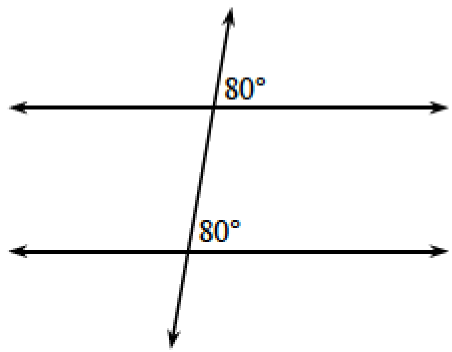


a. In this diagram, must $\overleftrightarrow{FG}$ and $\overleftrightarrow{HI}$ be parallel? Explain how you know.

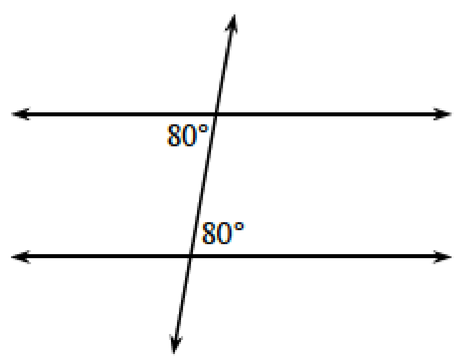
b. Write a theorem based on your conclusion to this problem.

2-49. Use your theorem from problem 2-48 to explain why lines must be parallel in the diagrams below.

a.



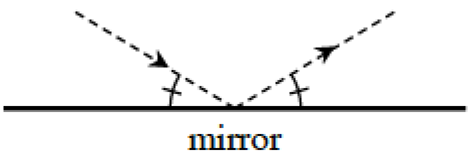
b.



c. Looking back at the diagrams in parts (a) and (b), write two new theorems that begin, "*If corresponding angles are congruent*, …" and "*If the measures of alternate interior angles are congruent*, …".

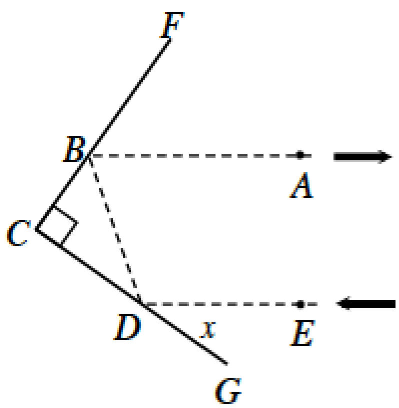
2-50. SOMEBODY'S WATCHING ME, Part Two

Remember Mr. Douglas' trick from problem 2-1? You now know enough about angle and line relationships to analyze why a hinged mirror set so the angle between the mirrors is 90° will reflect your image back to you from any angle. Since your reflection is actually light that travels from your face to the mirror, you will need to study the path of the light. Remember that a mirror reflects light, and that the angle the light hits the mirror will equal the angle it bounces off the mirror.



Your Task: Explain why the mirror bounces your image back to you from any angle. Include in your analysis:

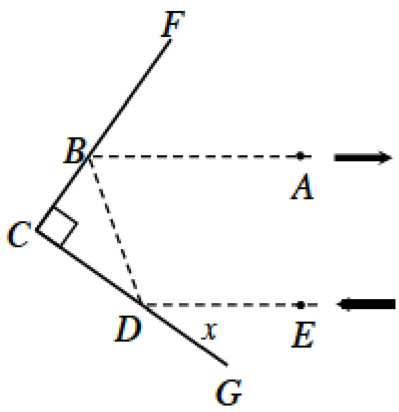
- Use angle relationships to find the measures of all the angles in the figure. (Each team member should choose a different *x*-value and calculate all of the other angle measures using his or her selected value of *x*.)



- What do you know about the paths the light takes as it leaves you and as it returns to you? That is, what is the relationship between $\overline{BA}$ and $\overline{DE}$?
- Does Mr. Douglas' trick work if the angle between the mirrors is not 90°?

Further Guidance

2-51. Since you are trying to show that the trick works for *any* angle at which the light could hit the mirror, each team will work with a different angle measure for x in this problem. Your teacher will tell you what angle x your team should use in the diagram below.



a. Using angle relationships and what you know about how light bounces off mirrors, find the measure of every other angle in the diagram.

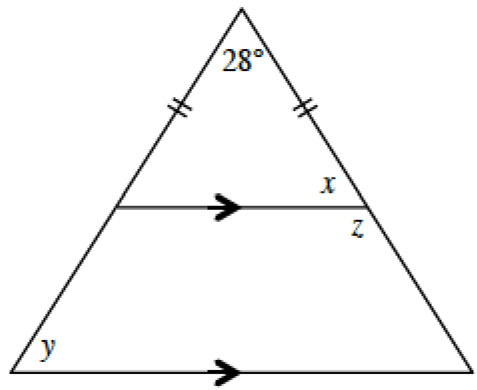
b. What is the relationship between $\angle ABD$ and $\angle EDB$? What does this tell you about the relationship between $\overline{BA}$ and $\overline{DE}$?

c. What if $m\angle C = 89°$? Does the trick still work?

