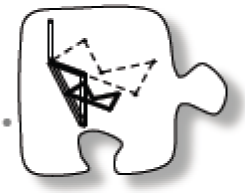
3.1.1 What do these shapes have in common?



Dilations

In Section 1.3, you organized shapes into groups based on their size, angles, sides, and other characteristics. You identified shapes using their characteristics and investigated relationships between different kinds of shapes, so that now you can tell if two figures are either parallelograms or trapezoids, for example. But what makes two figures look alike?

Today you will be introduced to a new transformation that enlarges a figure while maintaining its shape, called a **dilation**. After creating new enlarged figures, you and your team will explore the interesting relationships that exist between figures that have the same shape. In your teams, you should keep the following questions in mind as you work together today:

3-1. WARM-UP STRETCH

Before computers and copy machines existed, it sometimes took hours to enlarge documents or to shrink text on items such as jewelry. A pantograph device (like the one shown below) was often used to duplicate written documents and artistic drawings. You will now employ the same geometric principles by using rubber bands to draw enlarged copies of a design. Your teacher will show you how to do this.

During this activity, discuss the following questions:

What do the figures have in common?

What do you predict?

What specifically is different about the figures?

3-2. In problem 3-1, you created designs that were similar, meaning that they have the same shape. But how can you determine if two figures are similar? What do similar shapes have in common? To find out, your team will need to create similar shapes that you can measure and compare.

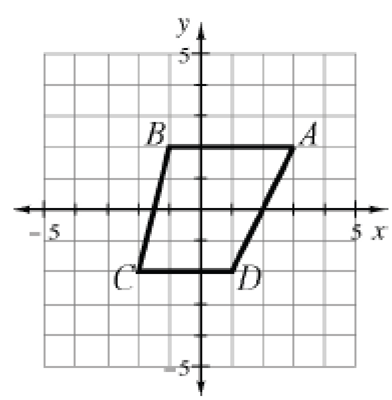


Diagram #1

a. Obtain a Lesson 3.1.1 Resource Page from your teacher. On it, find the quadrilateral shown in Diagram #1 at right.

   Dilate (stretch) the quadrilateral from the origin by a factor of 2, 3, 4, or 5 to form $A'B'C'D'$. Each team member should pick a different enlargement factor. You may want to imagine that your rubber band chain is stretched from the origin so that the knot traces the perimeter of the original figure.

For example, if your job is to stretch *ABCD* by a factor of 3, then *A′* would be located as shown in Diagram #2 at right.

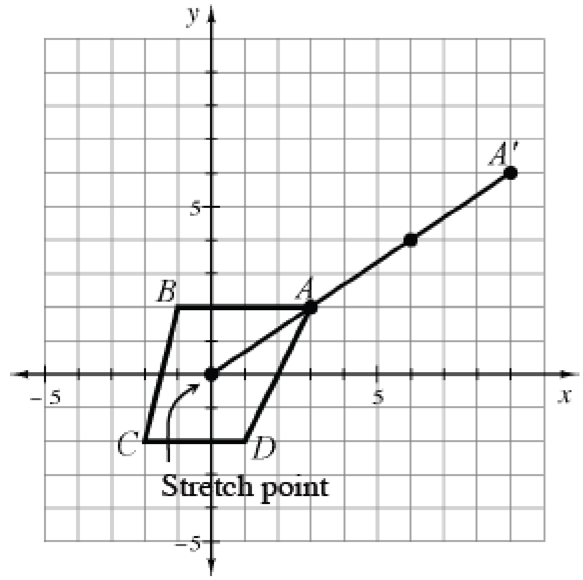
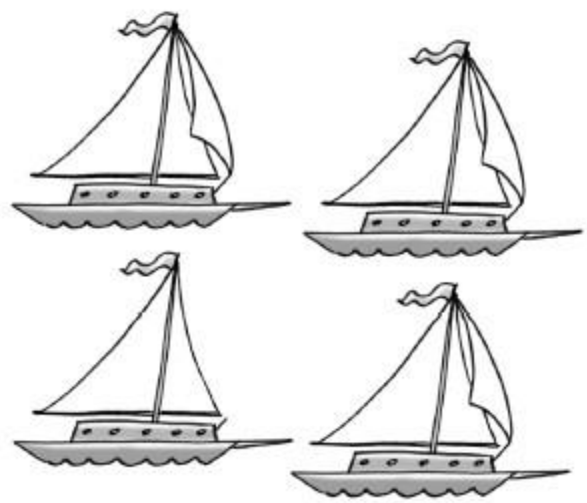


Diagram #2

b. Carefully cut out your enlarged figure and compare it to your teammates' figures. How are the four enlargements different? How are they the same? As you investigate, make sure you compare both angles and side lengths of the similar figures. Be ready to report your conclusions to the class.

- **3-3.** WHICH SHAPE IS THE EXCEPTION?

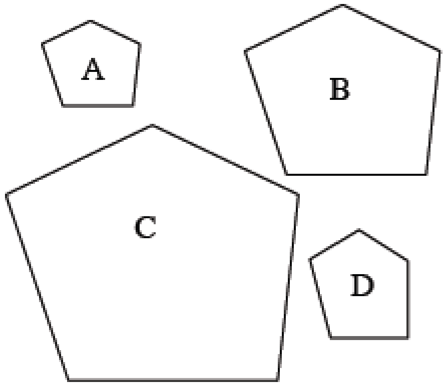
Sometimes figures look the same and sometimes they look very different. What characteristics make figures alike so that you can say that they are the same shape? How are figures that look the same but are different sizes related to each other? Understanding these relationships will allow us to know if figures that appear to have the same shape actually do have the same shape.

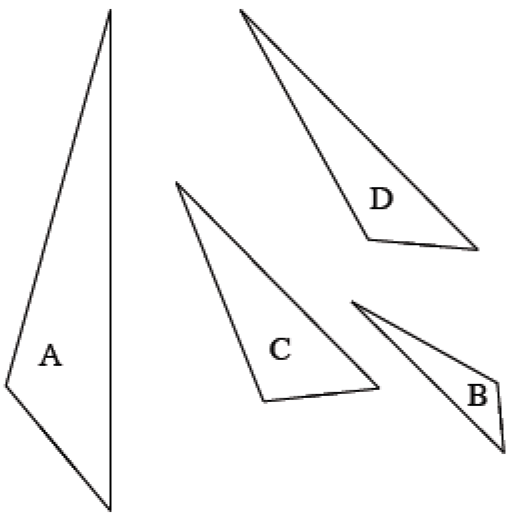


Your Task: For each set of figures below, three are **similar** (meaning that they are related through a sequence of transformations including dilation), and one is an exception. Find the exception in each set of figures.

Use tracing paper to answer each of these questions for both sets of shapes below:

- Which figure appears to be the exception? What makes that shape different from the others?
- What do the other three shapes have in common?
- Are there commonalities in the angles? Are there differences?
- Are there commonalities in the sides? Are there differences?

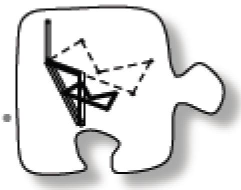
a. 

b. 

3-4. LEARNING LOG

Write an entry in your Learning Log about the characteristics that figures with different sizes need to have in order to maintain the same shape. Add your own diagrams to illustrate the description. Title this entry "Same Shape, Different Size" and include today's date.

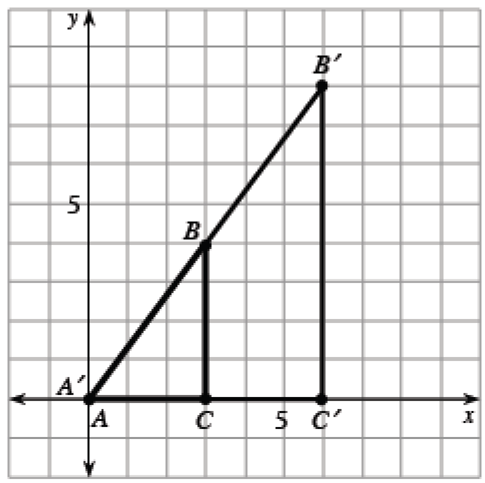
3.1.2 How can I maintain the shape?



Similarity

So far you have studied several shapes that appear to be **similar** (exactly the same shape but not necessarily the same size). But how can we know for sure that two shapes are similar? Today you will focus on the relationship between the lengths of sides of similar figures by enlarging and reducing shapes and looking for patterns.

