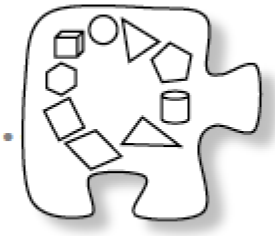
1.1.1 How can I design it?



Creating a Quilt Using Symmetry

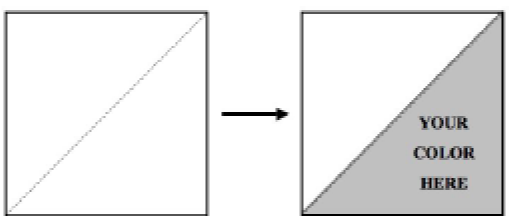
Welcome to Geometry! But what is geometry? At the end of this chapter you will have a better understanding of what geometry is. To start, you will focus on several activities that will hopefully challenge you and introduce you to important concepts in geometry that you will study in this course. While all of the problems are solvable with your current math skills, some will be revisited later in the course so that you may apply new geometric tools to solve and extend them.

Today you will consider an example of how geometry is applied in the world around you. A very popular American tradition is to create quilts by sewing together remnants of cloth in intricate geometric designs. These quilts often integrate geometric shapes in repeated patterns that show symmetry. For centuries, quilts have been designed to tell stories, document special occasions, or decorate homes.

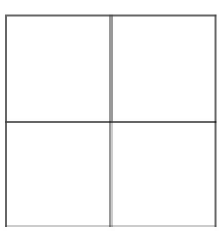
1-1. DESIGNING A QUILT, Part One

How can you use symmetry to design a quilt? Today you will work with your team to design a patch that will be combined with other team patches to make a class quilt. Before you start, review the Team Roles, which are outlined following this problem.

a. Each team member will receive four small squares. With a colored pencil or marker, shade in *half* of each square (one triangle) as shown below. Each team member should use a different color.



b. Next arrange your squares to make a larger 2-by-2 square (as shown below) with a design that has **reflection symmetry**. A design has reflection symmetry if it can be folded in half so that both sides match perfectly. Make sure that you have arranged your pieces into a different symmetrical pattern than the rest of your team.



c. Next, create a 4-by-4 square using the designs created by each team member as shown on the Lesson 1.1.1B Resource Page. Ask your teacher to verify that your designs are all symmetrical and unique. Then glue (or tape) all sixteen pieces carefully to the resource page and cut along the surrounding dashed square so that you have a blank border around your 4-by-4 square.

d. Finally, discuss with your team what all of you personally have in common. Come up with a team sentence that captures the most interesting facts. Write your names and this sentence in the border so they wrap around your 4-by-4 design.

To help you work together today, each member of your team has a specific job, assigned by your first name (or last name if team members have the same first name).

Team Roles

Resource Manager – If your name comes first alphabetically:

- Make sure the team has all of the necessary materials, such as colored pencils or markers and the Lesson 1.1.1A and 1.1.1B Resource Pages.
- Ask the teacher when the *entire* team has a question. You might ask, *"No one has an idea? Should I ask the teacher?"*
- Make sure your team cleans up by delegating tasks. You could say, *"I will put away the ________ while you _________ ."*

Facilitator – If your name comes second alphabetically:

- Start the team's discussion by asking, *"What are some possible designs?"* or *"How can we make sure that all of our designs are symmetrical?"* or *"Are all of our designs different?"*
- Make sure that all of the team members get any necessary help. You don't have to answer all the questions yourself. A good facilitator regularly asks, *"Do you understand what you are supposed to do?"* and *"Who can answer ________'s question?"*

Recorder/Reporter – If your name comes third alphabetically:

- Coordinate the taping or gluing of the quilt pieces together onto the resource page in the orientation everyone agreed to.
- Take notes for the team. The notes should include phrases like, *"We found that we all had in common …"* and explanations like, *"Each of our designs was found to be unique and symmetrical because …"*
- Help the team agree on a team sentence: *"What do we all have in common?"* and *"How can I write that on our quilt?"*

Task Manager – If your name comes fourth alphabetically:

- Remind the team to stay on task and not to talk to students in other teams. You can suggest, *"Let's try coming up with different symmetrical patterns."*
- Keep track of time. Give your team reminders, such as "*I think we need to decide now so that we will have enough time to …*"

1-2. DESIGNING A QUILT, Part Two

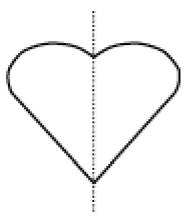
Your teacher will ask the Recorder/Reporters from each team to bring their finished quilt patches up to the board one at a time and tape them to the other patches. Be prepared to explain how you came up with your unique designs and interesting ideas about symmetry. Also be prepared to read your team sentence to the class. As you listen to the presentations, look for relationships between your designs and the other team designs.

METHODS AND MEANINGS

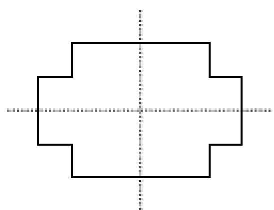
MATH NOTES

Lines of Symmetry

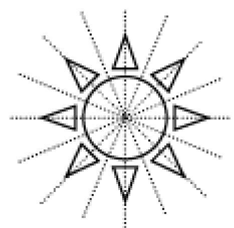
When a graph or picture can be folded so that both sides of the fold will perfectly match, it is said to have **reflective symmetry**. The line where the fold would be is called the **line of symmetry**. Some shapes have more than one line of symmetry. See the examples below.



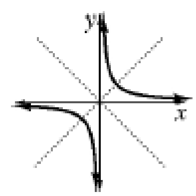
This shape has one line of symmetry.



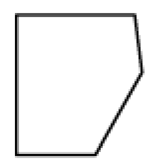
This shape has two lines of symmetry.



This shape has eight lines of symmetry.

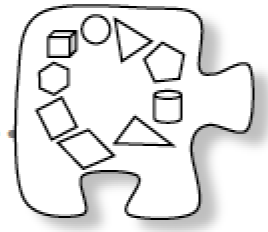


This graph has two lines of symmetry.



This shape has no lines of symmetry.

1.1.2 Can you predict the results?



Making Predictions and Investigating Results

Today you will investigate what happens when you change the attributes of a Möbius strip. As you investigate, you will record data in a table. You will then analyze this data and use your results to brainstorm further experiments. As you look back at your data, you may start to consider other related questions that can help you understand a pattern and learn more about what is happening. This way of thinking, called investigating, includes not only generating new questions, but also rethinking when the results are not what you expected.

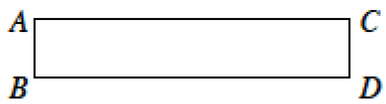
1-8. Working effectively with your study team will be an important part of the learning process throughout this course. Choose a member of your team to read aloud these Study Team Expectations:

STUDY TEAM EXPECTATIONS

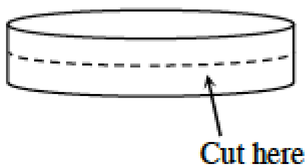
Throughout this course you will regularly work with a team of students. This collaboration will allow you to develop new ways of thinking about mathematics, increase your ability to communicate with others about math, and help you strengthen your understanding by having you explain your thinking to someone else. As you work together,

- You are expected to share your ideas and contribute to the team's work.
- You are expected to ask your teammates questions and to offer help to your teammates. Questions can move your team's thinking forward and help others to understand ideas more clearly.
- Remember that a team that functions well works on the same problem together and discusses the problem while it works.
- Remember that one student on the team should not dominate the discussion and thinking process.
- Your team should regularly stop and verify that everyone on the team agrees with a suggestion or a solution.
- Everyone on your team should be consulted before calling on the teacher to answer a question.

1-9. On a piece of paper provided by your teacher, make a “bracelet” by taping the two ends securely together. Putting tape on both sides of the bracelet will help to make sure the bracelet is secure. In the diagram of the rectangular strip shown below you would tape the ends together so that point *A* would attach to point *C*, and point *B* would attach to point *D*.



Now predict what you think would happen if you were to cut the bracelet down the middle, as shown in the diagram below. Record your prediction in a table like the one shown below or on your Lesson 1.1.2 Resource Page.

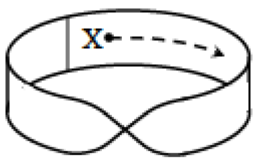


	Experiment	Prediction	Result
1-9	Cut bracelet in half as shown in the diagram.		
1-10			
1-11			
1-12a			
1-12b			
1-12c			
1-12d			

Now cut your strip as described above and record your result in the first row of your table. Make sure to include a short description of your result.

1-10. On a second strip of paper, label a point X in the center of the strip at least one inch away from one end.

Now turn this strip into a **Mobius strip** by attaching the ends together securely after making one twist. For the strip shown in the diagram at right, the paper would be twisted once so that point *A* would attach to point*D*. The result should look like the diagram below.



A Möbius Strip

Predict what would happen if you were to draw a line down the center of the strip from point X until you ran out of paper. Record your prediction, conduct the experiment, and record your result.

1-11. What do you think would happen if you were to cut your Möbius strip along the central line you drew in problem 1-10? Record your prediction in your table.

Cut just one of your team's Möbius strips. Record your result in your table. Consider the original strip of paper drawn in problem 1-9 to help you explain why cutting the Möbius strip had this result.

1-12. What else can you learn about Möbius strips? For each experiment below, first record your expectation. Then record your result in your table after conducting the experiment. Use a new Möbius strip for each experiment.

a. What if the result from problem 1-11 is cut in half down the middle again?

b. What would happen if the Möbius strip is cut one-third of the way from one of the sides of the strip? Be sure to cut a constant distance from the side of the strip.

c. What if a strip is formed by 2 twists instead of one? What would happen if it were cut down the middle?

d. If time allows, make up your own experiment. You might change how many twists you make, where you make your cuts, etc. Try to generalize your findings as you conduct your experiment. Be prepared to share your results with the class.

1-13. LEARNING REFLECTION

Think over how you and your study team worked today, and what you learned about Möbius strips. What questions did you or your teammates ask that helped move the team forward? What questions do you still have about Möbius strips? What would you like to know more about?

METHODS AND MEANINGS

MATH NOTES

The Investigative Process

The **investigative process** is a way to study and learn new mathematical ideas. Mathematicians have used this process for many years to make sense of new concepts and to broaden their understanding of older ideas.

In general, this process begins with a **question** that helps you frame what you are looking for. For example, a question such as, "*What if the Möbius strip has 2 half-twists? What will happen when that strip is cut in half down the middle?*" can help start an investigation to find out what happens when the Möbius strip is slightly altered.

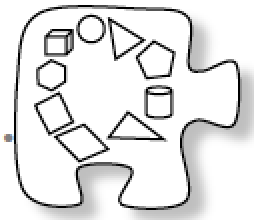
Once a question is asked, you can make an educated guess, called a **conjecture**. This is a mathematical statement that has not yet been proven.

Next, **exploration** begins. This part of the process may last awhile as you gather more information about the mathematical concept. For example, you may first have an idea about the diagonals of a rectangle, but as you draw and measure a rectangle on graph paper, you find out that your conjecture was incorrect.

When a conjecture seems to be true, the final step is to **prove** that the conjecture is always true. A proof is a convincing logical argument that uses definitions and previously proven conjectures in an organized sequence.

1.1.3 How can I predict the area?

Perimeters and Areas of Enlarging Tile Patterns



One of the core ideas of geometry is the measurement of shapes. Often in this course it will be important to find the areas and perimeters of shapes. How these measurements change as a shape is enlarged or reduced in size is especially interesting. Today your team will apply algebraic skills as you investigate the areas and perimeters of similar shapes.

1-19. CARPETMART

Your friend Alonzo has come to your team for help. His family owns a rug manufacturing company, which is famous for its unique and versatile designs. One of their most popular designs is shown below. Each rug design has an "original" size as well as enlargements that are exactly the same shape.

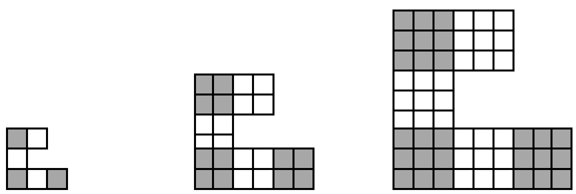


Figure #1: "The Original" Figure #2 Figure #3

Alonzo is excited because his family found out that the king of a far-away land is going to order an extremely large rug for one of his immense banquet halls. Unfortunately, the king is fickle and won't decide which rug he will order until the very last minute. The day before the banquet, the king will tell Alonzo which rug he wants and how big it will need to be. The king's palace is huge, so the rug will be VERY big!