2-52. Explain why the 90° hinged mirror always sends your image back to you, no matter which angle you look into it from.

Further Guidance section ends here.

2-53. Use what you have learned in Section 2.1 to find the measures of *x*, *y*, and *z* below. Justify each conclusion with the name of a geometric relationship from your Angle Relationships Toolkit.



2-54. EXTENSION

Hold a 90° hinged mirror at arm's length and find your own image. Now close your right eye. Which eye closes in the mirror? Look back at the diagram from problem 2-50. Can you explain why this eye is the one that closes?

Methods and Meanings

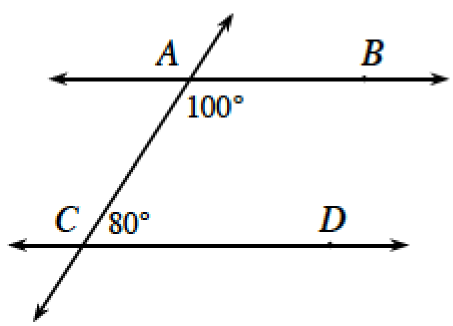
Math Notes

Proof by Contradiction

The kind of argument you used in Lesson 2.1.5 to justify “If same-side interior angles are supplementary, then lines are parallel” is sometimes called a **proof by contradiction**. In a proof by contradiction, you prove a claim by thinking about what the consequences would be if it were false. If the claim’s being false would lead to an impossibility, this shows that the claim must be true.
For example, suppose you know Mary’s brother is seven years younger than Mary. Can you argue that Mary is at least five years old? A proof by contradiction of this claim would go:

Suppose Mary is less than five years old.
Then her brother’s age is negative!
But this is impossible, so Mary must be at least five years old.

To show that lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ must be parallel in the diagram below, you used a proof by contradiction. You argued:

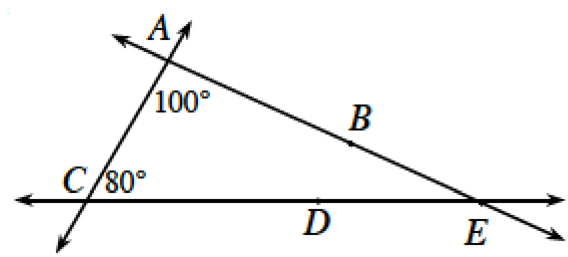


Suppose $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at some point *E*.

Then the angles in Δ*AEC* add up to more than 180°.

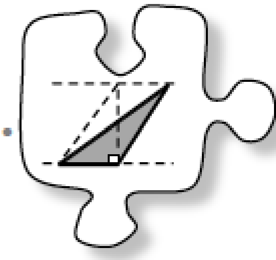
But this is impossible, so $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ must be parallel.

This is true no matter on which side of $\overleftrightarrow{AC}$ point *E* is assumed to be.



2.2.2 How can I find the area?

Areas of Triangles and Composite Shapes



How much grass would it take to cover a football field? How much paint would it take to cover a stop sign? How many sequins does it take to cover a dress? Finding the area of different types of shapes enables us to answer many questions. However, different people will see a shape differently. Therefore, during this lesson, be especially careful to look for different strategies that can be used to find area.

As you solve these problems, ask yourself the following focus questions:

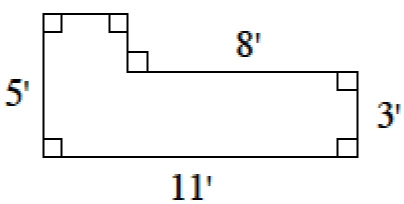
What shapes do I see in the diagram?
Does this problem remind me of one I have seen before?
Is there another way to find the area?

2-70. STRATEGIES TO MEASURE AREA

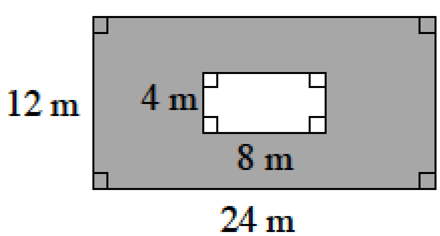
In Lesson 2.2.1 you used a grid to measure area. But what if a grid is not available? Or what if you want an exact measurement?

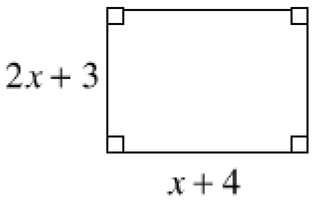
Examine the variety of shapes below. Work with your team to find the area of each one. If a shape has shading, then find the area of the shaded region. Be sure to listen to your teammates carefully and look for different strategies. Be prepared to share your team's method with the class.

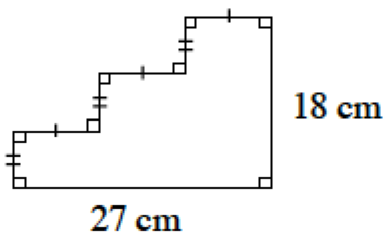
a.