3-11. Find your work from problem 3-5. The graph you created should resemble the graph below.



a. In problem 3-5, you dilated (stretched) ΔABC to create $\Delta A'B'C'$. Which side of $\Delta A'B'C'$ corresponds to $\overline{CB}$? Which side corresponds to $\overline{AB}$?

b. What is the relationship of the corresponding sides? Write down all of your observations. How could you get the lengths of $\Delta A'B'C'$ from the lengths of ΔABC?

c. Why does $\overline{A'B'}$ lie directly on $\overline{AB}$ and $\overline{A'C'}$ lie directly on $\overline{AC}$, but $\overline{B'C'}$ does not lie directly on $\overline{BC}$?

d. Could you get the side lengths of $\Delta A'B'C'$ by adding the same amount to each side of ΔABC? Try this and explain what happened.

e. Monica dilated ΔABC to get a different triangle. She knows that $\overline{A''B''}$ is 20 units long. How many times larger than ΔABC is $\Delta A''B''C''$? (That is, how many "rubber bands" did she use?) And how long is $\overline{B''C''}$? Show how you know.

3-12. SIMILARITY

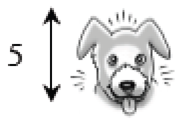
An important aspect of dilation is that the shape of a figure does not change even though its size may change. This is because dilations do not affect angles while they change lengths proportionally.

a. If Monica rotated $\Delta A''B''C''$ about a point, would it still remain the same shape as ΔABC? Why or why not? What if she translated it or reflected it?

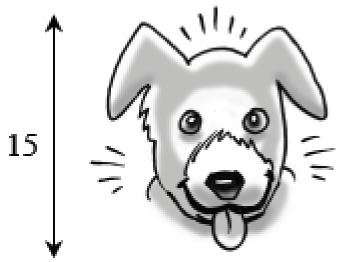
b. When two figures are related by a series of transformations (including dilation), they are called **similar**. Similar figures have the same shape but not necessarily the same size. Similar figures can be created by multiplying each side length by the same number. This number is called the **zoom factor**.

You may have used a zoom factor before when using a copy machine. For example, if you set the zoom factor on a copier to 50%, the machine shrinks the image in half (that is, multiplies it by 0.5) but keeps the shape the same. In this course, the zoom factor will be used to describe the ratio of the new figure to the original figure.

What zoom factor was used to enlarge the puppy shown below?

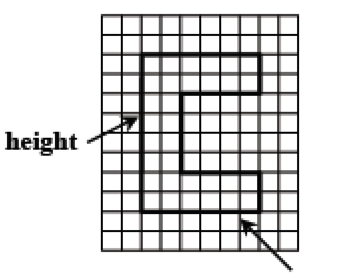


Original



Enlarged

3-13. Casey decided to enlarge her favorite letter: C, of course! Your team is going to help her out. Have each member of your team choose a different zoom factor below. Then on graph paper, enlarge (or reduce) the block "C" at right by your zoom factor (insert another page with graph paper).



Original

a. 3

b. 2

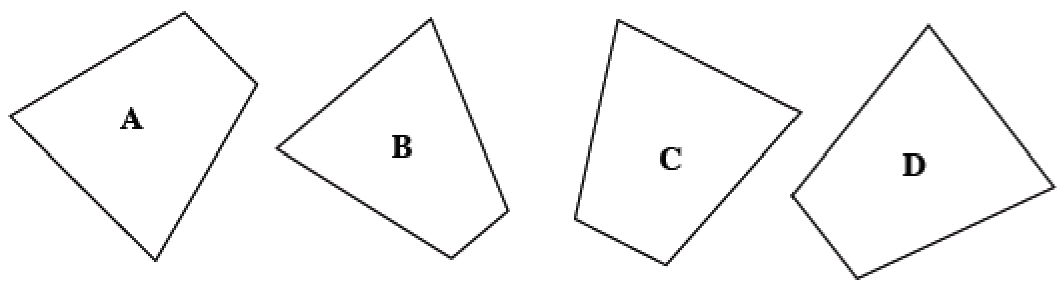
c. 1

d. $\frac{1}{2}$

3-14. Look at the different "C's" that were created in problem 3-13.

a. What happened when the zoom factor was 1?

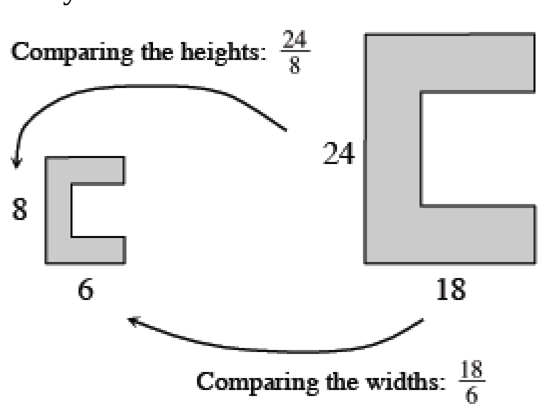
b. When there is a sequence of rigid transformations that carries one figure onto another, then the two figures are the same shape *and* the same size (that is, the zoom factor is 1) and they are called **congruent**. Compare the shapes below with tracing paper and determine which shapes are congruent.



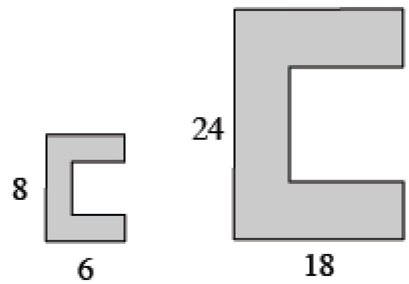
3-15. EQUAL RATIOS OF SIMILARITY

Casey wants to learn more about her enlarged "C"s. Return to your work from problem 3-13.

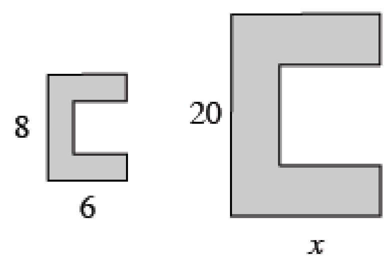
a. Since the zoom factor multiplies each part of the original shape, then the ratio of the widths must equal the ratio of the lengths.
Casey decided to show these ratios in the diagram below. Verify that her ratios are equal.



b. When looking at Casey's work, her brother wrote the equation $\frac{8}{6} = \frac{24}{18}$. Are his ratios, in fact, equal? And how could he show his work on his diagram? Copy his diagram below and add arrows to show what sides Casey's brother compared.



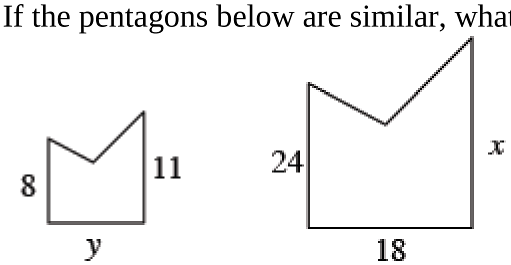
c. She has decided to create an enlarged "C" for the door of her bedroom. To fit, it needs to be 20 units tall. If x is the width of this "C", write and solve an equation to find out how wide the "C" on Casey's door must be. Be ready to share your equation and solution with the class.



3-16. Use your observations about ratios between similar figures to answer the following questions.

a. Assume one triangle has side lengths 6, 7, and 10 units while another has side lengths 3, 4, and 5 units. Are these triangles similar? How do you know?

b. If the pentagons below are similar, what are the values of x and y?



3-17. LEARNING LOG

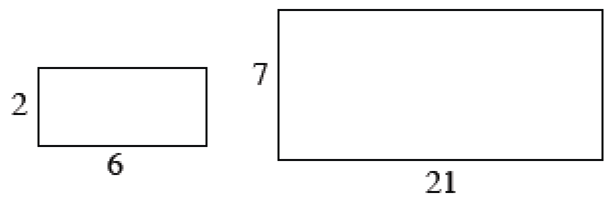
In a new entry of your Learning Log, explain what you know about the side lengths and angle measures of similar figures. If you know the dimensions of one triangle, how can you find the dimensions of another triangle that is similar to it? Also, how can you decide, when given side lengths, if the two triangles are similar? Title this entry, "Similar Figures" and include today's date.

3.1.3 How are the figures related?

Using Ratios of Similarity

You have learned that when you enlarge or reduce a shape so that it remains similar (that is, it maintains the same shape), each of the side lengths have been multiplied by a common zoom factor. You can also set up ratios within shapes and make comparisons to other similar shapes. Today you will learn about how changing the size of an object affects its perimeter. You will also learn how ratios can help solve similarity problems when drawing the figures is not practical.

3-24. Trace the rectangles below onto your paper.



a. Show that there is a sequence of one or more transformations that can carry one rectangle onto the other.

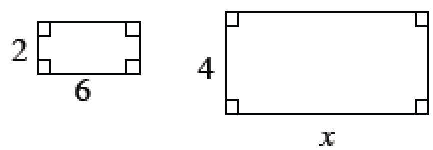
b. Use ratios to show that these rectangles are similar (figures that have the same shape, but not necessarily the same size).

c. What other ratios could you use?