Since the rugs are different sizes, and since each rug requires wool for the interior and fringe to wrap around the outside, Alonzo will need to quickly find the area and perimeter of each rug in order to obtain the correct quantities of wool and fringe.

Your Task: Your teacher will assign your team one of the rug designs to investigate (labeled (a) through (f) below). The "original" rug is shown in Figure 1, while Figures 2 and 3 are the next enlarged rugs of the series. With your team, create a table, graph, and equation for both the area and perimeter of your rug design. Then decide which representation will best help Alonzo find the area and perimeter for *any* figure number.

Be ready to share your analysis with the rest of the class. Your work must include the following:

- Diagrams for the rugs of the next two sizes (Figures 4 and 5) following the pattern shown in Figures 1, 2, and 3.
- A description of Figure 20. What will it look like? What are its area and perimeter?
- A table, graph, and equation representing the perimeter of your rug design.
- A table, graph, and equation representing the area of your rug design.

Rug Designs:

a.

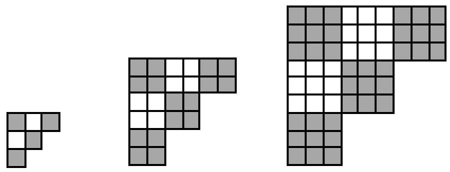


Figure 1 Figure 2 Figure 3

b.

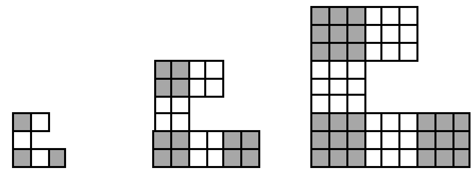


Figure 1 Figure 2 Figure 3

c.

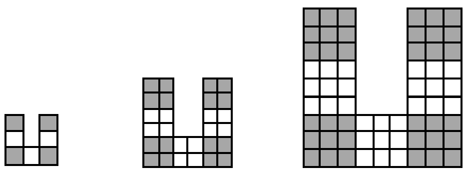


Figure 1 Figure 2 Figure 3

d.

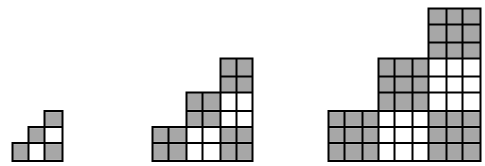


Figure 1 Figure 2 Figure 3

e.

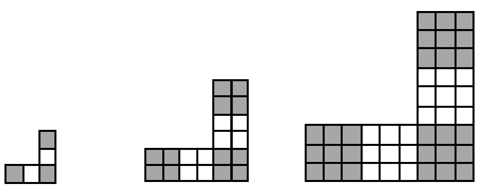


Figure 1 Figure 2 Figure 3

f.

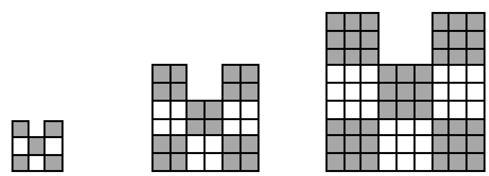


Figure 1 Figure 2 Figure 3

Further Guidance

1-20. To start problem 1-19, first analyze the pattern your team has been assigned on graph paper, draw diagrams of Figures 4 and 5 for your rug design. Remember to shade Figures 4 and 5 the same way Figures 1 through 3 are shaded.

1-21. Describe Figure 20 of your design. Give as much information as you can. What will it look like? How will the squares be arranged? How will it be shaded?

1-22. A table can help you learn more about how the perimeter changes as the rugs get bigger.

a. Organize your perimeter data in a table like the one shown below.

Figure number	1	2	3	4	5	20
Perimeter (in units)						

b. Graph the perimeter data for Figures 1 through 5. (You do not need to include Figure 20.) What shape is the graph?

c. How does the perimeter grow? Examine your table and graph and describe how the perimeter changes as the rugs get bigger.

d. Generalize the patterns you have found by writing an equation that will find the perimeter of any size rug in your design. That is, what is the perimeter of Figure *n*? Show how you got your answer.

1-23. Now analyze how the area changes with a table and graph.

a. Make a new table, like the one below, to organize information about the area of each rug in your design.

Figure number	1	2	3	4	5	20
Area (in square units)						

b. On a new set of axes, graph the area data for Figures 1 through 5. (You do not need to include Figure 20.) What shape is the graph?

c. How does the area grow? Does it grow the same way as the perimeter? Examine your table and graph and describe how the area changes as the rugs get bigger.

d. Write an equation that will find the area of Figure *n*. How did you find your equation? Be ready to share your strategy with the class.

Further Guidance section ends here.

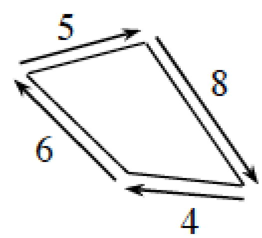
1-24. The King has arrived! He demands a Rug #100, which is Figure 100 in your design. What will its perimeter be? Its area? Justify your answer.

Methods and Meanings

Math Notes

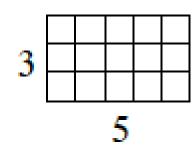
The Perimeter and Area of a Figure

The **perimeter** of a two-dimensional figure is the distance around its exterior (outside) on a flat surface. It is the total length of the boundary that encloses the interior (inside) region. See the example at right.



$$\text{Perimeter} = 5 + 8 + 4 + 6 = 23 \text{ units}$$

The **area** indicates the number of square units needed to fill up a region on a flat surface. For a rectangle, the area is computed by multiplying its length and width. The rectangle at right has a length of 5 units and a width of 3 units, so the area of the rectangle is 15 square units.



$$\text{Area} = 5 \cdot 3 = 15 \text{ square units}$$

1.1.4 Are you convinced?

Logical Arguments

"I don't have my homework today because…" Is your teacher going to be convinced? Will it make a difference whether you say that the dog ate your homework or whether you bring in a note from the doctor? Imagine your friend says, "I know that shape is a square because it has four right angles." Did your friend tell you enough to convince you?

Many jobs depend on your ability to convince other people that your ideas are correct. For instance, a defense lawyer must be able to form logical arguments to persuade the jury or judge that his or her client is innocent.

1-30. TRIAL OF THE CENTURY

The musical group Apple Core has accused your math teacher, Mr. Bosky, of stealing its newest pop CD, "Rotten Gala." According to the police, someone stole the CD from the BigCD Store last Saturday at some time between 6:00 p.m. and 7:00 p.m. Because your class is so well known for only reaching conclusions when sufficient evidence is presented, the judge has made you the jury! You are responsible for determining whether or not there is enough evidence to convict Mr. Bosky.

Carefully listen to the evidence that is presented. As each statement is read, decide:

- *Does the statement convince you? Why or why not?*
- *What could be changed or added to the statement to make it more convincing?*

Testimony

Mr. Bosky: "*But I don't like that CD! I wouldn't take it even if you paid me.*"

Mr. Bosky: "*I don't have the CD. Search me.*"

Mr. Bosky: "*I was at home having dinner Saturday.*"

Casey: "*There were several of us having dinner with Mr. Bosky at his house. He made us a wonderful lasagna.*"

Mrs. Thomas: "*All of us at dinner with Mr. Bosky left his house at 6:10 p.m.*"

Police Officer Yates: "*Driving as quickly as I could, it took me 30 minutes to go from Mr. Bosky's house to the BigCD store.*"

Coach Teller: "*Mr. Bosky made a wonderful goal right at the beginning of our soccer game, which started at 7:00 p.m. You can check the score in the local paper.*"

Police Officer Yates: "*I also drove from the BigCD store to the field where the soccer game was. It would take him at least 40 minutes to get there.*"

1-31. THE FAMILY FORTUNE

You are at home when the phone rings. It is a good friend of yours who says, "Hey, your last name is Marston. Any chance you have a grandmother named Molly Marston who was REALLY wealthy? Check out today's paper." You glance at the front page:

Family Fortune Unclaimed

City officials are amazed that the county's largest family fortune may go unclaimed. Molly "Ol' Granny" Marston died earlier this week and it appears that she was survived by no living relatives. According to her last will and testament, "Upon my death, my entire fortune is to be divided among my children and grandchildren." Family members have until noon tomorrow to come forward with a written statement giving evidence that they are related to Ms. Marston or the money will be turned over to the city.

You're amazed – Molly is your grandmother, so your friend is right! However, you may not be able to collect your inheritance unless you can convince city officials that you are a relative. You rush into your attic where you keep a trunk full of family memorabilia.

a. You find several items that you think might be important in an old trunk in the attic. With your team, decide which of the items listed below will help prove that Ol' Molly was your grandmother.

- **Family Portrait** − a photo showing three young children. On the back you see the date 1968.
- **Newspaper Clipping** − an article from 1972 titled "Triplets Make Music History." The first sentence catches your eye: "Jake, Judy, and Jeremiah Marston, all eight years old, were the first triplets ever to perform a six-handed piano piece at Carnegie Hall."
- **Jake Marston's Birth Certificate** − showing that Jake was born in 1964, and identifying his parents as Phillip and Molly Marston.
- **Your Learner's Permit** − signed by your father, Jeremiah Marston.
- **Wilbert Marston's Passport** − issued when Wilbert was fifteen.

b. Your team will now write a statement that will convince the city official (played by your faithful teacher!) that Ol' Molly was your grandmother. Be sure to support any claims that you make with appropriate evidence. Sometimes it pays to be convincing!

METHODS AND MEANINGS

MATH NOTES

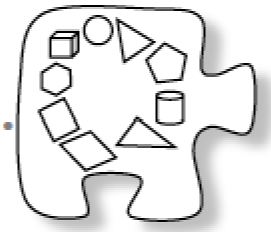
Solving Linear Equations

In Algebra, you learned how to solve a linear equation. This course will help you apply your algebra skills to solve geometric problems. Review how to solve equations by reading the example below.

Simplify. Combine like terms on each side of the equation whenever possible.	$3x - 2 + 4 = x - 6$	Combine like terms
	$3x + 2 = x - 6$ $-x = -x$	Subtract x on both sides
	$2x + 2 = -6$ $-2 = -2$	Subtract 2 on both sides
Keep equations balanced. The equal sign in an equation tells you that the expressions on the left and right are balanced. Anything done to the equation must keep that balance.	$\frac{2x}{2} = \frac{-8}{2}$	Divide both sides by 2
Move your *x*-terms to one side of the equation. Isolate all variables on one side of the equation and the constants on the other.	$x = -4$	
Undo operations. Use the fact that addition is the opposite of subtraction and that multiplication is the opposite of division to solve for *x*. For example, in the equation $2x = -8$, since the 2 and the *x* are multiplied, then dividing both sides by 2 will get *x* alone.		

1.1.5 What shapes can you find?

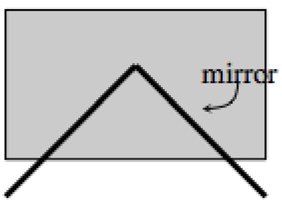
Building a Kaleidoscope



Today you will learn about angles and shapes as you study how a kaleidoscope works.

1-37. BUILDING A KALEIDOSCOPE

How does a kaleidoscope create the complicated, colorful images you see when you look inside? A hinged mirror and a piece of colored paper can demonstrate how a simple kaleidoscope creates its beautiful repeating designs.



Your Task: Place a hinged mirror on a piece of colored, unlined paper so that its sides extend beyond the edge of the paper as shown at right. Explore what shapes you see when you look directly at the mirror, and how those shapes change when you change the angle of the mirror. Discuss the questions below with your team. Be ready to share your responses with the rest of the class. View these two videos The Power of X (YouTube) and Behind the Power of X (YouTube).

Discussion Points

- What do you notice?
- What happens when you change the angle (opening) formed by the sides of the mirror?
- How can you describe the shapes you see in the mirror?

1-38. To complete your exploration, answer these questions together as a team.

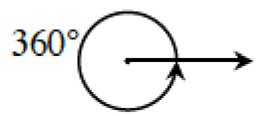
a. What happens to the shape you see as the angle formed by the mirror gets bigger (wider)? What happens as the angle gets smaller?

b. What is the smallest number of sides the shape you see in the mirror can have? What is the largest?

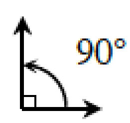
c. With your team, find a way to form a **regular hexagon** (a shape with six equal sides and equal angles).

d. How might you describe to another team how you set the mirrors to form a hexagon? What types of information would be useful to have?