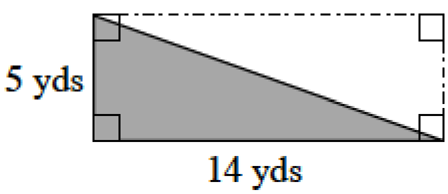


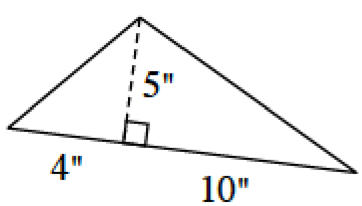
b.



c. 

d. 

e. 

f. 

2-71. Ismael claimed that he did not need to calculate the area for part (f) in problem 2-70 because it must be the same as the area for the triangle in part (e). Explore using Area of a Triangle (Desmos).

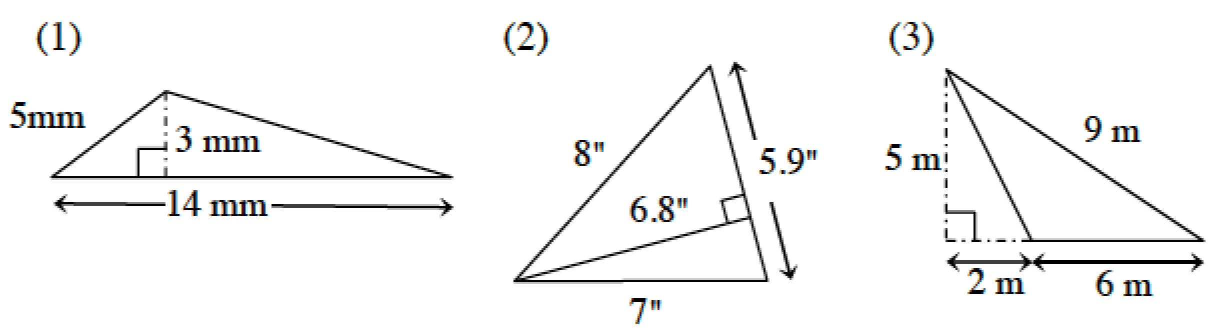
a. Is Ismael's claim correct? How do you know? Draw diagrams that show your thinking.

b. Do all triangles with the same bases and heights have the same areas? Use your technology tool to investigate. If no technology is available, obtain the Lesson 2.2.2 Resource Page and compare the areas of the given triangles.

c. Explain why the area of any triangle is half the area of a rectangle that has the same base and height. That is, show that the area of a triangle must be $\frac{1}{2}$ *bh*.

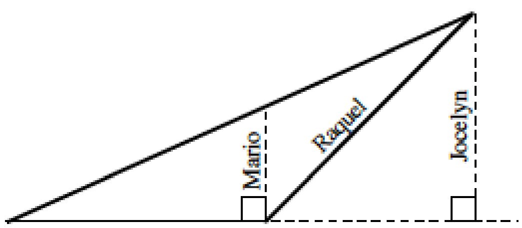
2-72. How do you know which dimensions to use when finding the area of a triangle?

a. Copy each triangle below onto your paper. Then find the area of each triangle. Draw any lines on the diagram that will help. Turning the triangles may help you discover a way to find their areas.



b. Look back at your work from part (a). Which numbers from each triangle did you use to find the area? For instance, in the center triangle, you probably used only the 6.8" and 5.9". Write an explanation and/or draw a diagram that would help another student understand how to choose which lengths to use when calculating the area.

c. Mario, Raquel, and Jocelyn are arguing about where the height is for the triangle below. The three have written their names along the part they think should be the height. Determine which person is correct. Explain why the one you chose is correct and why the other two are incorrect.



2-73. LEARNING LOG

In a Learning Log entry, describe at least two different strategies that were used today to find the area of irregular shapes. For each method, be sure to include an example. Title this entry "Areas of Composite Figures" and include today's date.

Methods and Meanings

Math Notes

Multiplying Binomials

One method for multiplying binomials is to use a generic rectangle. That is, use each factor of the product as a dimension of a rectangle and find its area. If $(2x + 5)$ is the base of a rectangle and $(3x - 1)$ is the height, then the expression $(2x + 5)(3x - 1)$ is the area of the rectangle. See the example below.

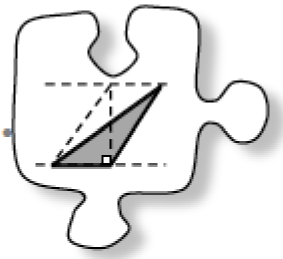
	$2x$	$+5$	
-1	$-2x$	-5	-1
$3x$	$6x^2$	$15x$	$3x$
	$2x$	$+5$	

Multiply: $(2x + 5)(3x - 1) = 6x^2 - 2x + 15x - 5$

$= 6x^2 + 13x - 5$

2.2.3 What is the area?

Areas of Parallelograms and Trapezoids



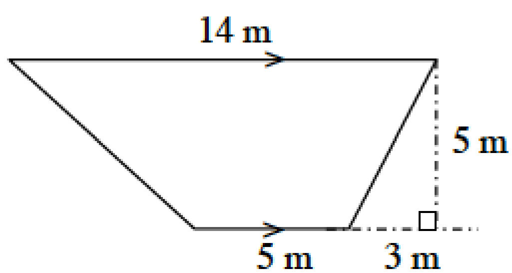
In Lesson 2.2.2, you used your knowledge of the area of a rectangle to develop a method to find the exact area of a triangle that works for all triangles. How can your understanding of the area of triangles and rectangles help with the study of other shapes?