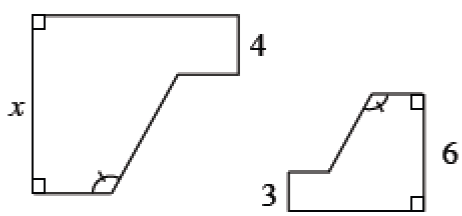
d. Linh claims that these figures are not similar. When she compared the heights, she wrote $\frac{2}{7}$. Then she compared the bases and got $\frac{21}{6}$. Why is Linh having trouble? Explain completely.

3-25. Each pair of figures below is similar. Review what you have learned so far about similarity as you solve for *x*.

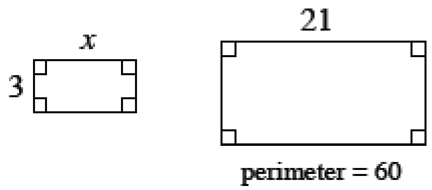
a.



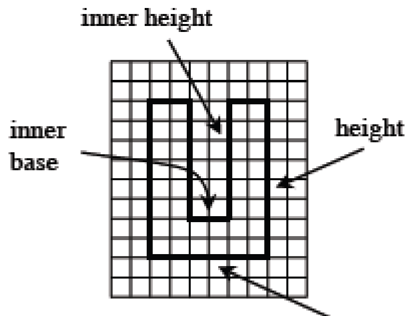
b.



c.



3-26. Casey's back at it! Now she wants you to enlarge the block "U" for her spirit flag.



Original

a. Copy her "U" onto graph paper (new page with graph paper works!).

b. Now draw a larger "U" with a zoom factor of $\frac{3}{2} = 1.5$. What is the height of the new "U"?

c. Find the ratio of the perimeters. That is, find $\frac{\text{Perimeter New}}{\text{Perimeter Original}}$. What do you notice?

d. Casey enlarged "U" proportionally so that it has a height of 10. What was her zoom factor? What is the base of this new "U"? Justify your conclusion.

3-27. After enlarging his "U" in problem 3-26, Al has an idea. He drew a 60° angle, as shown in Diagram #1 below. Then, he extended the sides of the angle so that they are twice as long, as shown in Diagram #2. "*Therefore, the new angle must have measure 120°,*" he explained. Do you agree? Discuss this with your team and write a response to Al.

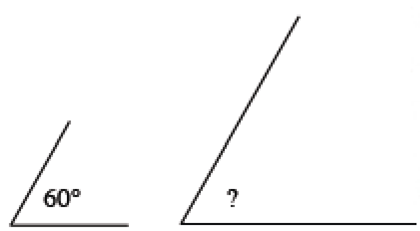
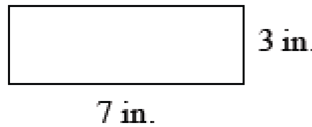


Diagram #1 Diagram #2

3-28. Al noticed that the ratio of the perimeters of two similar figures is equal to the ratio of the side lengths. *"What about the area? Does it grow the same way?"* he wondered.



a. Find the area and perimeter of the rectangle above.

b. Test Al's question by enlarging the rectangle by a zoom factor of 2. Then find the new area and perimeter.

c. Answer Al's question: Does the perimeter double? Does the area double? Explain what happened.

3.1.4 How can I use equivalent ratios?

Applications and Notation

Now that you have a good understanding of how to use ratios in similar figures to solve problems, how can you extend these ideas to situations outside the classroom? You will start by considering a situation for which you want to find the length of something that would be difficult to physically measure.

3-35. GEORGE WASHINGTON'S NOSE

On her way to visit Horace Mann University, Casey stopped by Mount Rushmore in South Dakota. The park ranger gave a talk that described the history of the monument and provided some interesting facts. Casey could not believe that the carving of George Washington's face is 60 feet tall from his chin to the top of his head!

However, when a tourist asked about the length of Washington's nose, the ranger was stumped! Casey came to her rescue by measuring, calculating and getting an answer. How did Casey get an answer? View this video about Mt. Rushmore's unveiling (YouTube).

Your Task: Figure out the length of George Washington's nose on the monument. Work with your team to come up with a strategy. Show all measurements and calculations on your paper with clear labels so anyone could understand your work.

Discussion Points

What is this question asking you to find?

How can you use similarity to solve this problem?

Is there something in this room that you can use to compare to the monument?

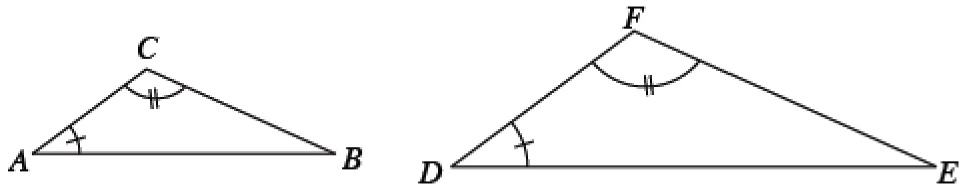
What parts do you need to compare?

Do you have any math tools that can help you gather information?

3-36. When solving problem 3-35, you may have written a proportional equation like the one below. When solving proportional situations, it is very important that parts be labeled to help you follow your work.

$$\frac{\text{Length of George's Nose}}{\text{Length of George's Head}} = \frac{\text{Length of Student's Nose}}{\text{Length of Student's Head}}$$

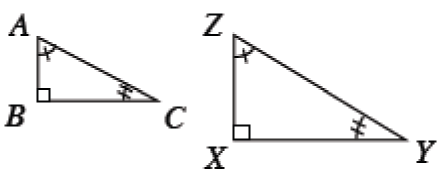
Likewise, when working with geometric shapes such as the similar triangles below, it is easier to explain which sides you are comparing by using notation that everyone understands



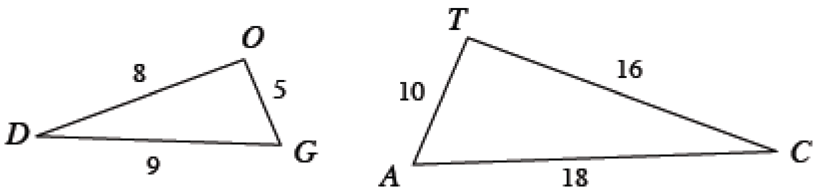
a. One possible proportional equation for these triangles is $\frac{AC}{AB} = \frac{DF}{DE}$. Write at least three more proportional equations based on the similar triangles above.

b. Jeb noticed that $m\angle A = m\angle D$ and $m\angle C = m\angle F$. But what about $m\angle B$ and $m\angle E$? Do these angles have the same measure? Or is there not enough information? Justify your conclusions.

3-37. The two triangles below are similar. Read the Math Notes box for this lesson to learn about how to write a statement to show that two shapes are similar.



Then examine the two triangles below.



a. Describe a sequence of transformations that carries one triangle onto the other. Use tracing paper to help.

b. Which of the following statements are correctly written and which are not? Note that more than one statement may be correct. Discuss your answers with your team.

i. $\Delta DOG \sim \Delta CAT$

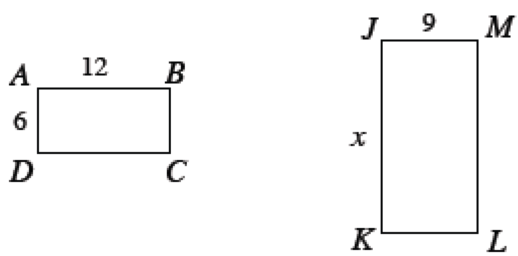
ii. $\Delta DOG \sim \Delta \mathrm{C}TA$

iii. $\Delta \mathrm{OG}D \sim \Delta ATC$

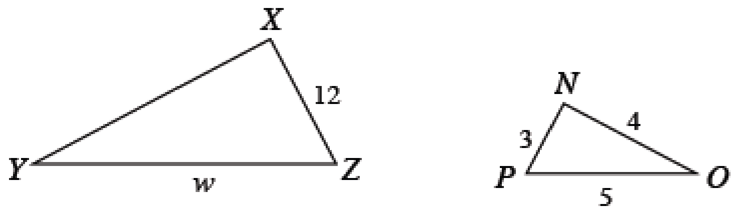
iv. $\Delta DGO \sim \Delta CAT$

3-38. Find the value of the variable in each pair of similar figures below. You may want to set up tables to help you write equations.