1-39. A good way to describe an angle is by measuring how *wide* or *spread apart* the angle is. For this course, you can think of the **measure of an angle** as the measure of rotation of the two sides of the mirror from a closed position. The largest angle you can represent with a hinged mirror is 360°. This is formed when you open a mirror all the way so that the backs of the mirror touch. This is a called a **circular angle** and is represented by the diagram below.

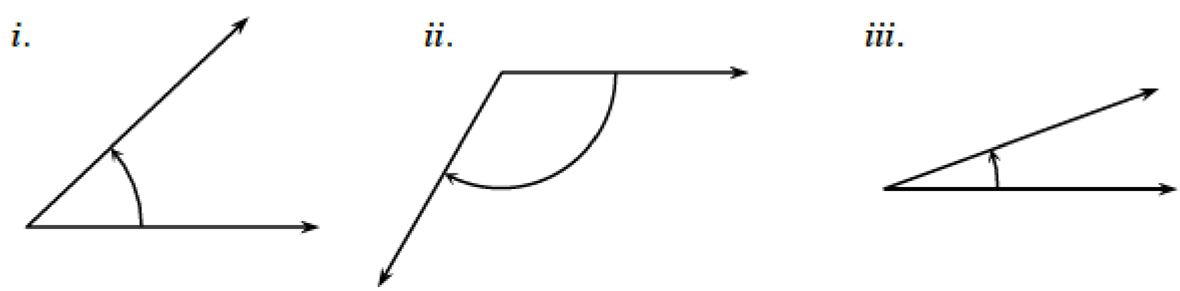


a. Other angles may be familiar to you. For example, an angle that forms a perfect "L" or a quarter turn is a 90° angle, called a **right angle** (shown at right). Four right angles can together form a circular angle.

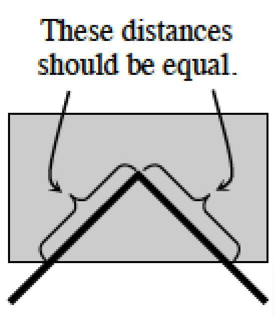


What if the two mirrors are opened to form a straight line? What measure would that angle have? Draw this angle and label its degrees. How is this angle related to a circular angle?

b. Based on the examples above, estimate the measures of the angles shown below. Then confirm your answer using a **protractor**, a tool that measures angles.



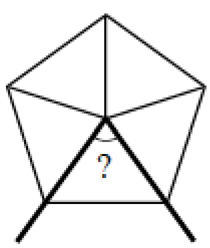
1-40. Now use your understanding of angle measurement to create some specific shapes using your hinged mirror. Be sure that both mirrors have the same length on the paper, as shown in the diagram below.



a. Antonio says he can form an **equilateral triangle** (a triangle with three equal sides and three equal angles) using his hinged mirror. How did he do this? Once you can see the triangle in your mirror, place the protractor on top of the mirror. What is the measure of the angle formed by the sides of the mirror?

b. Use your protractor to set your mirror so that the angle formed is 90°. Be sure that the sides of the mirror intersect the edge of the paper at equal lengths. What is this shape called? Draw and label a picture of the shape on your paper.

c. Carmen's mirror shows the image below, called a **regular pentagon**. She noticed that the five triangles in this design all meet at the hinge of her mirrors. She also noticed that the triangles must all be the same size and shape, because they are reflections of the triangle formed by the mirrors and the paper.



What must the sum of these five angles at the hinge be? And what is the angle formed by Carmen's mirrors? Test your conclusion with your mirror.

d. Discuss with your team and predict how many sides a shape would have if the angle that the mirror forms measures 40°. Explain how you made your prediction. Then check your prediction using the mirror and a protractor. Describe the shape you see with as much detail as possible.

1-41. Reflect on what you learned during today's activity.

a. Based on this activity, what are some things that you think you will be studying in Geometry?

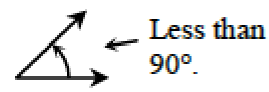
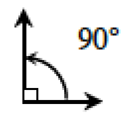
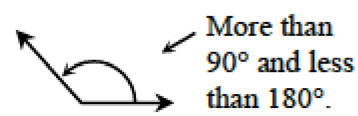
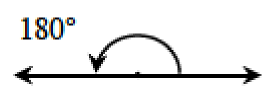
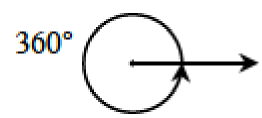
b. This activity was based on the question, *"What shapes can be created using reflections?"* What ideas from this activity would you want to learn more about? Write a question that could prompt a different, but related, future investigation.

Methods and Meanings

Math Notes

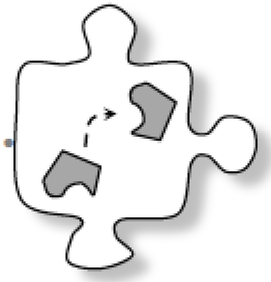
Types of Angles

When trying to describe shapes, it is convenient to classify types of angles. An angle is formed by two rays joined at a common endpoint. The measure of an angle represents the number of degrees of rotation from one ray to the other about the vertex. This course will use the following terms to refer to angles:

ACUTE:	Any angle with measure *between* (but not including) 0°and 90°.	
RIGHT:	Any angle that measures 90˚.	
OBTUSE:	Any angle with measure *between* (but not including) 90°and 180°.	
STRAIGHT:	Straight angles have a measure of 180° and are formed when the sides of the angle form a straight line.	
CIRCULAR:	Any angle that measures 360°.	

1.2.1 How do you see it?

Spatial Visualization and Reflections



Were you surprised when you looked into the hinged mirror during the Kaleidoscope Investigation of Lesson 1.1.5? Reflection can create many beautiful and interesting shapes and can help you learn more about the characteristics of other shapes. However, one reason you may have been surprised is because it is sometimes difficult to predict what a reflection will be. This is where spatial visualization plays an important role. Visualizing, the act of "picturing" something in your mind, is often helpful when working with shapes. In order to be able to investigate and describe a geometric concept, it is first useful to visualize a shape or action.

Today you will be visualizing in a variety of ways and will develop the ability to find reflections. As you work today, keep the following focus questions in mind:

How do I see it?

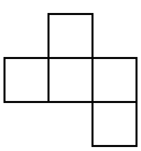
How can I verify my answer?

How can I describe it?

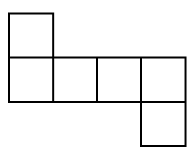
1-47. BUILDING BOXES

Which of the nets (diagrams) below would form a box with a lid if folded along the interior lines? Be prepared to defend your answer. After your team has discussed how each would fold, explore using *3D Nets* (CPM). Note: all of the nets are in the same eTool. Click the 'Gear' in the upper left corner to access additional nets.

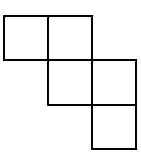
a.



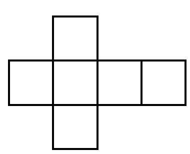
b.



c.



d.



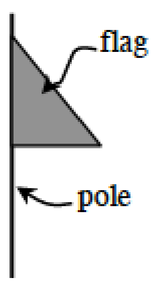
1-48. Have you ever noticed what happens when you look in a mirror? Have you ever tried to read words while looking in a mirror? What happens? Discuss this with your team. Then re-write the following words as they would look if you held this book up to a mirror. Do you notice anything interesting?

a. GEO

b. STAR

c. WOW

1-49. When Kenji spun the flag shown below very quickly about its pole, he noticed a three-dimensional shape emerge.

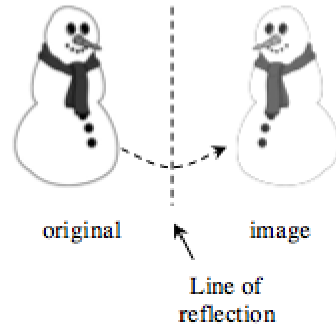


a. What shape did he see? Draw a picture of the three-dimensional shape on your paper and be prepared to defend your answer.

b. What would the flag need to look like so that a **sphere** (the shape of a basketball) is formed when the flag is rotated about its pole? Draw an example.

1-50. REFLECTIONS

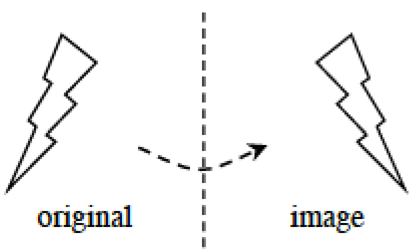
The shapes created in the Kaleidoscope Investigation in Lesson 1.1.5 were the result of reflecting a triangle several times in a hinged mirror. However, other shapes can also be created by a reflection. For example, the diagram at right shows the result of reflecting a snowman across a line.



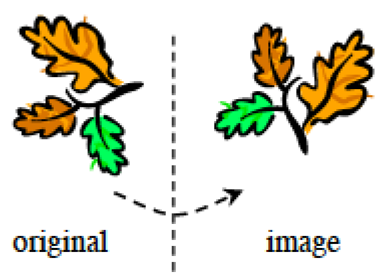
a. Why do you think the image is called a reflection? How is the image different from the original?

b. On the Lesson 1.2.1 Resource Page provided by your teacher, use your visualization skills to predict the reflection of each figure across the given line of reflection. Then draw the reflection. Check your work by folding the paper along the line of reflection.

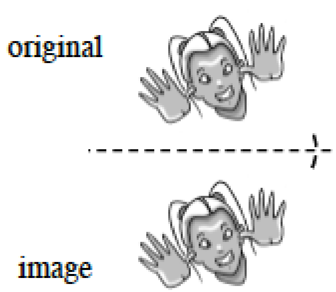
1-51. Sometimes, a motion appears to be a reflection when it really isn't. How can you tell if a motion is a reflection? Consider each pair of objects below. Which diagrams represent reflections across the given lines of reflection? Study each situation carefully and be ready to explain your thinking.



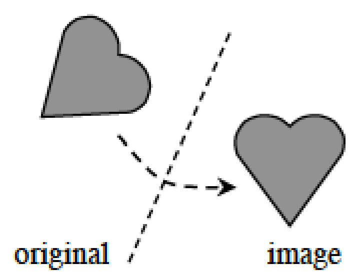
(1)



(2)



(3)



(4)

1-52. CONNECTIONS WITH ALGEBRA

What other ways can you use reflections? Consider how to reflect a graph as you answer the questions below.

a. On your Lesson 1.2.1 Resource Page, graph the parabola $y = x^2 + 3$ and the line $y = x$ for $x = -3, -2, -1, 0, 1, 2, 3$ on the same set of axes.

b. Now reflect the parabola over the line $y = x$. What do you observe? What happens to the x- and y-values of the original parabola?

1-53. LEARNING LOG

Throughout this course, you will be asked to reflect on your understanding of mathematical concepts in a Learning Log. Your Learning Log will contain explanations and examples to help you remember what you have learned throughout the course. It is important to write each entry of the Learning Log in your own words so that later you can use your Learning Log as a resource to refresh your memory. Your teacher will tell you where to write your Learning Log entries and how to structure or label them. Remember to label each entry with a title and a date so that it can be referred to later.

In this first Learning Log entry, describe what you learned today. For example, is it possible to reflect any shape? Is it possible to have a shape that, when reflected, doesn't change? How does reflection work? If it helps you to explain, sketch and label pictures to illustrate what you write. Title this entry "Reflections" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Event: Any outcome, or set of outcomes, from a probabilistic situation. A **successful event** is the set of all outcomes that are of interest in a given situation. For example, rolling a die is a probabilistic situation. Rolling a 5 is an event. If you win a prize for rolling an even number, you can consider the set of three outcomes {2, 4, 6} a successful event.

Sample space: All possible outcomes from a probabilistic situation. For example, the sample space for flipping a coin is heads and tails; rolling a die has a sample space of {1, 2, 3, 4, 5, 6}.

Probability: The likelihood that an event will occur. Probabilities may be written as ratios (fractions), decimals, or percents. An event that is certain to happen has a probability of 1, or 100%. An event that has no chance of happening has a probability of 0, or 0%. Events that "might happen" have probabilities between 0 and 1, or between 0% and 100%. The more likely an event is to happen, the greater its probability.