As you work today, ask yourself and your team members these focus questions:

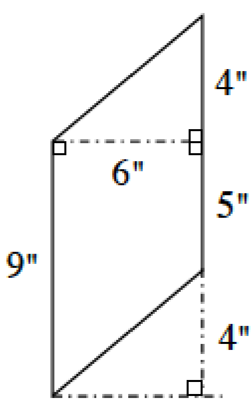
What do you see?
What shapes make up the composite figure?
Is there another way?

2-79. Find the areas of the figures below. Can you find more than one method for each shape?

a.



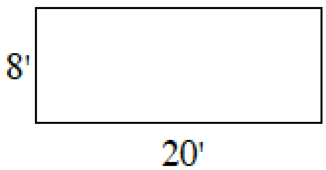
b.



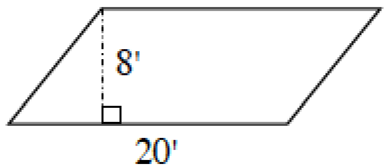
2-80. FINDING THE AREA OF A PARALLELOGRAM

One of the shapes in your Shape Bucket is shown below. It is called a **parallelogram**: a four-sided shape with two pairs of parallel sides. How can you find the area of a parallelogram? Consider this question as you answer the questions below.

a. Kenisha thinks that the rectangle and parallelogram below have the same area. Her teammate Shaundra disagrees. Who is correct? Justify your conclusion.



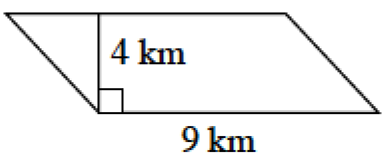
Rectangle



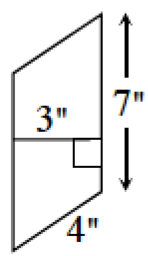
Parallelogram

b. In the parallelogram shown in part (a), the two lengths that you were given are often called the **base** and **height**. Several more parallelograms are shown below. In each case, find a related rectangle for which you know both the base and height. Rotating your book might help. Use what you know about rectangles to find the area of each parallelogram.

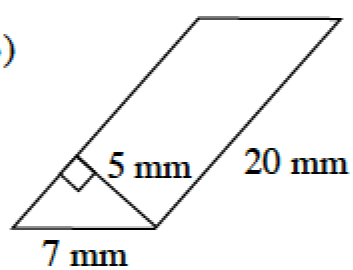
(1)



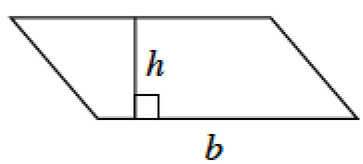
(2)



(3)



(4)

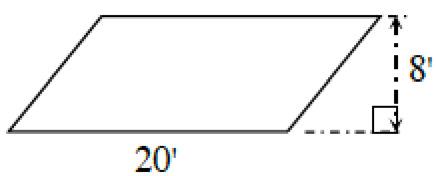


c. Describe how to find the area of a parallelogram when given its base and height.

d. Does the angle at which the parallelogram slants matter? Does every parallelogram have a related rectangle with equal area? Why or why not? Explain how you know.

2-81. Shaundra claims that the area of a parallelogram can be found by *only* using triangles.

a. Do you agree? Trace the parallelogram at right onto your paper. Then divide it into two triangles. (Do you see more than one way to do this? If you do, ask some team members to divide the parallelogram one way, and the others a second way.)



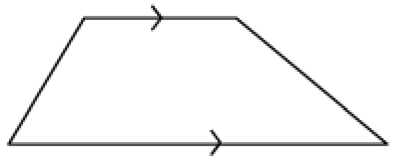
b. Use what you know about calculating the area of a triangle to find the area of the parallelogram. It may help to trace each triangle separately onto tracing paper so that you can rotate them and label any lengths that you know.

c. How does the answer to part (b) compare to the area you found in part (a) of problem 2-80?

2-82. FINDING THE AREA OF A TRAPEZOID

Another shape you will study from the Shape Bucket is a **trapezoid**: a four-sided shape that has at least one pair of parallel sides. The sides that are parallel are called **bases**, as shown in the diagram below. Answer the questions below with your team to develop a method to find the area of a trapezoid.

base #1



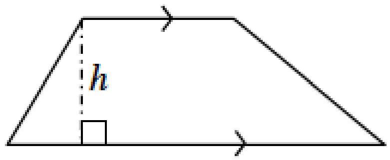
base #2

Trapezoid

a. While playing with the shapes in her Shape Bucket, Shaundra noticed that two identical trapezoids could be arranged to form a parallelogram. Is she correct?