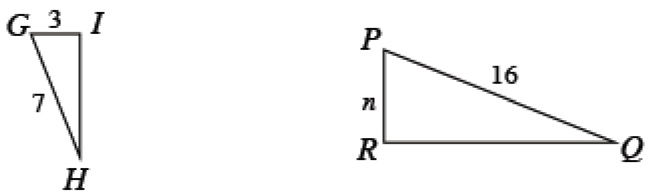
a. $ABCD \sim JKLM$



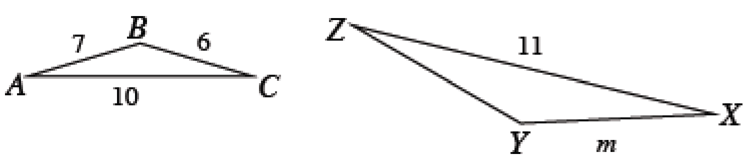
b. $\Delta NOP \sim \Delta XYZ$



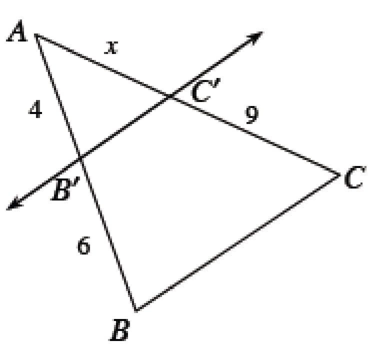
c. $\Delta GHI \sim \Delta PQR$



d. $\Delta ABC \sim \Delta XYZ$



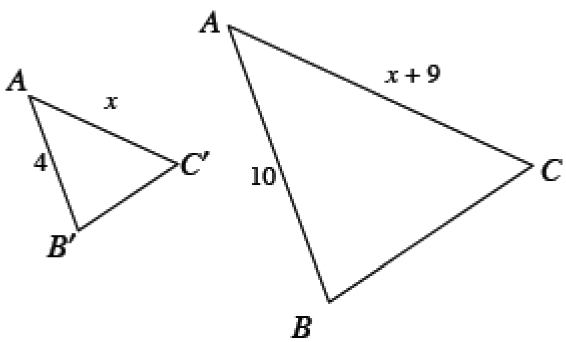
3-39. Rochida drew ΔABC *below* and then dilated it to create $\Delta AB'C'$.



a. Why are the two triangles similar?

b. What is the relationship of the lengths $\overline{AB'}$ and $\overline{AB}$? What about between $\overline{AC'}$ and $\overline{AC}$? Justify your answer.

c. Rochida decides to redraw the shape as two separate triangles, as shown below. Write and solve a proportional equation to find *x* using the corresponding sides.



d. How long is $\overline{AC}$? How long is $\overline{AC'}$?

e. What must the ratio of the original segment $\overline{BC}$ to its image $\overline{B'C'}$ be? Explain.

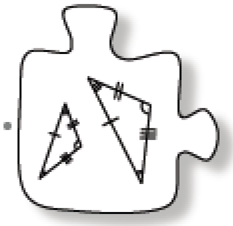
f. What is the relationship between $\overleftrightarrow{B'C'}$ and $\overleftrightarrow{BC}$?

3-40. LEARNING LOG

Write a Learning Log entry describing the different ways you can compare two similar objects or quantities with equivalent ratios. Title this entry “Comparing With Ratios” and include today’s date.

3.2.1 What information do I need?

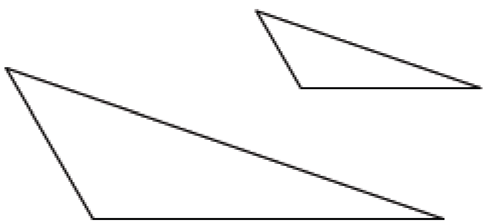
Conditions for Triangle Similarity



Now that you know what similar shapes have in common, you are ready to turn to a related question: How much information do I need to conclude that two triangles are similar? As you work through today's lesson, remember that similar polygons have corresponding angles that are congruent and corresponding sides that are proportional.

3-47. ARE THEY SIMILAR?

Erica thinks the triangles below might be similar. However, she knows not to trust the way figures look in a diagram, so she asks for your help.

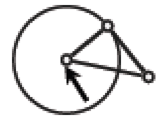


a. If two shapes are similar, what must be true about their angles and sides?

b. Obtain the Lesson 3.2.1 Resource Page from your teacher. Measure the angles and sides of Erica's triangles and help her decide if the triangles are similar or not.

c. Assuming that the corresponding sides of these similar triangles are parallel, demonstrate that there is a dilation that carries one onto the other by finding the point of dilation (the stretch point) for the two triangles on the resource page.

3-48. HOW MUCH IS ENOUGH?



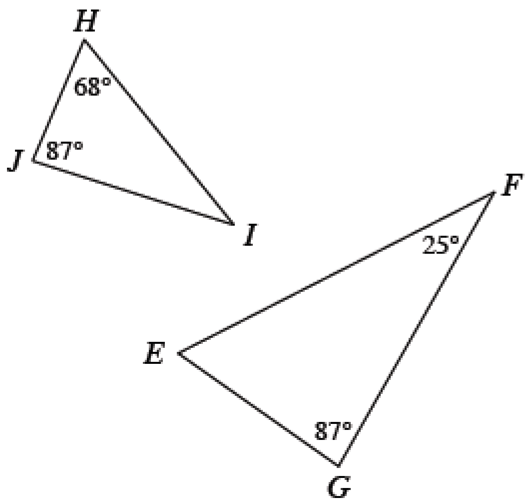
Jovan is tired of measuring all the angles and sides to determine if two triangles are similar. "*There must be an easier way*," he thinks. "*What if I know that all of the angles are congruent? Does that mean that the triangles are similar?*"

a. Before experimenting, make a prediction. Do you think that the triangles have to be similar if all three corresponding angles are congruent?

b. Experiment with Jovan's idea. To do this, use the 3-48b Student eTool (CPM), a dynamic geometry tool, or straws with protractors, to make a triangle with angles 38°, 62°, and 80°. Can you make another triangle, with the same angles, that is *not* similar to your original triangle? Can you create any two triangles with the same three angle measures that are *not* similar? Investigate, sketch your shapes, and write down your conclusion.

c. Describe a sequence of transformation to show that two triangles that have the same three angles are similar. Explore by changing the angles and using rigid transformations using the 3-48c Student eTool (CPM).

3-49. Scott is looking at the set of shapes below. He thinks that $\Delta EFG \sim \Delta HIJ$ but he is not sure that the shapes are drawn to scale.

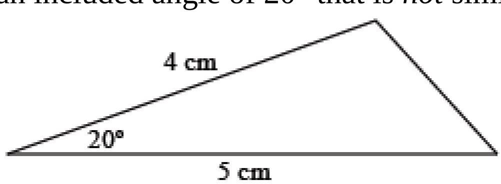


a. Are the corresponding angles equal? Convince Scott that these triangles are similar.

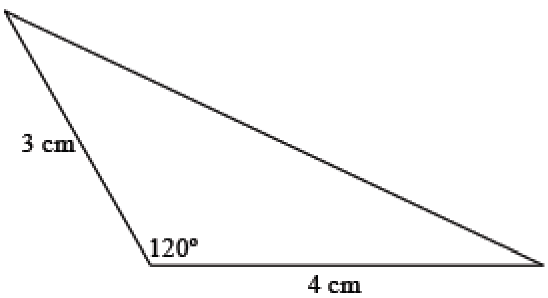
b. How many pairs of angles need to be congruent to be sure that triangles are similar? How could you abbreviate this similarity condition?

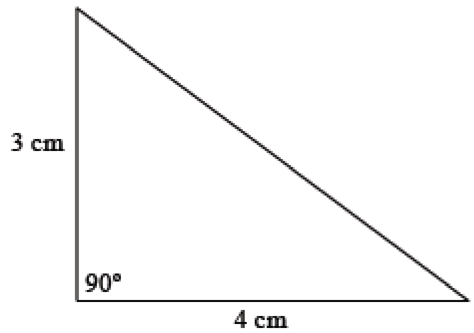
3-50. Carlos then asks, "*What if we only know that one angle is congruent, but the two pairs of corresponding sides that make the angle are proportional? Does that mean the triangles are similar?*"

a. Use a dynamic geometry, [3-50a Student eTool]() (CPM), or straws and protractors to test triangles with two pairs of corresponding sides that are proportional, with the included angles congruent as follows. Create a triangle with side lengths 4 cm and 5 cm and an angle of 20° between these two sides, as shown below. If a second triangle has an angle of 20°, and the two sides that make the angle share a common ratio with 4 cm and 5 cm (such as 8 cm and 10 cm), is the second triangle always similar to the first triangle? That is, is it possible to make a second triangle with two sides proportional to 4 cm and 5 cm, and an included angle of 20° that is *not* similar?