Experimental probability: The probability based on data collected in experiments.

$$\text{Experimental probability} = \frac{\text{number of successful outcomes in the experiment}}{\text{total number of outcomes in the experiment}}$$

Theoretical probability: Probability that is mathematically calculated. When each of the outcomes in the sample space has an *equally likely chance* of occurring, then

$$\text{Theoretical probability} = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$$

For example, to calculate the probability of rolling an even number on a die, first figure out how many possible (equally likely) outcomes there are. Since there are six faces on the number cube, the total number of possible outcomes is 6. Of the six faces, three of the faces are even numbers—there are three successful outcomes. Thus, to find the probability of rolling an even number, you would write:

$$P(\text{even}) = \frac{\text{number of ways to roll an even number}}{\text{number of faces on a number cube}} = \frac{3}{6} = 0.5 = 50\%$$

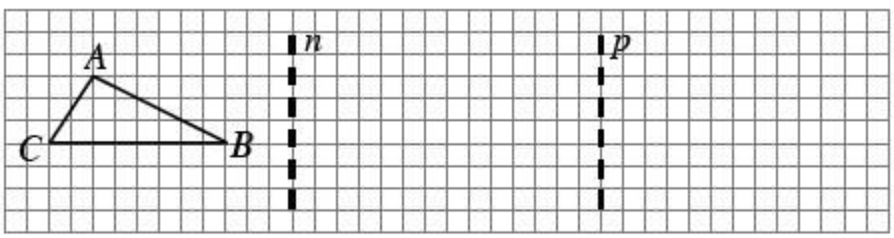
1.2.2 What if it is reflected more than once?

Rigid Transformations: Rotations and Translations

In Lesson 1.2.1, you learned how to change a shape by reflecting it across a line, like the ice cream cones shown at right. Today you will learn more about reflections and learn about two new types of transformations: rotations and translations.

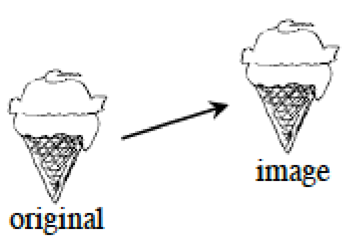
1-59. As Amanda was finding reflections, she wondered, "*What if I reflect a shape twice over parallel lines?*" Investigate her question as you answer the questions below.

a. On the Lesson 1.2.2 Resource Page provided by your teacher or using the 1-59a Student eTool (Desmos), find ΔABC and lines n and p (shown below). What happens when ΔABC is reflected across line n to form $\Delta A'B'C'$ and then $\Delta A'B'C'$ is reflected across line p to form $\Delta A''B''C''$? First visualize the reflections and then test your idea of the result by drawing both reflections.



b. Examine your result from part (a). Compare the original triangle ΔABC with the final result, $\Delta A''B''C''$. What single motion would change ΔABC to $\Delta A''B''C''$?

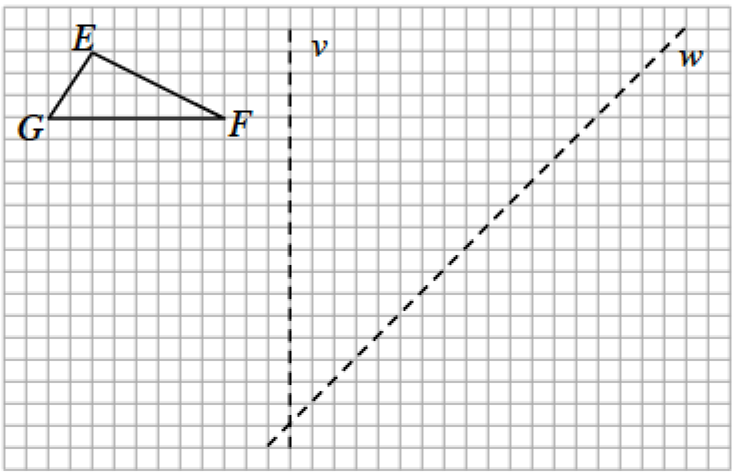
c. Amanda analyzed her results from part (a). "*It looks like I could have just slid ΔABC over!*" Sliding a shape from its original position to a new position is called **translating**. For example, the ice cream cone at right has been translated. Notice that the image of the ice cream cone has the same *orientation* as the original (that is, it is not turned or flipped). What words can you use to describe a translation?



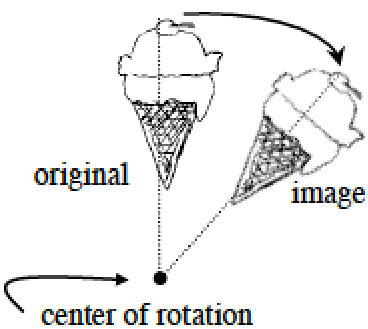
d. The words "transformation" and "translation" sound alike and can easily be confused. Discuss in your team what these words mean and how they are related to each other.

1-60. After answering Amanda's question in problem 1-59, her teammate asks, *"What if the lines of reflection are not parallel? Is the result still a translation?"* Find Δ*EFG* and lines *v* and *w* on the Lesson 1.2.2 Resource Page or using the 1-60a Student eTool (Desmos) and the 1-60c Student eTool(Desmos).

a. First visualize the result when Δ*EFG* is reflected over *v* to form Δ*E′F′G′*, and then Δ*E′F′G′* is reflected over *w* to form Δ*E″F″G″*. Then draw the resulting reflections on the resource page. Is the final image a translation of the original triangle? If not, describe the result.



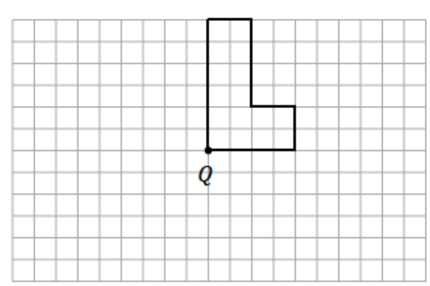
b. Amanda noticed that when the reflecting lines are not parallel, the original shape is rotated, or turned, to get the new image. For example, the diagram at right shows the result when an ice cream cone is rotated about a point.



In part (a), the center of rotation is at point *P,* the point of intersection of the lines of reflection. Use a piece of tracing paper to test that Δ*E″F″G″* can be obtained by rotating Δ*EFG* about point *P.* To do this, first trace Δ*EFG* and point *P* on the tracing paper. While the tracing paper and resource page are aligned, apply pressure on *P* so that the tracing paper can rotate about this point. Then turn your tracing paper until Δ*EFG* rests atop Δ*E″F″G″*.

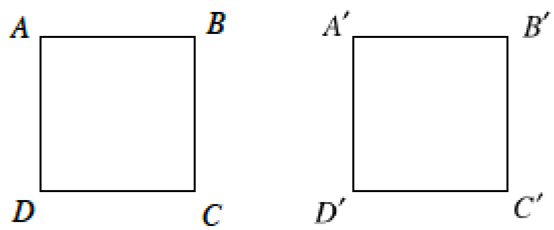
c. The rotation of Δ*EFG* in part (a) is an example of a 90˚ clockwise rotation. The term "clockwise" refers to a rotation that follows the direction of the hands of a clock, namely ↻. A rotation in the opposite direction ↺ is called "counter-clockwise."

On your resource page, rotate the "L" 90° counter-clockwise (↺) about point *Q*.



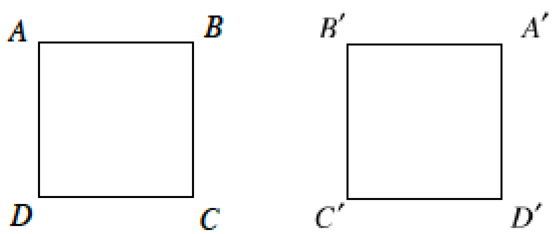
1-61. NOTATION FOR TRANSFORMATIONS

Notice that the figure labels can help you recognize what transformations are involved. For example, in the diagram at right, the original square *ABCD* on the left was *translated* to the image square on the right. The image location is different from the original, so different letters are used to label its vertices (corners).

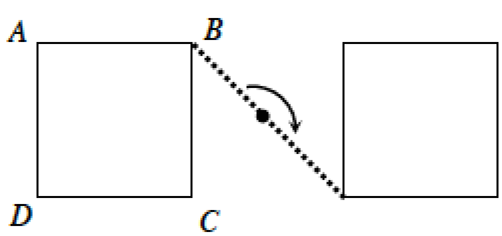


To keep track of how the vertices correspond, we call the image *A′B′C′D′*. The ′ symbol is read as "prime," so the shape on the right is called, "A prime *B* prime *C* prime *D* prime." *A′* is the image of A, *B′* is the image of *B,* etc. This notation tells you which vertices correspond.

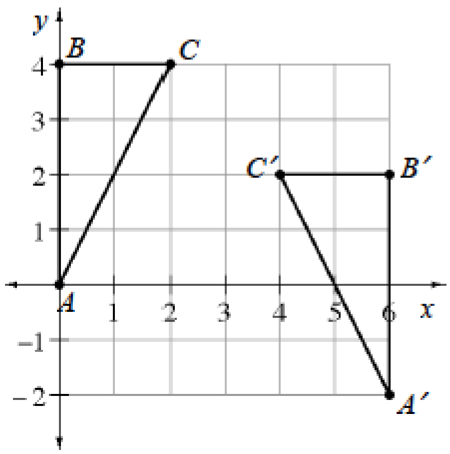
a. The diagram below shows a different transformation of *ABCD*. Look carefully at the correspondence between the vertices. Can you rotate or reflect the original square to make the letters correspond as shown? If you can reflect, where would the line of reflection be? If you can rotate, where would the point of rotation be?



b. This time, *ABCD* is rotated 180° about the point as shown. Copy the diagram (both squares and the point) and label the vertices of the image square below. If you have trouble, ask your teacher for tracing paper.



1-62. What if you have the original figure and its image after a sequence of transformations? Examine ΔABC and $\Delta A'B'C'$ in the graph below.



a. With your team, describe at least two different ways to move ΔABC onto $\Delta A'B'C'$.

b. Are there always multiple ways to describe any transformation that does not change the shape or size of the figure? Discuss this question with your team and be prepared to share your reasons with the class.

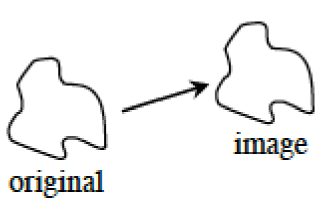
METHODS AND MEANINGS

MATH NOTES

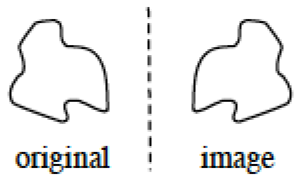
Rigid Transformations

A **rigid transformation** maps each point of a figure to a new point, so that the resulting image has the same size and shape of the original. There are three types of rigid transformations, described below.

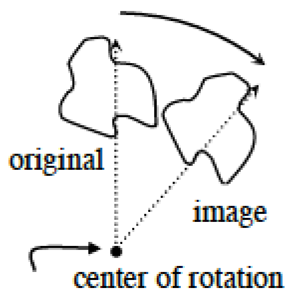
A transformation that preserves the size, shape, and orientation of a figure while *sliding* it to a new location is called a **translation**.



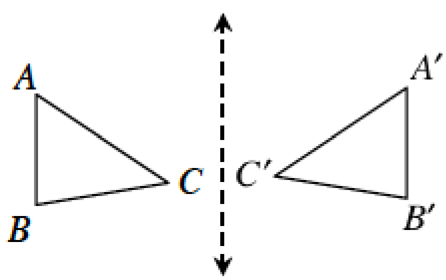
A transformation that preserves the size and shape of a figure across a line to form a mirror image is called a **reflection**. The mirror line is a **line of reflection**. One way to find a reflection is to *flip* a figure over the line of reflection.



A transformation that preserves the size and shape while *turning* an entire figure about a fixed point is called a **rotation**. Figures can be rotated either clockwise (↻) or counterclockwise (↺).



When labeling a transformation, the new figure (image) is often labeled with **prime notation.** For example, if ΔABC is reflected across the vertical dashed line, its image can be labeled $\Delta A'B'C'$ to show exactly how the new points correspond to the points in the original shape. We also say that ΔABC is **mapped** to $\Delta A'B'C'$.

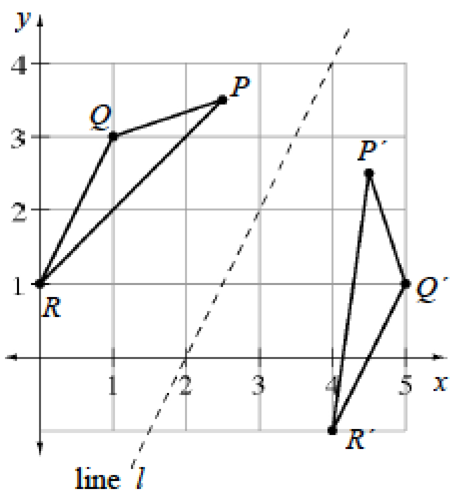


1.2.3 What is the relationship?

Slopes of Parallel and Perpendicular Lines

In Lesson 1.2.2, you learned how to label vertices in an image to show how those vertices correspond to vertices in the original figure. Today, you will learn about relationships between an object and its image that will help you to predict the image's position. These relationships are described using algebra and are just one example of a connection between the new concepts you are studying in geometry and the math you learned in previous courses.