Trace the trapezoid shown below onto a piece of tracing paper. Be sure to label its bases and height as shown in the diagram. Work with a team member to move and rearrange the trapezoid on each piece of tracing paper so that they create a parallelogram.

b_1



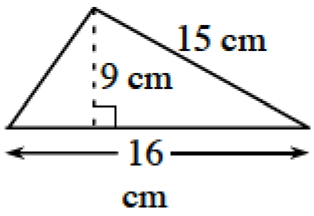
b_2

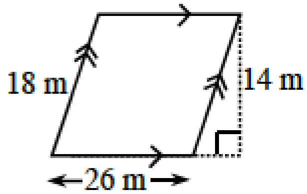
b. Since you built a parallelogram from two trapezoids, you can use what you know about finding the area of a parallelogram to find the area of the trapezoid. If the bases of each trapezoid are b_1 and b_2 and the height of each is h, then find the area of the parallelogram. Then use this area to find the area of the original trapezoid.

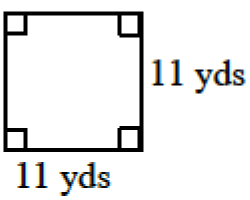
c. Kenisha sees it differently. She sees two triangles inside the trapezoid. If she divides a trapezoid into two triangles, what area will she get? Again assume that the bases of the trapezoid are b_1 and b^2 and the height is h.

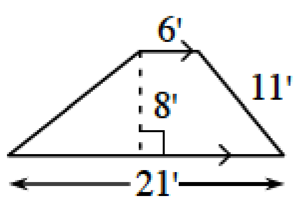
d. Are the area expressions you created in parts (b) and (c) **equivalent**? That is, will they calculate the same area? Use your algebra skills to demonstrate that they are equivalent.

2-83. Calculate the exact areas of the shapes below.

a. 

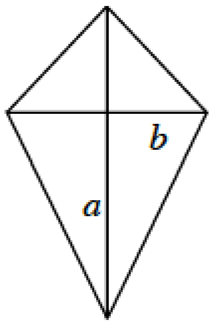
b. 

c. 

d. 

2-84. EXTENSION

Examine the diagram of the kite below. Work with your team to find a way to show that its area must be half of the product of the diagonals. That is, if the length of the diagonals are *a* and *b*, provide a diagram or explanation of why the area of the kite must be $\frac{1}{2}ab$.



METHODS AND MEANINGS

MATH NOTES

Conditional Statements

A **conditional statement** is written in the form "**If …, then ….**" Here are some examples of conditional statements:

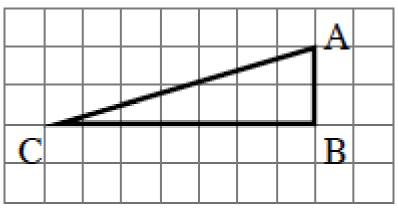
> *If a shape is a rhombus, then it has four sides of equal length.*
> *If it is February 14th, then it is Valentine's Day.*
> *If a shape is a parallelogram, then its area is A = bh.*

2.3.1 Is the answer reasonable?

Triangle Inequality

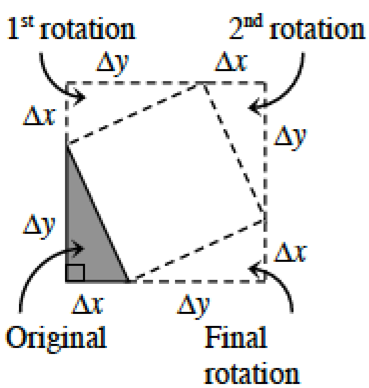
You now have several tools for describing triangles (lengths, areas, and angle measures), but can *any* three line segments create a triangle? Or are there restrictions on the side lengths of a triangle? And how can you know that the length you found for the side of a triangle is accurate? Today you will investigate the relationship between the side lengths of a triangle.

2-99. Roiri (pronounced "ROAR-ree") loves right triangles and has provided the one below to analyze. He wants your team to find the length of the **hypotenuse** (the longest side: $\overline{AC}$).



a. Estimate the length of $\overline{AC}$.

b. Roiri decided to repeatedly rotate the triangle on graph paper as shown below. He says the quadrilateral constructed on $\overline{AC}$ is a square. Do you agree? Justify your answer.



c. With your team, find a strategy to determine the area of the central square. Then use the area of the central square to find the length of the hypotenuse (the longest side of the right triangle). Was your result for the hypotenuse close to your estimate? Why or why not?

d. Will Roiri's strategy of rotating the triangle and finding the areas always work to find the longest length of a right triangle? Explain.

2-100. For a different triangle Δ*ABC* where *AB* = 3 units and *BC* = 4 units, Roiri found that *AC* = 25. Donna is not sure that is possible. What do you think? Visualize this triangle and explain if you think this triangle is possible or not.

2-101. PINK SLIP

Oh no! During your last shift at the Shape Factory everything seemed to be going fine--until the machine that was producing triangles made a huge CLUNK and then stopped. Since your team was on duty, all of you will be held responsible for the machine's breakdown.