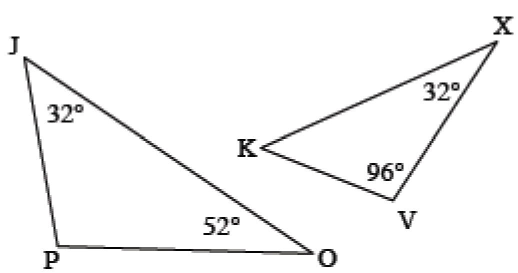
b. Test your ideas from part (a) on the additional triangles below. Is it possible to make a second triangle with two of the sides proportional to the given triangle and with the same included angle that is *not* similar? Explore using the 3-50b #1 Student eTool (CPM) and the 3-50b #2 Student eTool (CPM).

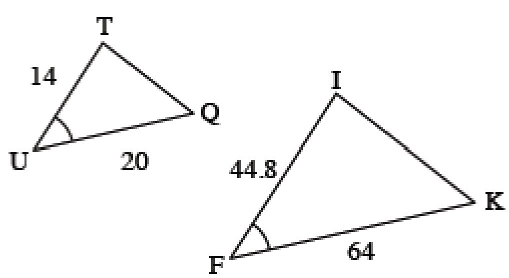




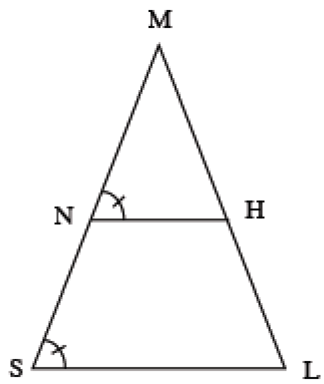
c. Based on your investigations from parts (a) and (b), what can you conclude about triangles with SAS similarity (SAS ~)? Justify your response with transformations.

3-51. Based on your conclusions from problems 3-49and 3-50, decide if each pair of triangles below is similar. If they are similar, describe a sequence of transformations that carry one onto the other. Explain your reasoning.

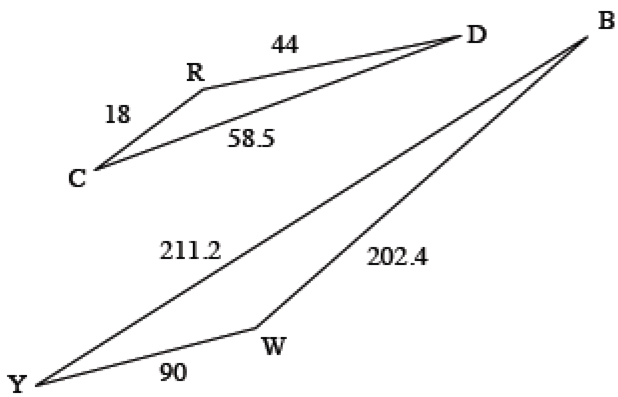
a. 

b. 

c.



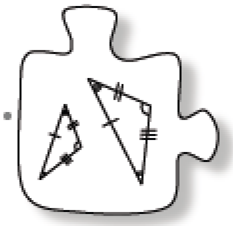
d.



3-52. LEARNING LOG

Read the Math Notes box for this lesson, which introduces new names for the observations you made in problems 3-49 and 3-50. Then write a Learning Log entry about what you learned today. Be sure to address the question: *How much information do I need about a pair of triangles in order to be sure that they are similar?* Title this entry "AA ~ and SAS ~" and include today's date.

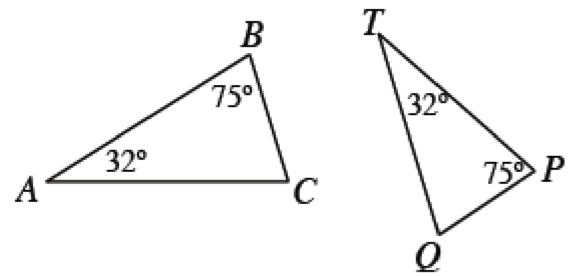
3.2.2 How can I organize my information?



Creating a Flowchart

In Lesson 3.2.1, you developed the AA ~ and SAS ~ conditions to help confirm that triangles are similar. Today you will continue working with similarity and will learn how to use flowcharts to organize your reasoning.

3-59. Examine the triangles at right.

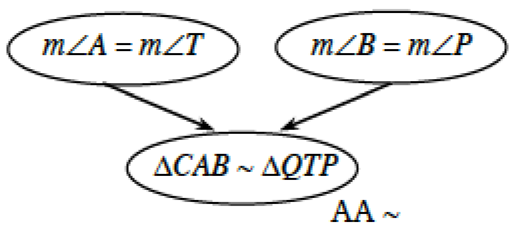


a. Are these triangles similar? Use full sentences to explain your reasoning.

b. Julio decided to use a diagram (called a **flowchart**) to explain his reasoning.

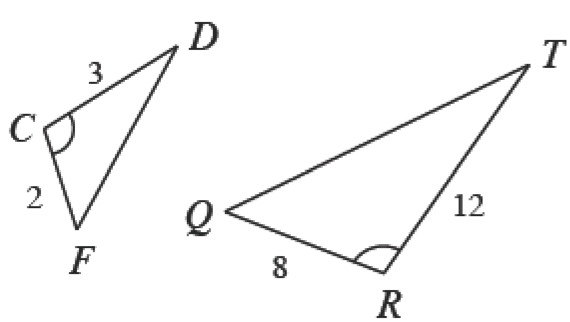
Compare your explanation to Julio's flowchart. Did Julio use the same reasoning you used?

JULIO'S FLOWCHART



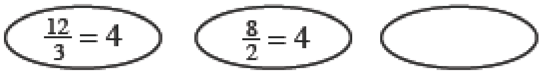
3-60. Besides showing your reasoning, a flowchart can be used to organize your work as you determine whether or not triangles are similar.

a. Are these triangles similar? Which triangle similarity condition can you use?

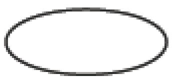


b. What facts must you know to use the triangle similarity condition you chose? Julio started to list the facts in a flowchart below. Copy them on your paper and complete the third oval.

Facts:

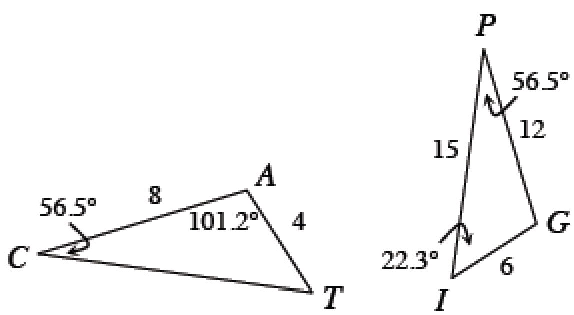


c. Once you have the needed facts in place, you can conclude that you have similar triangles. Add to your flowchart by making an oval and filling in your conclusion.

Conclusion: 

d. Finally, draw arrows to show the flow of the facts that lead to your conclusion and record the similarity condition you used, following Julio's example from problem 3-59.

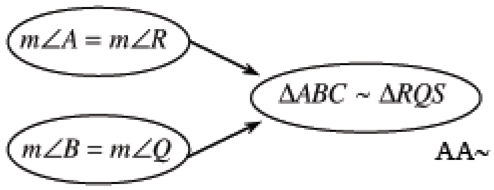
3-61. Now examine the triangles below.



a. Are these triangles similar? Justify your conclusion using a flowchart.

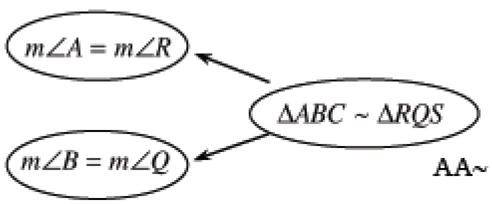
b. What is the length of CT? How do you know?

3-62. Lindsay was solving a math problem and drew the flowchart below:

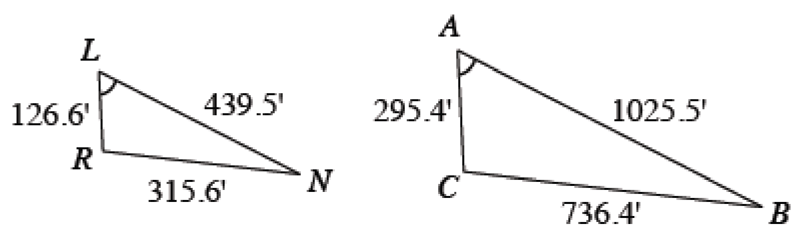


a. Draw and label two triangles that could represent Lindsay's problem. What question did the problem ask her? How can you tell?

b. Lindsay's teammate was working on the same problem and made a mistake in his flowchart: How is this flowchart different from Lindsay's? Why is this the wrong way to explain the reasoning in Lindsay's problem?



3-63. Ramon is examining the triangles below. He suspects they may be similar by SAS ~.



a. Which pair of corresponding sides do you think Ramon is relying on? Why?

b. Set up ovals for the facts you need to know to show that the triangles are similar. Complete any necessary calculations and fill in the ovals.