1-68. Below, ΔPQR was reflected across line *l* to form $\Delta P'Q'R'$. Describe the relationship of the original triangle and its image to the line of reflection. Specifically, how far away is each triangle from the line of reflection? What do you notice about the location of the image relative to the angle of reflection and the original triangle?



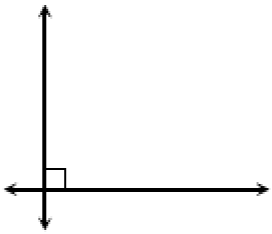
1-69. CONNECTIONS WITH ALGEBRA

In problem 1-68, you made some observations about reflections. Slope can help reveal more about transformations such as reflections.

a. Begin by graphing the equation $y = \frac{3}{5}x - 4$. Use tracing paper to translate the graph of $y = \frac{3}{5}x - 4$ up 5 units. Write the equation of the resulting image. What is the relationship between $y = \frac{3}{5}x - 4$ and its image? How do their slopes compare?

b. Now use tracing paper to rotate $y = \frac{3}{5}x - 4$ 90° clockwise (↻) about (0, 0). Write the equation of the result. Describe the relationship between $y = \frac{3}{5}x - 4$ and this new image.

c. The original line and the rotated line in part (b) are **perpendicular** because they form a 90° angle where they intersect. Look at the slopes of the original line and the rotated line and make any observations you can about the relationship between the slopes.



Perpendicular lines form a right angle.

1-70. INVESTIGATING SLOPES

Do you think that all perpendicular slopes are related in the way you observed in problem 1-69? Investigate this idea by drawing three different lines with slope triangles on graph paper. Use slopes $1, \frac{1}{3}$, and $\frac{-3}{4}$.

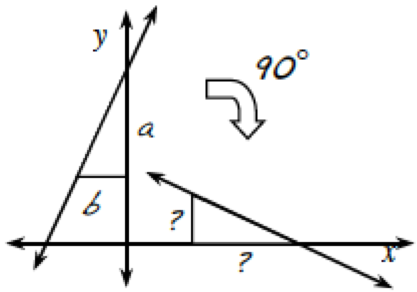
a. Use tracing paper to rotate each line with its slope triangle 90°, either counterclockwise (↺) or clockwise (↻). Find the slope of each new line.

b. How does the slope of each rotated line compare to the slope of its original line? Share any patterns you find with your teammates.

c. Use patterns from the work you have done to describe the general relationship of the slopes of perpendicular lines. That is, if you have two perpendicular lines and know the slope of one, how can you find the slope of the other?

1-71. SLOPES OF PERPENDICULAR LINES

Two lines are perpendicular whenever one line can be rotated 90° onto the other. However, a rotation does not only move the points on the line – it moves all the points on the graph! Therefore, the relationship of the slopes of perpendicular lines can be demonstrated as true by rotating a non-special line along with its slope triangle.

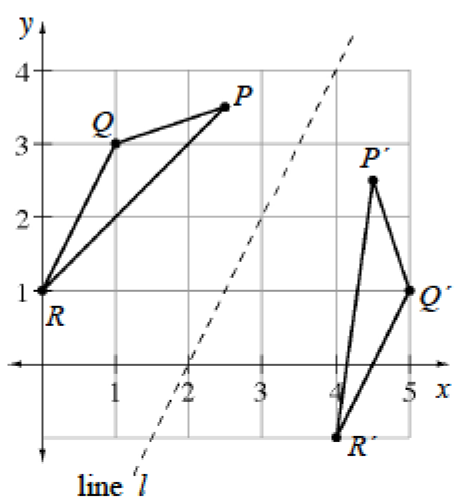


However, to prove something mathematically, you must be able to explain why it is true in all cases, and not just for particular numbers. To prove the relationship between perpendicular slopes, Sabrina drew the picture above.

a. Use Sabrina's drawing to explain why the slope of the perpendicular line must always be $\frac{-b}{a}$ if neither a nor b is zero.

b. What if the original line has a slope of 0? Explain what happens to a line with slope of 0 if it is rotated 90°, and what the slope of the perpendicular line would be.

1-72. Now that you know more about the slope of parallel and perpendicular lines, revisit the reflection from problem 1-68 and confirm the relationships using slope.



a. Graph the triangles onto graph paper so that $P(2.5, 3.5)$, $Q\,(1, 3)$, $R\,(0, 1)$ and $P'\,(4.5, 2.5)$, $Q'\,(5, 1)$, $R'\,(4, -1)$. Then use your ruler to draw three dashed lines: $\overleftrightarrow{PP'}$, $\overleftrightarrow{QQ'}$, and $\overleftrightarrow{RR'}$. What is the relationship of these dashed lines? Use your knowledge of slope to verify your observations.

b. Now focus on the relationship between the line of reflection and each of the segments connecting a point with its image. What do you notice about the lengths and angles? Be as specific as you can. Use what you know about reflections to explain why your observations must be true.

c. Use slope to confirm that the line of reflection is perpendicular to the line segments connecting each original point and its image.

1-73. Evan has graphed the point (5, 7) and he wants to reflect it over the line $y = \frac{-2}{5}x + 6$. He predicts that the reflected point will have coordinates (2, 2). *Without graphing*, can you confirm his answer or show that he cannot be correct?

1-74. EXTENSION

Suppose the equation for line A is $y = \frac{6}{5}x - 10$. Line A is parallel to line B, which is perpendicular to line C. If line D is perpendicular to line C and perpendicular to line E, what is the slope of line E? Justify your conclusion.

1-75. LEARNING LOG

In your Learning Log, summarize what you have learned today. Be sure to explain the relationship between the slopes of perpendicular lines and describe how to get the slope of one line when you know the slope of a line perpendicular to it. Title this entry "**Slopes of Perpendicular Lines**" and include today's date.

1.2.4 How can I move it?

Defining Transformations


Photo courtesy of the artist.
Sue Sales, *Hearts*.

In Lesson 1.1.1, your class made a quilt using designs based on a geometric shape. Similarly, throughout American history, quilters have created quilts that use transformations to create intricate geometric designs. For example, the quilt at right is an example of a design based on rotation and reflection, while the quilt at left contains translation, rotation, and reflection.

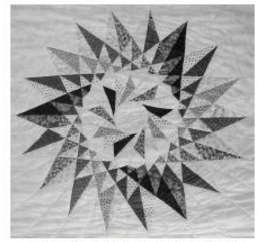


Photo courtesy of the artist.
Sue Sales. *Balance From Within*.

In Lesson 1.2.3, you found ways to locate the image of a shape after it is reflected. Today, you will work with your team to develop ways to describe the image of a shape after it is rotated or translated.

1-81. ROTATIONS ON A GRID

Consider what you know about rotation, a motion that turns a shape about a point. Does it make any difference if a rotation is clockwise (↻) versus counterclockwise (↺)? If so, when does it matter? Are there any circumstances when it does not matter? And are there any situations when the rotated image lies exactly on the original shape?

Investigate these questions as you rotate the shapes below about the given point on the [Lesson 1.2.4 Resource Page](). Use tracing paper if needed. Be prepared to share your answers to the questions posed above.

1-82. So what exactly is a rotation? If a figure is rotated, how can you describe it? Investigate this question below.

a. On graph paper, graph $\overline{AB}$ with coordinates $A(4, 1)$ and $B(2, 5)$. Then use tracing paper to rotate $\overline{AB}$ 90° counterclockwise (↺) about the point $O(0, 0)$ to graph $\overline{A'B'}$.

b. Compare the lengths of and angles formed by the line segments that connect points A, B, A' and B' to the point of rotation O. Which lengths are equal? Which angle measures are equal? Be specific. To help, you may want to draw the line segments connecting each point with O.

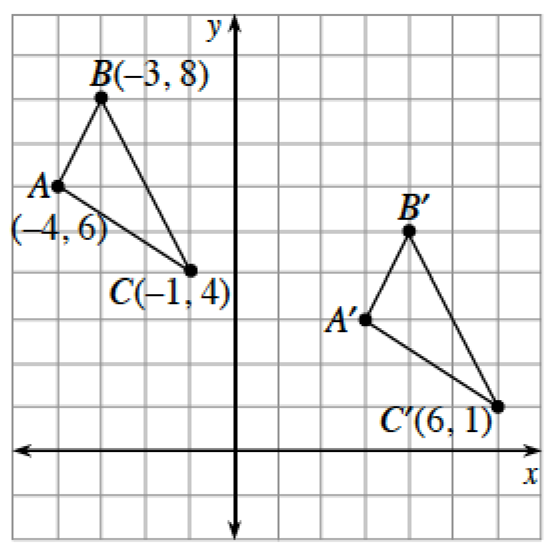
c. Why does it make sense that for all points P in the graph, a rotation about a point O moves it to a new point P' so that $OP = OP'$ and $m\angle POP'$ equals the measure of rotation? Use tracing paper to make sense of this relationship.

d. Use tracing paper to help explain why a rotation does not change any angles or lengths of a figure.

1-83. TRANSLATIONS ON A GRID

So what is a translation? The formal name for a slide is a translation. (Remember that translation and transformation are different words.) $\Delta A'B'C'$ below is the result of translating ΔABC.

a. Describe the translation. That is, how many units to the right and how many units down does the translation move the triangle?



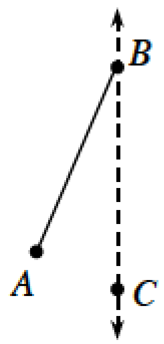
b. On graph paper, plot ΔEFG with coordinates $E(4, 2)$, $F(1, 7)$, and $G(2, 0)$. Find the coordinates of $\Delta E'F'G'$ if $\Delta E'F'G'$ is translated the same way as ΔABC was in part (a).

c. For the translated triangle in part (b), draw a line segment connecting each vertex to its translated image. What do you notice these line segments? What does this tell you about how a translation moves each point of the graph?

d. Use the tracing paper to help explain how you know that a translation does not change any angles or lengths of a figure.

1-84. FACTS ABOUT ISOSCELES TRIANGLES

How can transformations such as reflections help us to learn more about familiar shapes? Consider reflecting a **line segment** (the portion of a line between two points) across a line that passes through one of its endpoints. An example of this would be reflecting $\overline{AB}$ across $\overleftrightarrow{BC}$ in the picture below.



a. Copy $\overline{AB}$ and $\overleftrightarrow{BC}$ and draw $\overline{A'B}$, the reflection of $\overline{AB}$. When points *A* and *A'* are connected, what figure is formed by points *A*, *B*, and *A'*?

b. Use what you know about reflection to make as many statements as you can about the shape from part (a). For example, are there any sides that must be the same length? Are there any angles that must be equal? Is there anything else special about this shape?

c. **LEARNING LOG**
When two sides of a triangle have the same length, that triangle is called **isosceles**. In your Learning Log, describe all the facts you know about isosceles triangles based on the reflection. Be sure to include a diagram. Label this entry "Isosceles Triangles" and include today's date.

METHODS AND MEANINGS

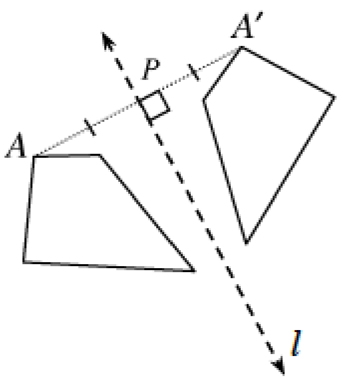
MATH NOTES

Formal Definitions of Rigid Transformations

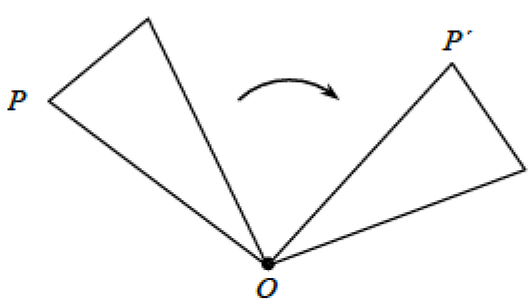
In algebra, you learned that a function is an equation that assigns each input a unique output. Most of these functions involved expressions and numbers such as $f(x) = 3x - 5$, so $f(2) = 1$.