Luckily, your boss has informed you that if you can figure out what happened and how to make sure it will not happen again, you will keep your job. The last order the machine was processing was for a triangle with sides 3 cm, 5 cm, and 10 cm.

a. Use the eTool, *Triangle Inequality* (Desmos), or pasta provided by your teacher to investigate what happened today at the factory. Can a triangle be made with *any* three side lengths? If not, what condition(s) would make it impossible to build a triangle? Try building triangles with the side lengths listed below:

(1) 3 cm, 5 cm, and 10 cm
(2) 4 cm, 9 cm, and 12 cm
(3) 2 cm, 4 cm, and 5 cm
(4) 3 cm, 5 cm, and 8 cm

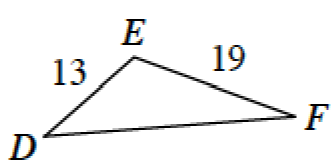
b. For those triangles that could not be built, what happened? Why were they impossible?

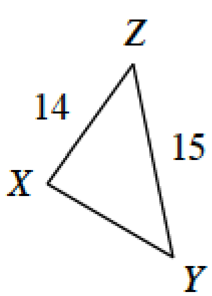
c. Use the eTool, *Triangle Inequality* (Desmos), or dry pasta to investigate the restrictions on the three side lengths that can form a triangle. For example, if two sides of a triangle are 5 cm and 12 cm long, respectively, what is the longest side that could join these two sides to form a triangle? (Could the third side be 12 cm long? 19 cm long?) What is the shortest possible length that could be used to form a triangle? (Does 5 cm work? 9 cm?)

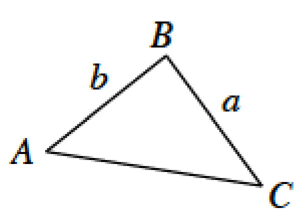
d. Write a memo to your boss explaining what happened. If you can convince your boss that the machine's breakdown was not your fault *and* show the company how to fix the machine so that this does not happen again, you might earn a promotion!

2-102. The values you found in parts (a) and (c) of problem 2-101 were the *minimum* and *maximum* limits for the length of the third side of any triangle with two sides of lengths 5 cm and 12 cm. The fact that there are restrictions on the side lengths that may be used to create a triangle is referred to as the **Triangle Inequality**.

Determine the minimum and maximum limits for each missing side length in the triangles below.

a. 

b. 

c. 

2-103. LEARNING LOG

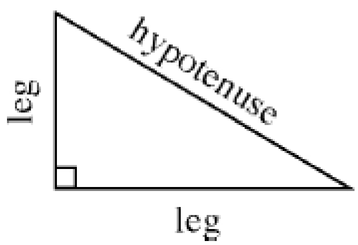
In a Learning Log entry, explain how you can tell if three sides will form a triangle or not. Draw diagrams to support your statements. Title this entry, "Triangle Inequality" and include today's date.

METHODS AND MEANINGS

MATH NOTES

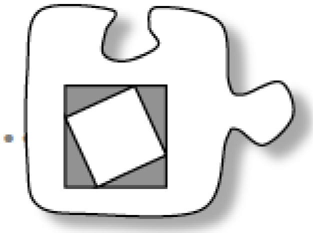
Right Triangle Vocabulary

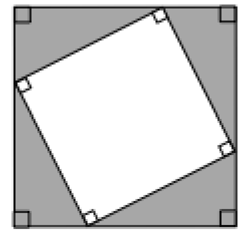
Several of the triangles that you have been working with in this section are right triangles, that is, triangles that contain a 90° angle. The two shortest sides of the right triangle (the sides that meet at the right angle) are called the **legs** of the triangle and the longest side (the side opposite the right angle) is called the **hypotenuse** of the triangle.



2.3.2 Is there a shortcut?

The Pythagorean Theorem



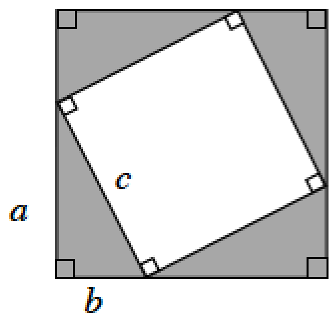


In Lesson 2.3.1, you learned a method to find the length of a hypotenuse of a right triangle by finding the area of the square built on the hypotenuse, as shown in the diagram at right. However, what if the sides of the triangle make it difficult to draw (such as very large numbers or decimal values)? Or what if you do not even know the lengths of one of the legs?

Today, you will work with your team to find the relationship between the legs and hypotenuse of a right triangle. By the end of this lesson, you should be able to find the side of *any* right triangle, when given the lengths of the other two sides.