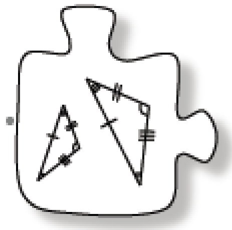
c. Are the triangles similar? If so, complete your flowchart and name an appropriate similarity condition. If not, explain how you know.

3-64. LEARNING LOG

In your Learning Log, explain how to set up a flowchart. For example, how do you know how many ovals you should use? How do you know what to put inside the ovals? Provide an example. Title this entry “Using Flowcharts” and include today’s date.

3.2.3 How can I use equivalent ratios?

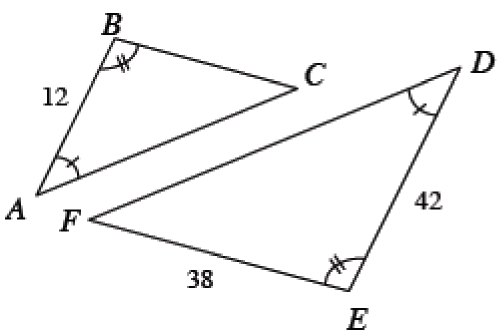
Triangle Similarity and Congruence



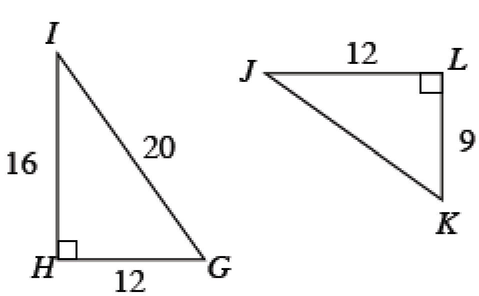
By examining and testing side ratios and angles, you are now able to determine whether two figures are similar. But how can you tell if two shapes are the same shape *and* the same size? In this lesson you will examine properties that guarantee that shapes are exact replicas of one another. Note that some figures in this lesson and throughout the course may not be drawn to scale. Always use the factual information stated about or marked on the figure(s) to make decisions.

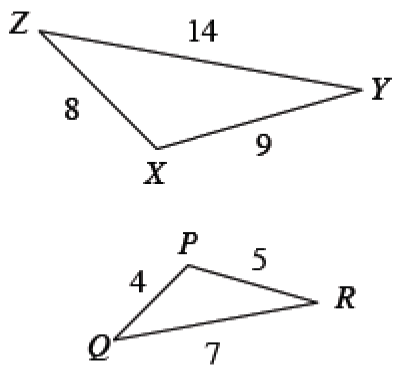
3-71. Decide if each pair of triangles below is similar. Use a flowchart to organize your facts and conclusion for each pair of triangles.

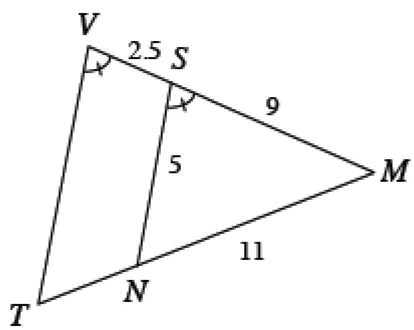
a.



b.



c. 

d. 

3-72. For the diagrams in problem 3-71, find the lengths of the segments listed below, if possible. If it is not possible, explain why not.

a. $\overline{BC}$

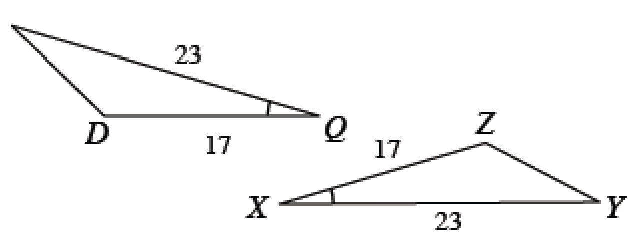
b. $\overline{AC}$

c. $\overline{VT}$

d. $\overline{TN}$

3-73. Kamraan offers you a challenge. Are the triangles at right similar? How do you know? Examine the triangles at below.

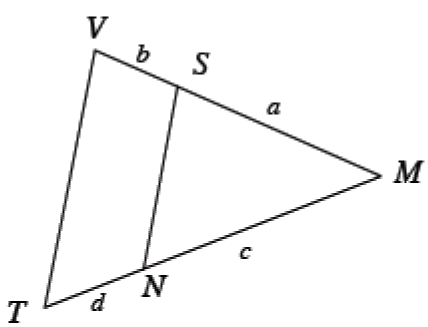
a. Use a flowchart to organize your explanation.



b. Kamraan says, "*These triangles aren't just similar—they're congruent!*" Is Kamraan correct? What special value in your flowchart indicates that the triangles are congruent?

c. Write a conjecture (in "If…, then…" form) for your observation in part (b). Then prove that it is true by justifying the conclusion.

3-74. Kamraan has a new challenge. He drew triangle *VTM* below and $\overleftrightarrow{SN}$ which intersects the sides so that the subdivided lengths are proportional, that is, so that $\frac{b}{a}=\frac{d}{c}$. He asks, "*Is* $\overleftrightarrow{VT}$ *parallel to* $\overleftrightarrow{SN}$ *? How do you know?*" Think about this question as you answer the questions below.



a. Kamraan's figure looks like two triangles on top of each other. Separate the triangles, label the vertices and all the side lengths you can.

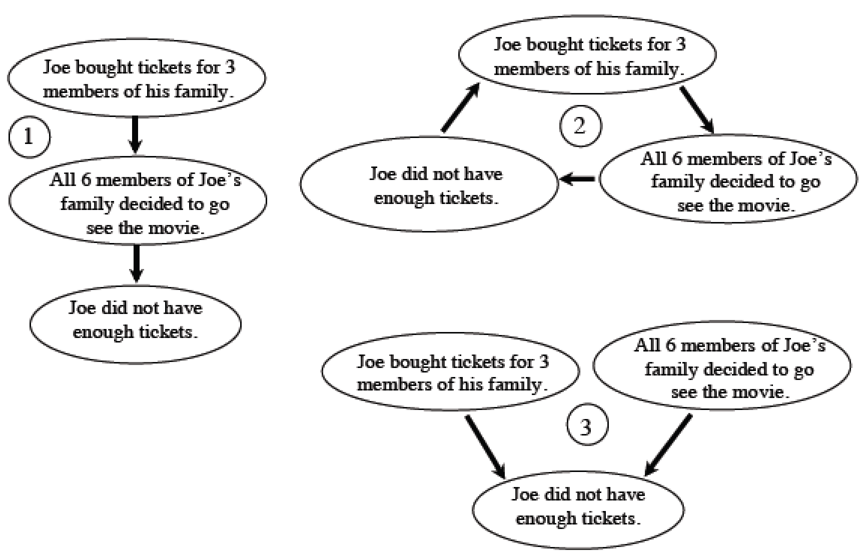
b. How can you prove that $\frac{a+b}{a} = \frac{c+d}{c}$? Talk about this with your team and be ready to share your reasoning with the class.

c. Are the triangles similar? How do you know?

d. Address Kamraan's challenge: Prove that $\overleftrightarrow{VT}$ is parallel to $\overleftrightarrow{SN}$. Record your reasoning.

3-75. Flowcharts can also be used to represent real-life situations. For example, yesterday Joe found out that three people in his family (including Joe) wanted to see a movie, so he went to the theater and bought three tickets. Unfortunately, while he was gone, three more family members decided to go. When everyone arrived at the theater, Joe did not have enough tickets.

Joe sat down later that night and tried to create a flowchart to describe what had happened. Here are the three possibilities he came up with:

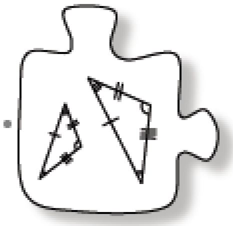


a. Which of these flowcharts best captures the situation that happened on Saturday? Why?

b. What is wrong with the other two flowcharts as descriptions of this situation?

3.2.4 What information do I need?

More Conditions for Triangle Similarity



So far, you have worked with two methods for determining that triangles are similar: the AA ~ and the SAS ~ conditions. Are these the only ways to determine if two triangles are similar? Today you will investigate similar triangles and complete your list of triangle similarity conditions.

Keep the following questions in mind as you work together today:

How much information is needed?

Are the triangles similar? How can you tell?

Can I find a triangle with this information that is not similar?