In this course, you are now studying functions that assign each point in the plane to a unique point in the plane. These functions are called **rigid transformations** (or motions) because they move the entire plane with any figures you have drawn so that all of the figures remain unchanged. Therefore, angles and distances are preserved. There are three basic rigid motions that we will consider: reflections, translations, and rotations. All rigid motions can be seen as a combination of them.

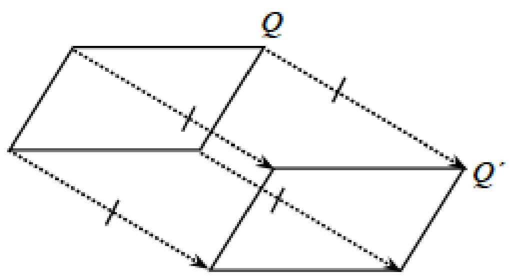
Reflections: When a figure is reflected across a line of reflection, such as the figure at right, it appears that the figure is "flipped" over the line. However, formally, a reflection across a line of reflection is defined as a function of each point (such as A) to a point (such as A') so that the line of reflection is the perpendicular bisector of the segments connecting the points and their images (such as $\overline{AA'}$). Therefore, $AP = A'P$.



Rotations: Formally, a rotation about a point O is a function that assigns each point (P) in the plane a unique point (P') so that all angles of rotation $\angle POP'$ have the same measure (which is the angle of rotation) and $OP = OP'$.

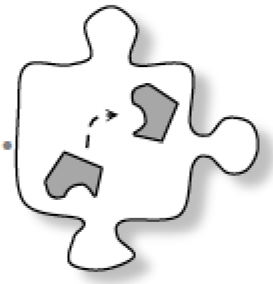


Translations: Formally, a translation is a function that assigns each point (Q) in the plane a unique point (Q') so that all line segments connecting an original point with its image have equal length and are parallel.



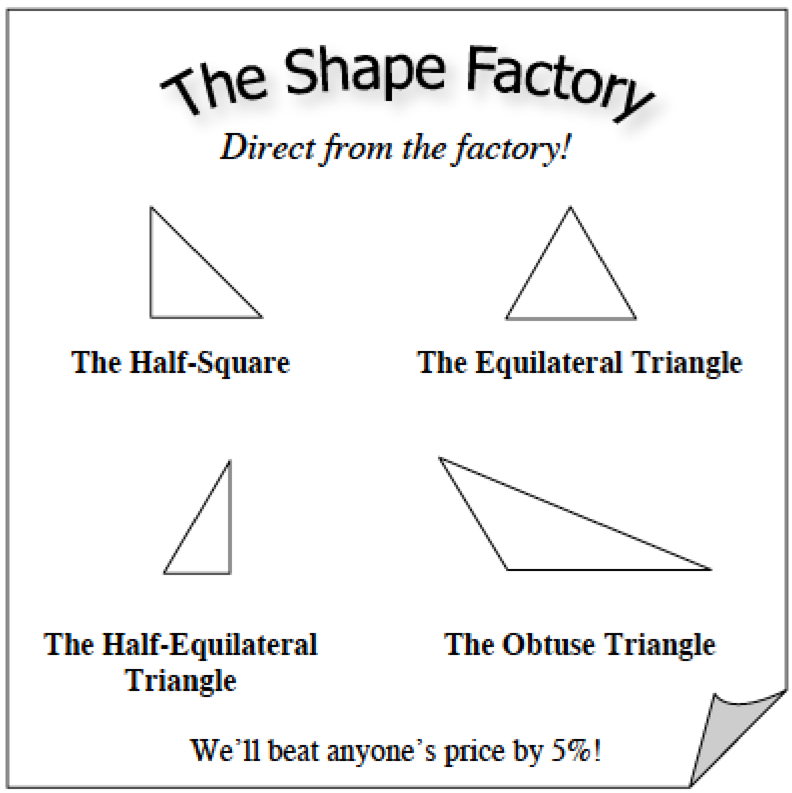
1.2.5 What shapes can I create with triangles?

Using Transformations to Create Shapes



In Lesson 1.2.4, you practiced reflecting, rotating and translating figures. Since these are rigid transformations, the image always had the same size and shape as the original. In this lesson, you will combine the image with the original to make new, larger shapes from four basic "building-block" shapes.

As you create new shapes, consider what information the transformation gives you about the resulting new shape. By the end of this lesson, you will have generated most of the shapes that will be the focus of this course.

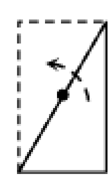


1-90. THE SHAPE FACTORY

The Shape Factory, an innovative new company, has decided to branch out to include new shapes. As Product Developers, your team is responsible for finding exciting new shapes to offer your customers. The current company catalog is shown at right.

Since your boss is concerned about production costs, you want to avoid buying new machines and instead want to reprogram your current machines.

The factory machines not only make all the shapes shown in the catalog, but they also are able to rotate or reflect a shape. For example, if the half-equilateral triangle is rotated 180° about the **midpoint** (the point in the middle) of its longest side, as shown at right, the result is a rectangle.



Your Task: Your boss has given your team until the end of this lesson to find as many new shapes as you can. Your team's reputation, which has suffered recently from a series of blunders, could really benefit by an impressive new line of shapes formed by a triangle and its transformations. For each triangle in the catalog, determine which new shapes are created when it is rotated or reflected so that the image shares a side with the original triangle. Be sure to make as many new shapes as possible. Use tracing paper or any other reflection tool to help.

1-92. EXTENSION

What other shapes can be created by reflection and rotation? Explore this as you answer the questions below. You can investigate these questions in any order. Remember that the resulting shape includes the original shape and all of its images. Remember to record and name each result.

- What if you reflect an equilateral triangle twice, once across one side and another time across a different side?
- What if an equilateral triangle is repeatedly rotated about one vertex so that each resulting triangle shares one side with another until new triangles are no longer possible? Describe the resulting shape.
- What if you rotate a trapezoid 180º around the midpoint of one if its non-parallel sides?

1-93. BUILDING A CATALOG

Your boss now needs you to create a catalog page that includes your shapes. Each entry should include a diagram, a name, and a description of the shape. List any special features of the shape, such as if any sides are the same length or if any angles must be equal. Use color and arrows to highlight these attributes in the diagram.

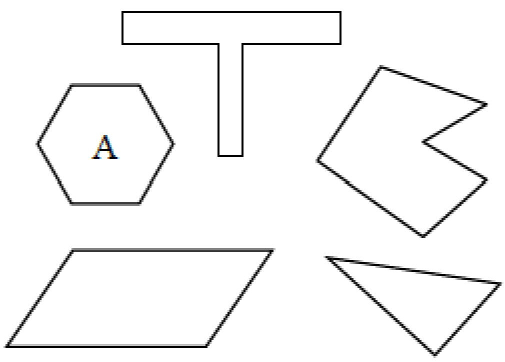
Methods and Meanings

Math Notes

Polygons

A **polygon** is defined as a two-dimensional closed figure made up of straight line segments connected end-to-end. These segments may not cross (intersect) at any other points.

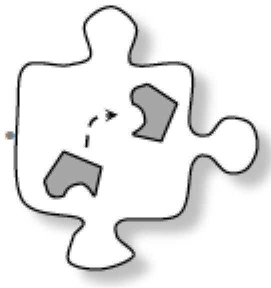
Below are some examples of polygons.



Shape A above is an example of a **regular polygon** because its sides are all the same length and its angles have equal measure.

Polygons are named according to the number of sides that they have. Polygons that have 3 sides are **triangles**, those with 4 sides are **quadrilaterals**, polygons with 5 sides are **pentagons**, polygons with 6 sides are **hexagons**, polygons with 8 sides are **octagons**, polygons with 10 sides are **decagons**. For most other polygons, people simply name the number of sides, such as “11-gon” to indicate a polygon with 11 sides.

1.2.6 What shapes have symmetry?



Symmetry

You have encountered symmetry several times in this chapter. For instance, the quilt your class created in Lesson 1.1.1 contained symmetry. The shapes you saw in the hinged mirrors during the kaleidoscope investigation (Lesson 1.1.5) were also symmetric. But so far, you have not developed a test for determining whether a polygon is symmetric. And since symmetry is related to transformations, how can you use transformations to describe this relationship? This lesson is designed to deepen your understanding of symmetry.

By the end of this lesson, you should be able to answer these target questions:

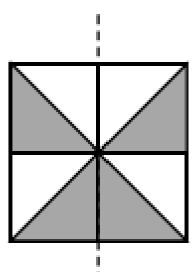
What is symmetry?

How can I determine whether or not a polygon has symmetry?

What types of symmetry can a shape have?

1-99. REFLECTION SYMMETRY

In problem 1-1, you created a quilt panel that had **reflection symmetry** because if the design were reflected across the line of symmetry, the image would be exactly the same as the original design. That is, a figure has reflection symmetry if a reflection carries it onto itself. See an example of a quilt design that has reflection symmetry below.



Obtain the Lesson 1.2.6 Resource Page. On it, examine the many shapes that will be our focus of study in this course. Which of these shapes have reflection symmetry? Consider this as you answer the questions below.

a. For each figure on the resource page, draw all the possible lines of symmetry. If you are not sure if a figure has reflection symmetry, use tracing paper or a reflective tool to explore.

b. Which types of triangles have reflection symmetry?

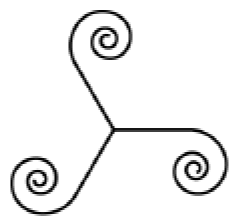
c. Which types of **quadrilaterals** (polygons with four sides) have reflection symmetry?

d. Which figures on your resource page have more than three lines of symmetry?

1-100. ROTATION SYMMETRY

In problem 1-99, you learned that many shapes have reflection symmetry. These shapes remain unaffected when they are reflected across a line of symmetry. Similarly, some figures can also be rotated onto themselves so that they remain unchanged.

a. Examine the diagram below. Can this figure be rotated onto itself? Trace this shape on tracing paper and/or use the Spiral eTool (Desmos) to test your conclusion. If it is possible, where is the point of rotation?



b. Jessica claims that she can rotate all figures in such a way that they will not change. How does she do it?

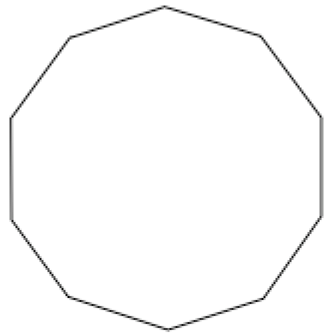
c. Since all figures can be rotated 360° without change, that is not a very special quality. However, the shape in part (a) above was special because it could be rotated less than 360° and still remain unchanged. A shape with this quality is said to have **rotation symmetry**.

But what shapes have rotation symmetry? Examine the figures on your Lesson 1.2.6 Resource Page and identify those that have rotation symmetry.

d. Which shapes on the resource page have 90° rotation symmetry? That is, which can be rotated about a point 90° and remain unchanged?

1-101. TRANSLATION SYMMETRY

In problems 1-99 and 1-100, you identified shapes that have reflection and rotation symmetry. What about translation symmetry? Is there an object that can be translated so that its end result is exactly the same as the original object? If so, draw an example and explain why it has **translation symmetry**.



1-102. DESCRIBING SYMMETRY

Now describe all types of symmetry for the same figure if possible. For example, assume you have a regular polygon with 10 sides (called a **decagon**).

a. What reflections carry this decagon onto itself? That is, describe its lines of reflections. If this polygon has no reflection symmetry, explain how you know.

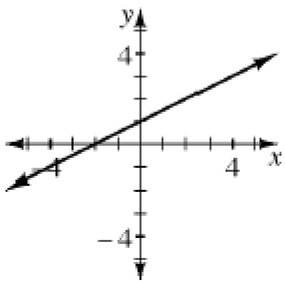
b. What rotations carry this polygon onto itself? That is, describe a point of rotation and angles for which this polygon has rotation symmetry. If this polygon has no rotation symmetry, explain how you know.

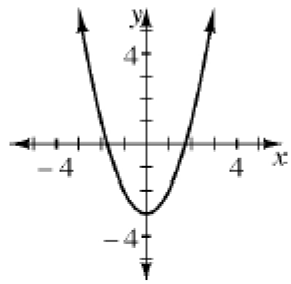
c. What translations carry this decagon onto itself? That is, describe a translation for which this polygon has translation symmetry. If this polygon has no translation symmetry, explain how you know.