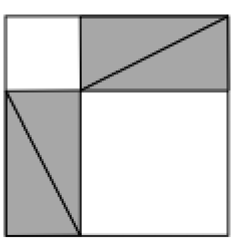
2-109. Roiri complained that while his method from problem 2-99 works, it seems like too much work! He remembers that rearranging a shape does not change its area and thinks he can find a shortcut. Obtain a [Lesson 2.3.2 Resource Page] for your team and cut out the shaded triangles. Note that the lengths of the sides of the triangles are a, b, and c units respectively. Explore using the interactive eTool: [Pythagorean Theorem] (Desmos).

a. First, arrange the triangles to look like Roiri's in the diagram below. Draw this diagram on your paper. What is the area of the unshaded square?



b. Roiri claims that moving the triangles within the outer square won't change the area of the unshaded square. Is Roiri correct? Why or why not?

c. Move the shaded triangles to match the diagram below. In this configuration, what is the total area that is unshaded?



d. Write an equation that relates the two ways that you found to represent the unshaded area in the figure.

2-110. The relationship between the square of the lengths of the legs and the square of the length of the hypotenuse in a right triangle that you found in problem 2-109 is known as the **Pythagorean Theorem**. This relationship is a powerful tool because once you know the lengths of any two sides of a right triangle, you can find the length of the third side.

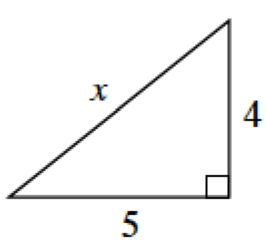
a. Use the eTool, *Pythagorean Theorem* (Desmos), to examine how the square of the hypotenuse always equals the sum of the squares of the legs of a right triangle. Think about it until it makes sense and you can explain it to someone else so that it will make sense to him or her.

b. LEARNING LOG
Add an entry in your Learning Log for the Pythagorean Theorem, explaining what it is and how to use it. Be sure to include a diagram. Title this entry, "Pythagorean Theorem" and include today's date.

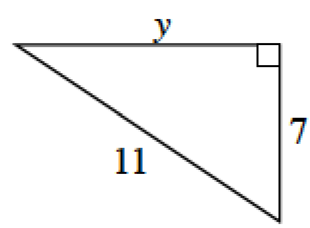
2-111. Apply the Pythagorean Theorem to answer the questions below.

a. For each triangle below, find the value of the variable.

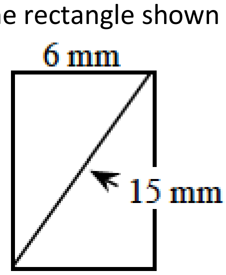
i.



ii.



b. Examine the rectangle shown below. Find its perimeter and area.



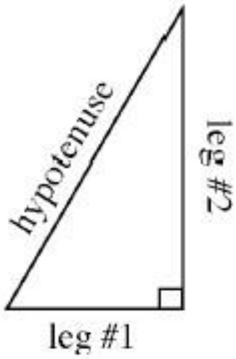
c. On graph paper, draw $\overline{AC}$ with coordinates *A*(2, 6) and *C*(5, –1). Then draw a slope triangle. Use the slope triangle to find the length of $\overline{AC}$.

2-112. The Garcia family took a day trip from Cowpoke Gulch. Their online directions told them to drive four miles north, six miles east, three miles north, and then one mile east to Big Horn Flat. Draw a diagram and calculate the direct distance (straight) from Cowpoke Gulch to Big Horn Flat.

METHODS AND MEANINGS

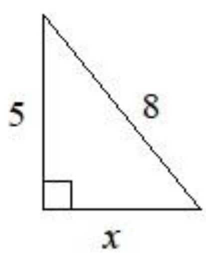
MATH NOTES

The Pythagorean Theorem



The **Pythagorean Theorem** states that in a right triangle,
(lengthof leg #1)^2 + (length of leg #2)^2 = (length of hypotenuse)^2

The Pythagorean Theorem can be used to find a missing side length in a right triangle. See the example below.



$5^2 + x^2 = 8^2$
$25 + x^2 = 64$
$x^2 = 39$
$x = \sqrt{39} \approx 6.24$

In the example above, $\sqrt{39}$ is an example of an **exact** answer, while 6.24 is an **approximate** answer.

4. WHAT HAVE I LEARNED?

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on. Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 2-118. Sandra's music bag contains:

3 traditional country songs
6 traditional rock songs
4 hip-hop rap songs
5 contemporary country songs
1 Latin rap song
3 traditional pop songs

a. What is the probability that the player will select some rap music next?

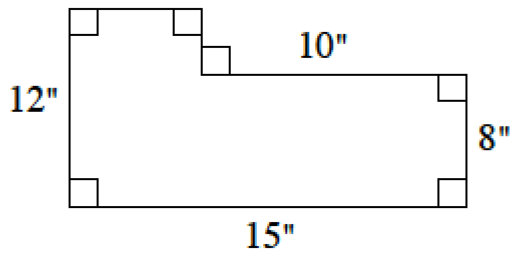
b. Find P(traditional), that is, the probability that the player will randomly select traditional music of any kind.

c. Find P(traditional pop).

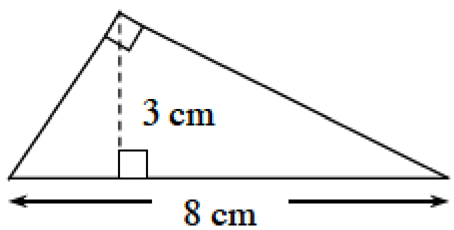
d. Find P(not country), the probability that the next song is *not* country music.