3-82. Robel's team is using the SAS ~ condition to show that two triangles are similar. *"This is too much work,"* Robel says. *"When we're using the AA ~ condition, we only need to look at two pairs of corresponding parts. Let's just calculate the ratios for two pairs of corresponding sides to determine that triangles are similar."*

If two pairs of corresponding side lengths share a common ratio, must the triangles be similar? If not, what additional information is needed? Investigate these questions below.

a. Robel has a triangle with side lengths 4 cm and 5 cm. If your triangle has two sides that share a common ratio with Robel's, does your triangle have to be similar to his? Is SS ~ a valid similarity condition? Explain how you know.

b. Kashi asks, *"I want to test ASA~, which means I start with two pairs of congruent angles and the lengths of the sides connecting these angles are proportional. Would that be enough to know the triangles are similar?"* Discuss this with your team and write Kashi an explanation.

3-83. What other triangle similarity conditions involving sides and angles might there be? List the names of every other possible triangle similarity condition you can think of that involve corresponding side lengths and angles.

3-84. In problem 3-82, Robel discovered that SS ~ was not a valid similarity condition. But Kendall wondered if SSS ~ was valid.

a. Before experimenting, make a prediction. Do you think that the triangles have to be similar if all three of the corresponding sides share a common ratio?

b. Experiment with Kendall's idea. To do this, use the dynamic geometry tools below to test triangles with proportional side lengths. Begin with those listed below, then try some others. If you do not have access to a dynamic tool, cut straws into the lengths below and create two triangles. Can you create two triangles with proportional sides that are *not* similar? Investigate, sketch your shapes, and write down your conclusion.

Triangle #1: side lengths 3, 5, and 7; Triangle #2: side lengths 6, 10, and 14 Explore with 3-84b #1 eTool (CPM).

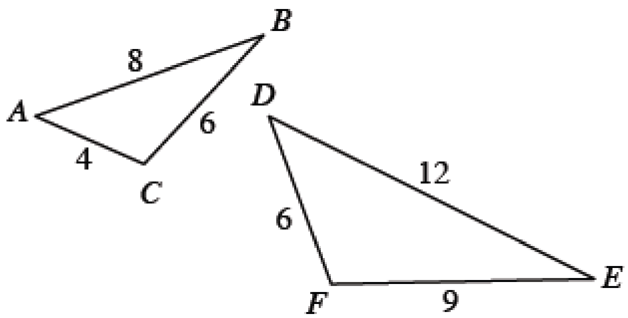
Triangle #1: side lengths 3, 4, and 5; Triangle #2: side lengths 6, 8, and 10 Explore with 3-84b #2 eTool (CPM).

3-85. SSS SIMILARITY?

Kendall observes that whenever she builds triangles with the same three lengths, the triangles always end up congruent.

a. Are two triangles with the same three side lengths always congruent?

b. Kendall now wants to figure out if three pairs of corresponding proportional side lengths (SSS ~) can be used to determine if triangles are similar. She decides to test triangles with side lengths 4, 6, and 8 units and 6, 9, and 12 units shown below. Explore using the [3-85b Student eTool](#) (CPM).



Start by drawing Δ*DEF* on your paper. Then mark the point *G* at the place 4 units away from point *D* on $\overline{DF}$. Then 8 units away from point *D* on $\overline{DE}$, mark the point *J*. Connect points *G* and *J*.

c. Your diagram looks like two triangles on top of each other. Draw the two triangles side by side and label the vertices on both triangles. Rename vertex *D* on the smaller triangle as vertex *H*.

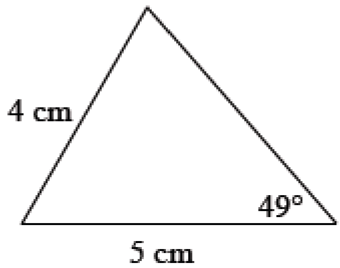
d. Now label the lengths of the sides of both triangles. Use the concepts of dilation and similarity to find $\overline{GJ}$. Justify your answer.

e. Knowing the dimensions of Δ*GHJ*, what can you conclude about Δ*DEF* and Δ*ABC?*

f. Can you extend this reasoning to other pairs of triangles with three pairs of proportional sides. Discuss this with your team and explain why or why not?

3-86. TESTING MORE SIMILARITY CONDITIONS

Cori's team put "SSA ~" on their list of possible triangle similarity conditions. To test their idea, Cori started by drawing the triangle below.



a. Use a dynamic geometry tool, 3-86 Student eTool (CPM), or straws and a protractor to investigate whether SSA ~ is a valid triangle similarity condition. If a triangle has two sides sharing a common ratio with Cori's, and has the same angle that is not in between those sides, must it be similar to Cori's triangle? In other words, can you create a triangle that is *not* similar to Cori's?

b. If you determine SSA ~ is not a valid similarity condition, cross it off your list!

c. Go through your list of possible triangle similarity conditions, crossing off all of the invalid ones and the ones that contain unnecessary information. How many valid triangle similarity conditions (without extra information) are there? List them.

3-87. LEARNING LOG

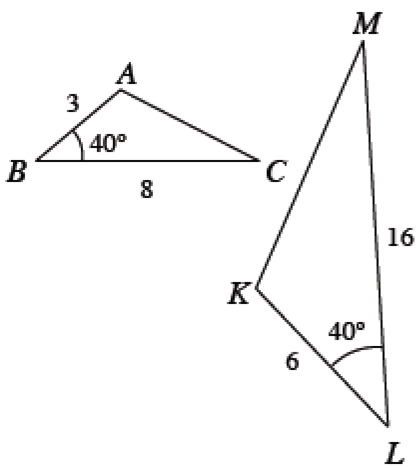
Reflect on what you have learned today. In your Learning Log, write down the triangle similarity conditions that help to determine if triangles are similar. You can write these conditions as conditional statements (in "If…, then…" form) or as arrow diagrams. Title this entry "Triangle Similarity Conditions" and include today's date.

3.2.5 Are the triangles similar?

Determining Similarity

You now have a complete list of the three triangle similarity conditions (AA ~, SAS ~, and SSS ~) that can be used to verify that two triangles are similar. Today you will continue to practice applying these conditions and using flowcharts to organize your reasoning.

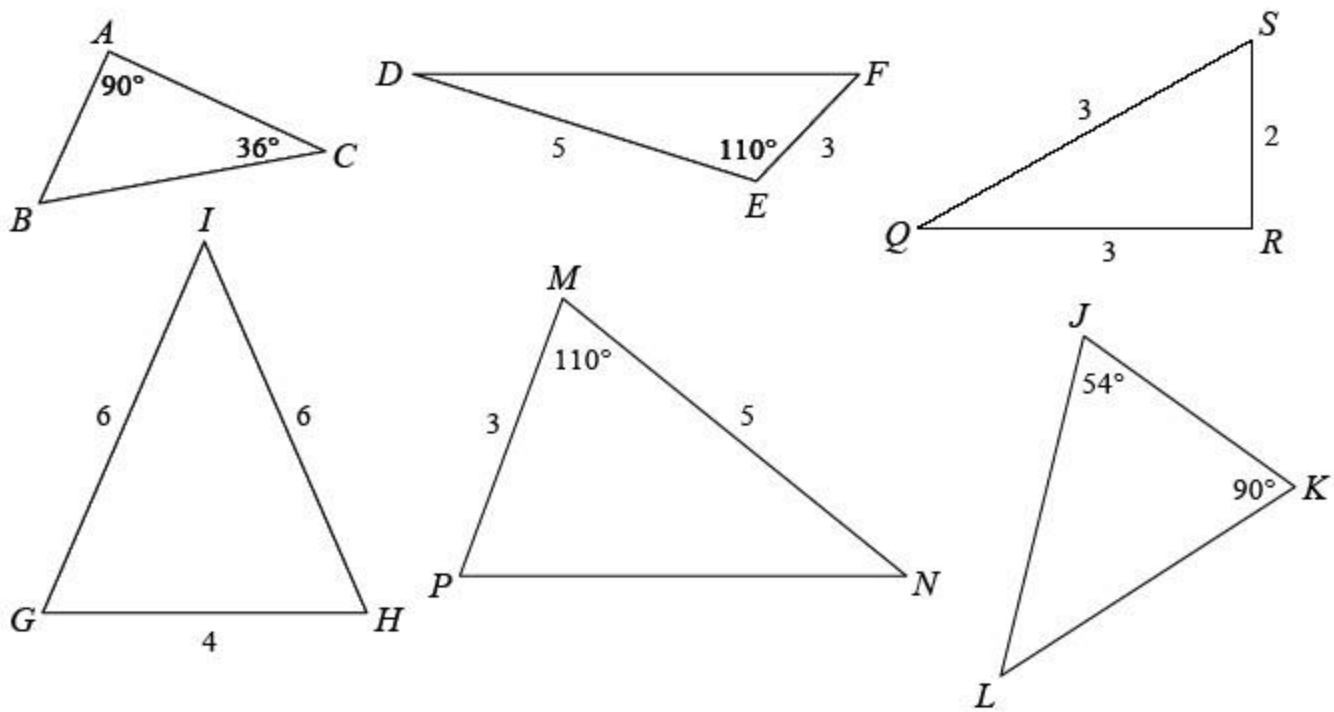
3-94. Lynn wants to show that the triangles below are similar.



a. What similarity condition should Lynn use?

b. Make a flowchart showing that these triangles are similar.