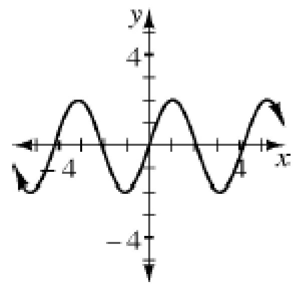
1-103. CONNECTIONS WITH ALGEBRA

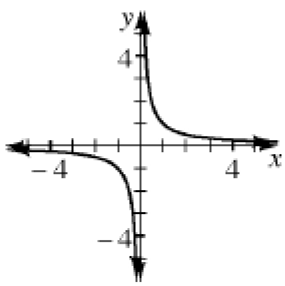
During this lesson, you have focused on the types of symmetry that can exist in geometric objects. But what about shapes that are created on graphs? What types of graphs have symmetry?

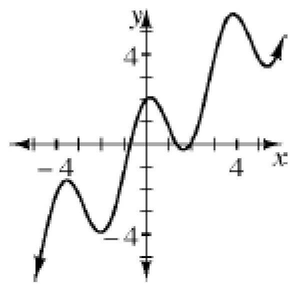
a. Examine the graphs below. Decide which have reflection symmetry, rotation symmetry, translation symmetry, or a combination of these.

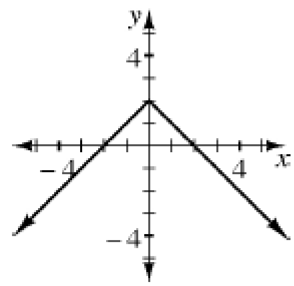
(1)


(2)


(3)


(4)


(5)


(6)


b. If the *y*-axis is a line of symmetry of a graph, then its function is referred to as **even**. Which of the graphs in part (a) are even functions?

c. If the graph has rotation symmetry about the origin (0, 0), its function is called **odd**. Which of the graphs in part (a) are odd functions?

1-104. Reflect on what you have learned during this lesson. In your Learning Log, answer the questions posed at the beginning of this lesson, reprinted below. When helpful, give examples and draw a diagram. Title your entry "Symmetry" and include today's date.

What is symmetry?

How can I determine whether or not a polygon has symmetry?

What types of symmetry can a shape have?

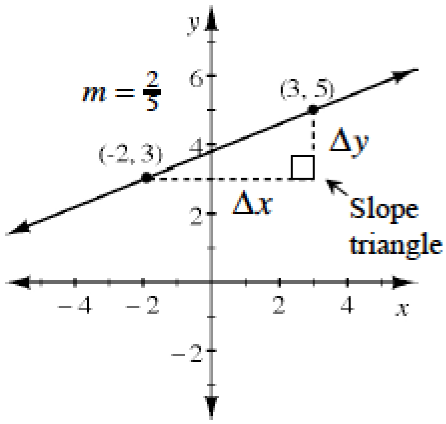
METHODS AND MEANINGS

MATH NOTES

Slope of a Line and Parallel and Perpendicular Slopes

During this course, you will use your algebra tools to learn more about shapes. One of your algebraic tools that can be used to learn about the relationship of lines is slope. Review what you know about slope below.

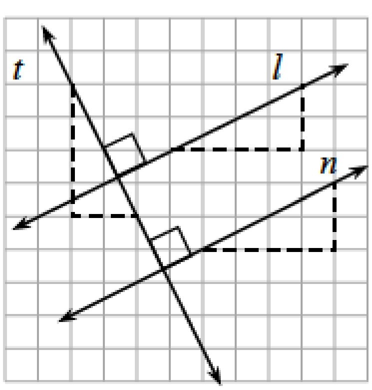
$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\Delta y}{\Delta x}$$



The **slope** of a line is the ratio of the change in y (Δy) to the change in x (Δx) between any two points on the line. It indicates both how steep the line is and its direction, upward or downward, left to right.

Lines that point upward from left to right have positive slope, while lines that point downward from left to right have negative slope. A horizontal line has zero slope, while a vertical line has undefined slope. The slope of a line is denoted by the letter m when using the $y = mx + b$ equation of a line.

One way to calculate the slope of a line is to pick two points on the line, draw a slope triangle (as shown in the example above), determine Δy and Δx, and then write the slope ratio. Be sure to verify that your slope correctly resulted in a negative or positive value based on its direction.



Parallel lines lie in the same plane (a flat surface) and never intersect. They have the same steepness, and therefore they grow at the same rate. Lines l and n below are examples of parallel lines.

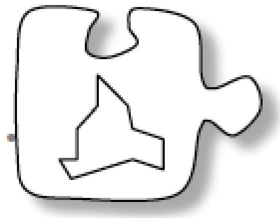
On the other hand, **perpendicular** lines are lines that intersect at a right angle. For example, lines *m* and *n* above are perpendicular, as are lines *m* and *l*. Note that the small square drawn at the point of intersection indicates a right angle.

The **slopes of parallel lines** are the same. In general, the slope of a line parallel to a line with slope *m* is *m*.

The **slopes of perpendicular lines** are opposite reciprocals. For example, if one line has slope $\frac{4}{5}$, then any line perpendicular to it has slope $-\frac{5}{4}$. If a line has slope −3, then any line perpendicular to it has slope $\frac{1}{3}$. In general, the slope of a line perpendicular to a line with slope *m* is $-\frac{1}{m}$.

1.3.1 How can I classify this shape?

Attributes and Characteristics of Shapes



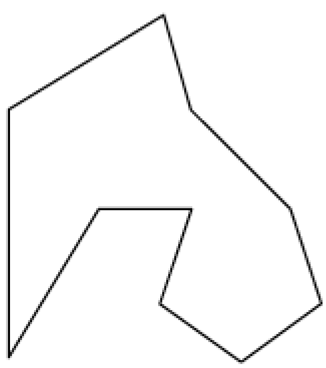
In Lesson 1.2.5, you generated a list of shapes formed by triangles and in Lesson 1.2.6, you studied the different types of symmetries that a shape can have. In Section 1.3, you will continue working with shapes to learn more about their attributes and characteristics. For example, which shapes have sides that are parallel? And which basic shapes are equilateral?

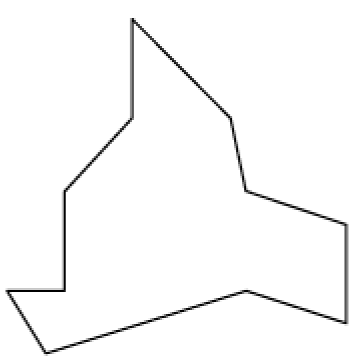
By the end of this lesson you should have a greater understanding about the attributes that make shapes alike and different. Throughout the rest of this course you will study these qualities that set shapes apart as well as learn how shapes are related.

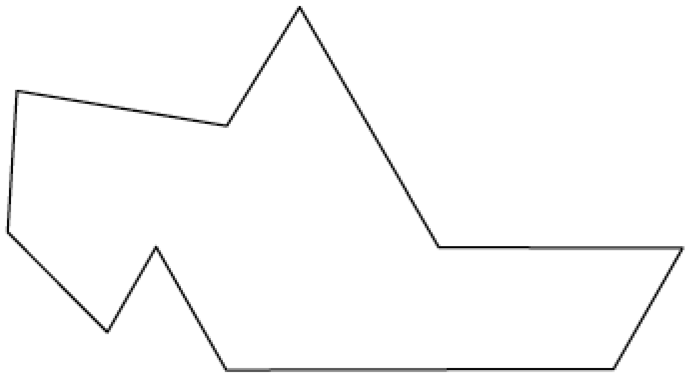
1-110. INTRODUCTION TO THE SHAPE BUCKET

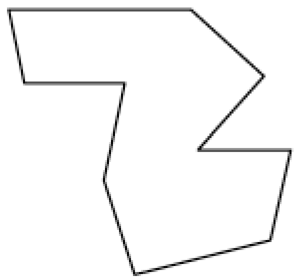
Obtain a Shape Bucket and a Lesson 1.3.1B Resource Page from your teacher. The Shape Bucket contains most of the basic geometric shapes you will study in this course. Count the items and verify that you have all 16 shapes. Take the shapes out and notice the differences between them. Are any alike? Are any strangely different?

Once you have examined the shapes in your bucket, work as a team to build the composite figures below (also shown on the resource page). Composite figures are made by combining two or more shapes to make a new figure. On the Lesson 1.3.1B Resource Page, show the shapes you used to build the composite shapes by filling in their outlines within each composite shape.

a. 

b. 

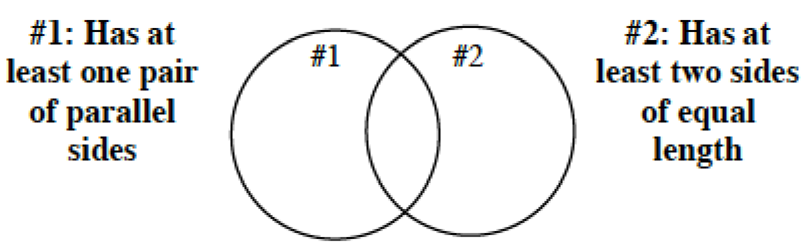
c. 

d. 

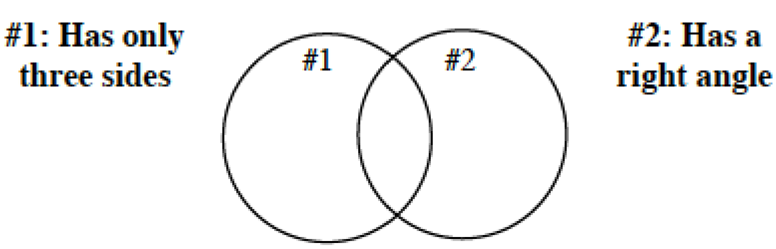
1-111. VENN DIAGRAMS

Obtain a Venn diagram (Lesson 1.3.1C Resource Page) from your teacher.

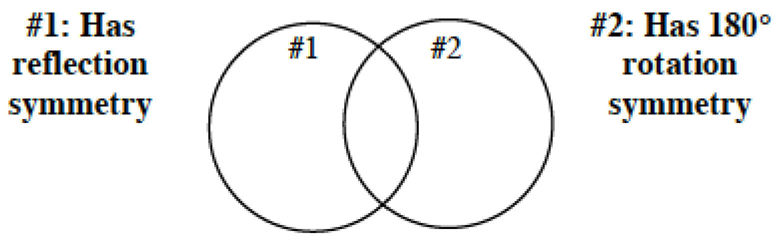
a. The *left* circle of the Venn diagram, Circle #1, will represent the attribute “has at least one pair of parallel sides” and the *right* side, Circle #2, will represent the attribute “has at least two sides of equal length” as shown below. Sort through the shapes in the Shape Bucket and decide as a team where each shape belongs. Be sure to record your solution on paper. As you discuss this problem with your teammates, justify your statements with reasons such as, “I think this shape goes here because…” Explore using the 1-111 Venn Diagram A: Student eTool (Desmos).



b. Next, reclassify the shapes for the new Venn diagram shown below. Describe each region in a sentence. Explore using the 1-111 Venn Diagram B: Student eTool (Desmos).



c. Finally, reclassify the shapes for the new Venn diagram shown below. Describe each region in a sentence. Explore using the 1-111 Venn Diagram C: Student eTool (Desmos).

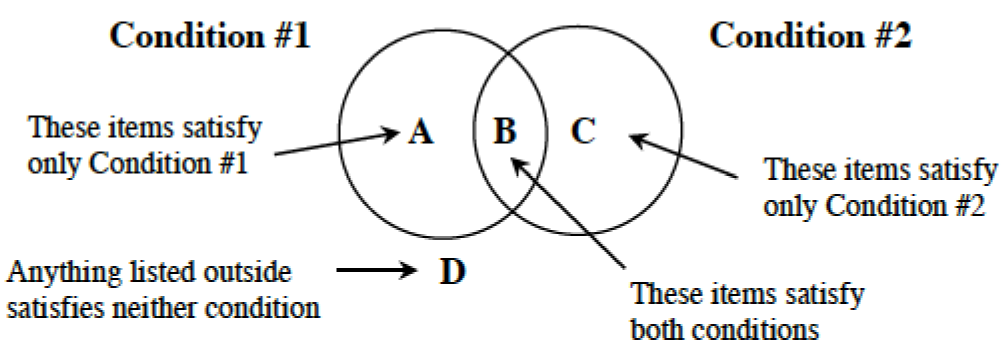


Methods and Meanings

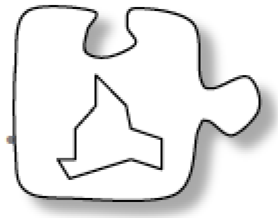
Math Notes

Venn Diagrams

A Venn diagram is a tool used to classify objects. It is usually composed of two or more circles that represent different conditions. An item is placed or represented in the Venn diagram in the appropriate position based on the conditions it meets. See the example below:



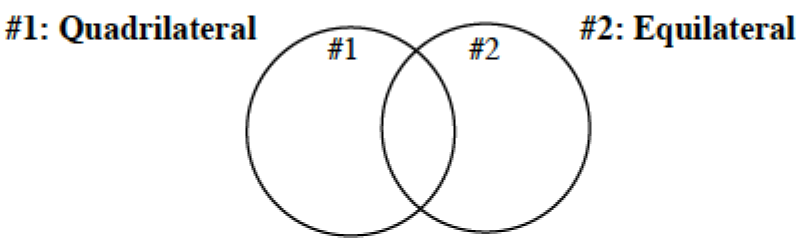
1.3.2 How can I describe it?