CL 2-119. Find the area of each figure.

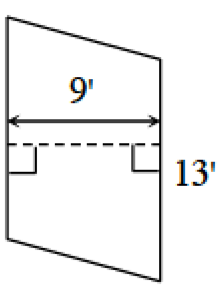
a.



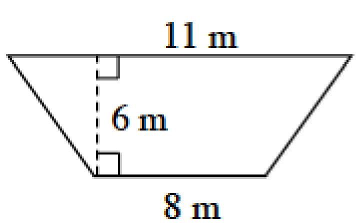
b.



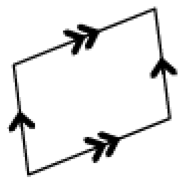
c.

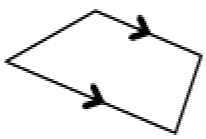


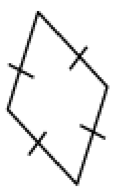
d.

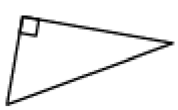


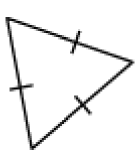
CL 2-120. Name each of the following shapes.

a. 

b. 

c. 

d. 

e. 

f. Graph the following points and then name the shape that is created when you connect the points in the given order.
$N(-2, 6)$, $A(-4, 6)$, $M(-4, 3)$, $E(-2, 3)$

CL 2-121. Graph the segment that connects the points *A*(−4, 8) and *B*(6, 3).

a. What is the slope of $\overline{AB}$?

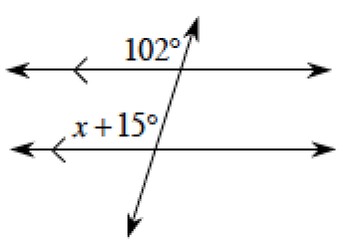
b. Write an equation for the line that connects points *A* and *B*.

c. Write an equation for a line that is parallel to $\overline{AB}$.

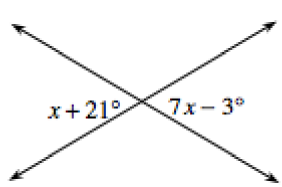
d. Write an equation for a line that is perpendicular to $\overline{AB}$.

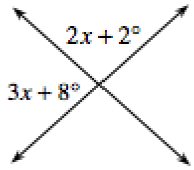
CL 2-122. Identify the geometric angle relationship(s) in each diagram. Use what you know about those relationships to write an equation and solve for *x*.

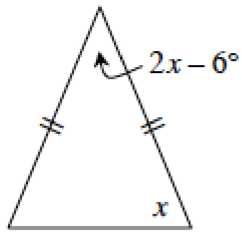
a.



b.



c. 

d. 

CL 2-123. Examine the system of equations below.

$y = -2x + 6$

$y = \frac{1}{2}x - 9$

a. Solve the system below *twice*: graphically and algebraically. Verify that your solutions from the different methods are the same.

b. What is the relationship between the two lines? How can you tell?

c. Solve the system below using your method of choice.

$2x + 3y = 18$

$4x - 3y = 6$

CL 2-124. Charlotte was transforming the hexagon *ABCDEF*.

a. What single transformation did she perform in Diagram #1?

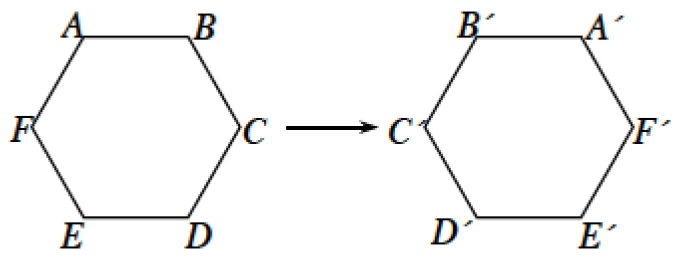


Diagram #1

b. What single transformation did she perform in Diagram #2?

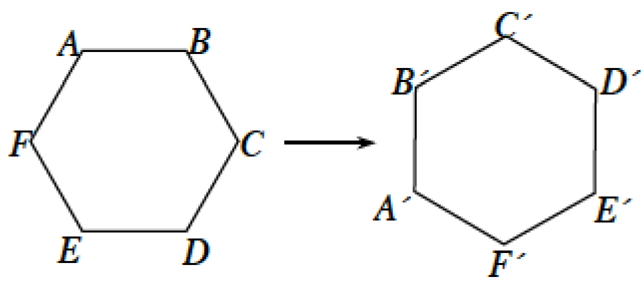


Diagram #2

c. What transformation didn't she do? Write directions for this type of transformation for hexagon *ABCDEF* and perform it.

CL 2-125. Explain what you are doing when you find the perimeter of a flat shape. How is that different than finding its area?

CL 2-126. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in math classes you have taken before? Use the table to make a list of topics you need help on and a list of topics you need to practice more.