3-95. Below are six triangles, none of which is drawn to scale. Among the six triangles are three pairs of similar triangles. Identify the similar triangles, then for each pair make a flowchart justifying the similarity.

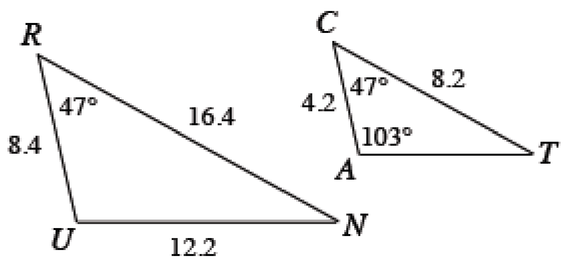


3-96. Revisit the similar triangles from problem 3-95.

a. Which pair of triangles are congruent? How do you know?

b. Suppose that in problem 3-95, AB = 3 cm, AC = 4 cm, and KJ = 12 cm. Find all the other side lengths in ΔABC and ΔJKL.

3-97. Examine the triangles below.



a. Are these triangles similar? If so, make a flowchart justifying their similarity.

b. Charles has $\Delta CAT \sim \Delta RUN$ as the conclusion of his flowchart. Leesa has $\Delta NRU \sim \Delta TCA$ as her conclusion. Who is correct? Why?

c. Are ΔCAT and ΔRUN congruent? Explain how you know.

d. Find all the missing side lengths and all the angle measures of ΔCAT and ΔRUN.

3-98. THE FAMILY FORTUNE, Part Two

In Lesson 1.1.4, you had to convince city officials that you were a relative of Molly "Ol' Granny" Marston, who had just passed away leaving a sizable inheritance. Below is the evidence you had available:

Family Portrait — a photo showing three young children. On the back you see the date 1968.

Newspaper Clipping — from 1972 titled "Triplets Make Music History." The first sentence catches your eye: "Jake, Judy, and Jeremiah Marston, all eight years old, were the first triplets ever to perform a six-handed piano piece at Carnegie Hall."

Jake Marston's Birth Certificate — showing that Jake was born in 1964, and identifying his parents as Phillip and Molly Marston.

Your Learner's Permit — signed by your father, Jeremiah Marston.

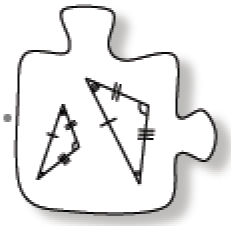
Wilbert Marston's Passport — issued when Wilbert was fifteen.

As their answer to this problem, one team wrote the following argument for the city officials:

> *The birth certificate shows that Jake Marston was Molly Marston's son. The newspaper clipping shows that Jeremiah Marston was Jake Marston's brother. Therefore, Jeremiah Marston was Molly Marston's son. The learner's permit shows that Jeremiah Marston is my father. Therefore, I am Molly Marston's grandchild.*

Your Task: Make a flowchart showing the reasoning in this team's argument. This flowchart will have more levels than the ones you have made in the past, because certain conclusions will be used as facts to support other conclusions. So plan carefully before you start to draw your chart.

3.2.6 What can I do with similar triangles?



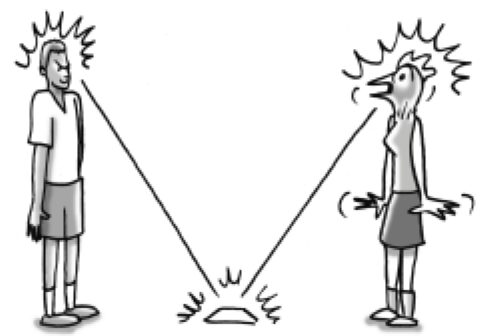
Applying Similarity

In previous lessons, you have learned methods for finding similar triangles. Once you find triangles are similar, how can that help you? Today you will apply similar triangles to analyze situations and solve new applications. As you work on today's problems, ask the following questions in your team:

What is the relationship?

Are any triangles similar? What similarity conjecture can we use?

3-105. YOU ARE GETTING SLEEPY…



Legend has it that if you stare into a person's eyes in a special way, you can hypnotize them into squawking like a chicken. Here's how it works.

Place a mirror on the floor. Your victim has to stand exactly 200 cm away from the mirror and stare into it. The only tricky part is that you need to figure out where you have to stand so that when you stare into the mirror, you are also staring into your victim's eyes.

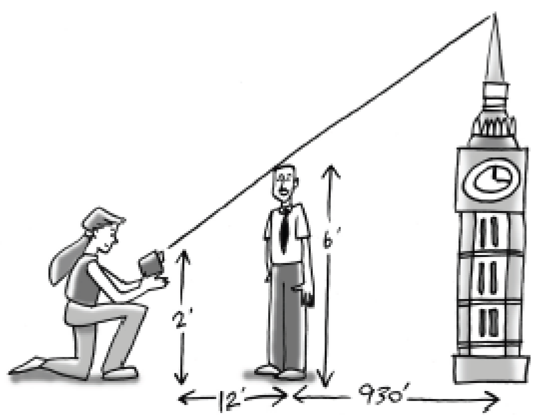
If your calculations are correct and you stand at the *exact* distance, your victim will squawk like a chicken!

a. Choose a member of your team to hypnotize. Before heading to the mirror, first analyze this situation. Draw a diagram showing you and your victim standing on opposite sides of a mirror. Measure the heights of both yourself and your victim (heights to the eyes, of course), and label all the lengths you can on the diagram. (Remember, your victim will need to stand 200 cm from the mirror.)

b. Are there similar triangles in your diagram? Justify your conclusion. (Hint: Remember what you know about how light reflects off mirrors.) Then calculate how far you will need to stand from the mirror to hypnotize your victim.

c. Now for the moment of truth! Have your teammate stand 200 cm away from the mirror, while you stand at your calculated distance from the mirror. Do you make eye contact? If not, check your measurements and calculations and try again.

3-106. LESSONS FROM ABROAD



Latoya was trying to take a picture of her family in front of the Big Ben clock tower in London. However, after she snapped the photo, she realized that the top of her father's head exactly blocked the top of the clock tower!

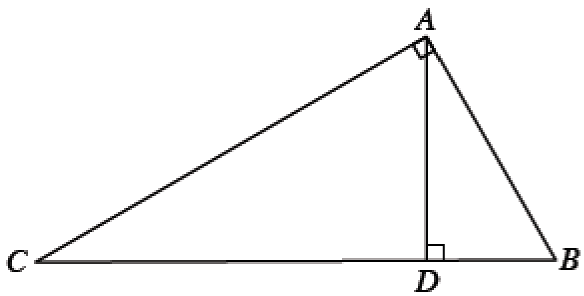
While disappointed with the picture, Latoya thought she might be able to estimate the height of the tower using her math knowledge. Since Latoya took the picture while kneeling, the camera was 2 feet above the ground. The camera was also 12 feet from her 6-foot tall father, and he was standing about 930 feet from the base of the tower.

a. Sketch the diagram above on your paper and locate as many triangles as you can. Can you find any triangles that must be similar? If so, explain how you know they are similar.

b. Use the similar triangles to determine the height of the Big Ben clock tower.

3-107. REVISITING THE PYTHAGOREAN THEOREM WITH SIMILARITY

In Lesson 2.3.2, you proved the Pythagorean Theorem using a proof involving the area of triangles and squares. Did you know that you can also prove the Pythagorean Theorem using triangle similarity? For example, in triangle *ABC* with right angle at *A*below, how can similarity be used to prove that $(AB)^2 + (AC)^2 = (BC)^2$? In this problem, you will support this proof by providing reasons for each part of the justification.



a. Find three different triangles in the diagram. Why are all of these triangles similar?

b. Carol argues that $\frac{DB}{AB} = \frac{AB}{CB}$ and $\frac{DC}{AC} = \frac{AC}{BC}$. Do you agree? Why or why not?

c. How can you use Carol's equations to show that $(AB)^2 = (CB)(DB)$ and $(AC)^2 = (BC)(DC)$?

d. Why does $(AB)^2 + (AC)^2 = (CB)(DB) + (BC)(DC)$?

e. Why does $(CB)(DB) + (BC)(DC) = BC(DB + DC) = (BC)^2$?

f. Finish the proof: Why does $(AB)^2 + (AC)^2 = (BC)^2$?