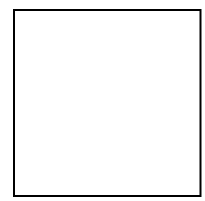
More Characteristics of Shapes

In Lesson 1.3.1, you used shapes to build new, unique, composite shapes. You also started to analyze the attributes (qualities) of shapes. Today you will continue to look at their attributes as you learn new vocabulary.

1-117. Using your Venn diagram Resource Page from Lesson 1.3.1, categorize the shapes from the Shape Bucket in the Venn diagram as shown below. Record your results on paper. Explore using the Generic Venn Diagram (Desmos).



1-118. DESCRIBING A SHAPE


Square

How can you describe a square? With your class, find a way to describe a square using its attributes (special qualities) so that anyone could draw a square based on your description. Be as complete as possible. You may not use the word “square” in the description.

1-119. Each shape in the bucket is unique; that is, it differs from the others. You will be assigned a few shapes to describe as completely as possible for the class. But what is a complete description? As you work with your team to create a complete description, consider the questions below.

- What do you notice about your shape?
- What makes it different from other shapes?
- If you wanted to describe your shape to a friend on the telephone who cannot see it, what would you need to include in the description?

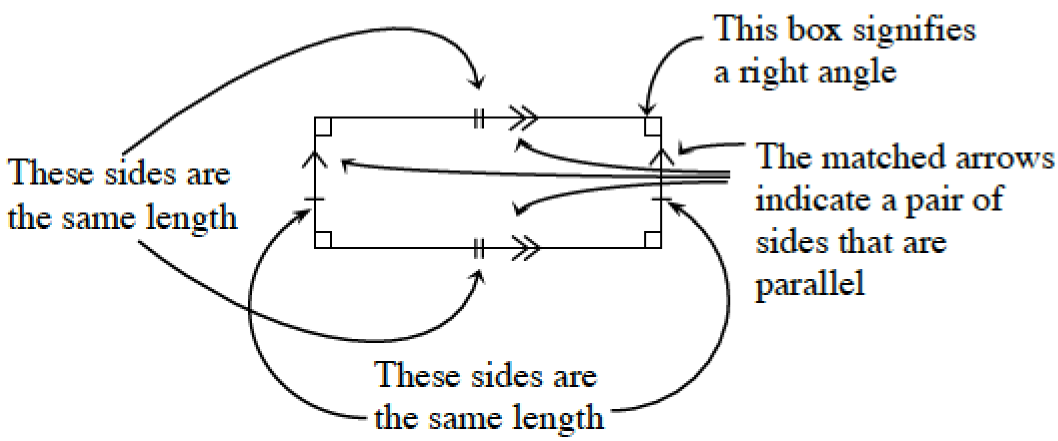
1-120. SHAPES TOOLKIT

Obtain the Lesson 1.3.2A Resource Page entitled "Shapes Toolkit".

a. In the space provided, describe the shape based on the descriptions generated from problem 1-119. Leave space so that later observations can be added for each shape. Note that the description for "rectangle" has been provided as an example.

b. On the diagram for each shape, mark sides that must have equal length or that must be parallel. Also mark any angles that measure 90°. See the descriptions for how to do this below.

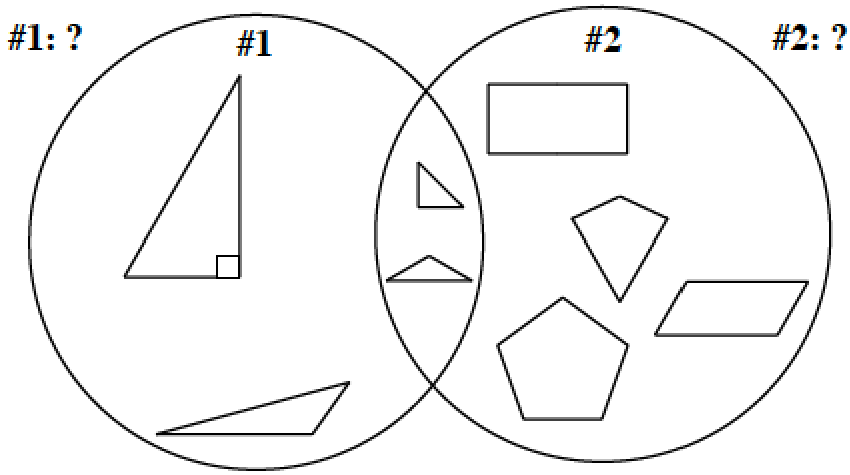
- To show that two sides have the same length, use "tick marks" on the sides. However, to show that one pair of equal sides may not be the same length as the other pair of equal sides, you should use one tick mark on each of the two opposite, equal sides and two tick marks on each of the other two opposite, equal sides, as shown below.



- To show that the rectangle has two pairs of parallel sides, use one ">" mark on each of one pair of parallel sides and two ">>" marks on each of the other two parallel sides, as shown above.
- Also mark any right angles by placing a small square at the right angle vertex (the corner). See the example above.

d. The Shapes Toolkit Resource Page is the first page of a special information organizer, called your Geometry Toolkit, which you will be using for this course. It is a reference tool that you can use when you need to remember the name or description of a shape. Find a safe place in your Geometry binder to keep your Toolkit.

1-121. Examine the Venn diagram below.

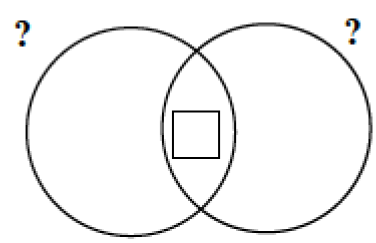


a. What attribute does each circle represent? How can you tell?

b. Where would the regular hexagon from your Shape Bucket go in this Venn diagram? What about the trapezoid? Justify your reasoning.

c. Create another shape that would belong outside both circles. Does your shape have a name that you have studied so far? If not, give it a new name.

1-122. Elizabeth has a Venn diagram that she started at right. It turns out that the only shape in the Shape Bucket that could go in the intersection (where the two circles overlap) is a square! What are the possible attributes that her circles could represent? Discuss this with your team and be ready to share your ideas with the class.



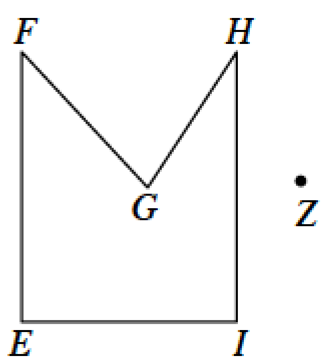
4. WHAT HAVE I LEARNED?

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

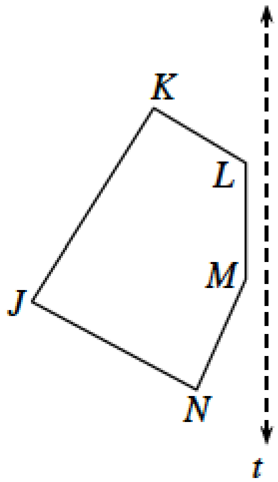
Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 1-128. Trace the figures in parts (a) and (b) onto your paper and perform the indicated transformations. Copy the figure from part (c) onto graph paper and perform the indicated transformation. Label each image with prime notation ($A \rightarrow A'$).

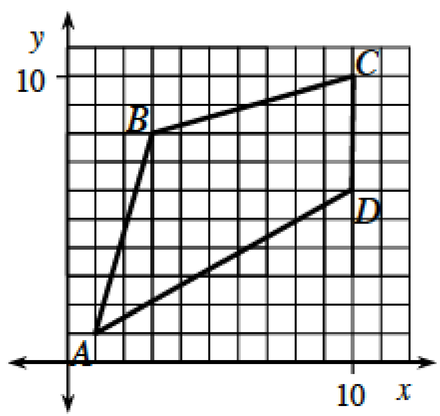
a. Rotate *EFGHI* 90° clockwise ↻ about point *Z*



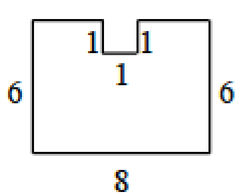
b. Reflect *JKLMN* over line *t*



c. Translate *ABCD* down 5 units and right 3 units

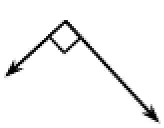


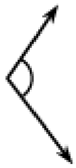
CL 1-129. Assume that all angles in the diagram below are right angles and that all the measurements are in centimeters. Find the perimeter of the figure.

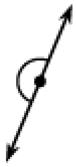


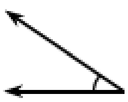
CL 1-130. Estimate the measures of the angles below. Are there any that you know for sure?

a.



b. 

c. 

d. 

CL 1-131. Examine the angles in problem CL 1-130. If these four angles are placed in a bag, what is the probability of randomly selecting:

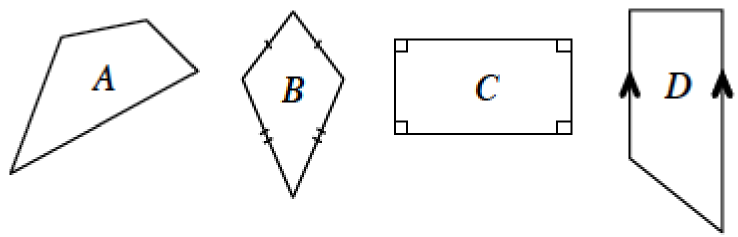
a. An acute angle

b. An angle greater than 60°

c. A 90° angle

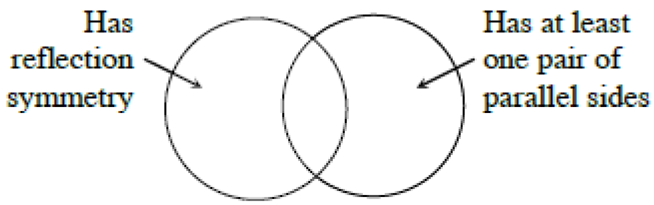
d. An angle less than or equal to 180°

CL 1-132. Examine the shapes below.



a. Describe what you know about each shape based on the information provided in the diagram. Then name the shape.

b. Decide where each shape would be placed in the Venn diagram below.



CL 1-133. Solve each equation below. Check your solution.

a. $3x - 12 + 10 = 8 - 2x$

b. $\frac{x}{7} = \frac{3}{2}$

c. $5 - (x + 7) + 4x = 7(x - 1)$

d. $x^2 + 11 = 36$

CL 1-134. Find the value of y for each equation twice: first for $x = 8$, then for .$x = -3$.

a. $y = x^2 + 13x + 8$

b. $y = 6x - 2$

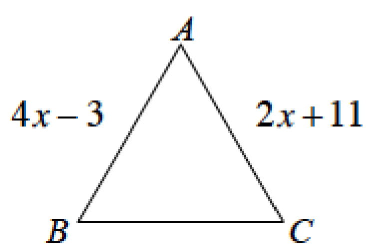
CL 1-135. Graph and connect the points in the table below. Then graph the equation in part (b) on the same set of axes. Also, find the equation for the data in the table.

a.

x	−4	−3	−2	−1	0	1	2	3	4	5	6
y	−5	−3	−1	1	3	5	7	9	11	13	15

b. $y = x^2 + x - 2$

CL 1-136. ΔABC below is equilateral. Use what you know about an equilateral triangle to write and solve an equation for x. Then find the perimeter of ΔABC.



CL 1-137. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in math classes you have taken before? Use the table to make a list of topics you need help on and a list of topics you need to practice more.