1.1.1 How can I design it?

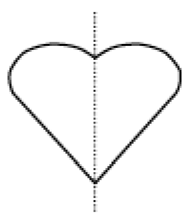
Creating a Quilt Using Symmetry

METHODS AND MEANINGS

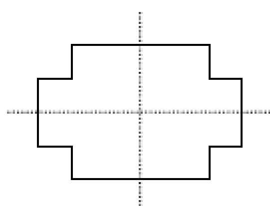
MATH NOTES

Lines of Symmetry

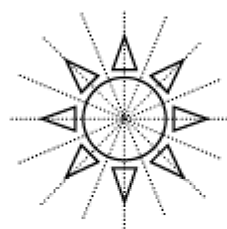
When a graph or picture can be folded so that both sides of the fold will perfectly match, it is said to have **reflective symmetry**. The line where the fold would be is called the **line of symmetry**. Some shapes have more than one line of symmetry. See the examples below.



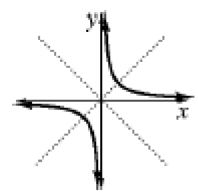
This shape has one line of symmetry.



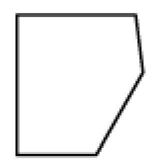
This shape has two lines of symmetry.



This shape has eight lines of symmetry.



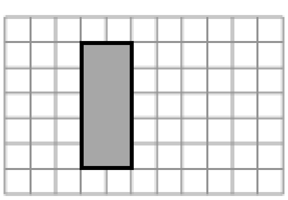
This graph has two lines of symmetry.



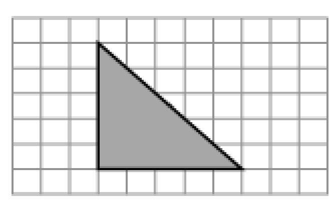
This shape has no lines of symmetry.

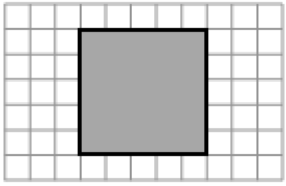
1-3. One focus of this Geometry course is to help you recognize and accurately identify a shape. For example, a **rectangle** is a four-sided shape with four right angles. Which of the shapes below can be called a rectangle? More than one answer is possible.

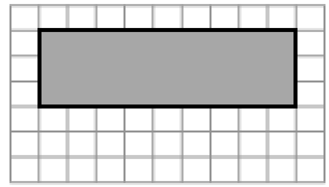
a.

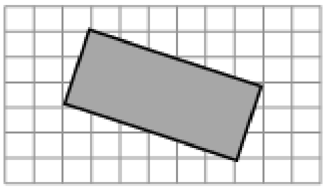


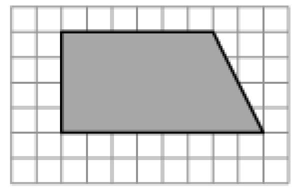
b.



c. 

e. 

d. 

f. 

1-4. Calculate the values of the expressions below. Show all steps in your process. The answers are provided for you to check your result. If you miss two or more of these and cannot find your errors, be sure to seek help from your team or teacher.

a. $2 \cdot (3(5 + 2) - 1)$

b. $6 - 2(4 + 5) + 6$

c. $3 \cdot 8 \div 2^2 + 1$

d. $5 - 2 \cdot 3 + 6(3^2 + 1)$

1-5. Match each table of data on the left with its equation on the right and briefly explain why it matches the data.

Rules: 1. $y = x$ 2. $y = 3x - 1$ 3. $y = x + 3$

4. $y = x^2$ 5. $y = -x^2$ 6. $y = x^2 + 3$

a.

x	1	0	−4	2	−2	−1
y	4	3	−1	5	1	2

b.

x	−1	3	1	0	−2	2
y	−1	−9	−1	0	−4	−4

c.

x	3	−2	1	0	2	−3
y	12	7	4	3	7	12

d.

x	−3	4	2	−2	0	−10
y	−10	11	5	−7	−1	−31

1-6. Simplify the expressions below as much as possible.

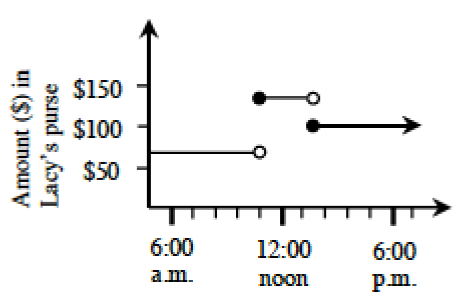
a. $2a + 4(7 + 5a)$

b. $4(3x + 2) - 5(7x + 5)$

c. $x(x + 5)$

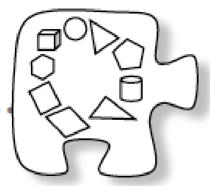
d. $2x + x(x + 6)$

1-7. Examine the graph below. Then, in a sentence or two, suggest reasons why the graph rises at 11:00 a.m. and then drops at 1:15 p.m.



1.1.2 Can you predict the results?

Making Predictions and Investigating Results



METHODS AND MEANINGS

MATH NOTES

The Investigative Process

The **investigative process** is a way to study and learn new mathematical ideas. Mathematicians have used this process for many years to make sense of new concepts and to broaden their understanding of older ideas.

In general, this process begins with a **question** that helps you frame what you are looking for. For example, a question such as, "*What if the Möbius strip has 2 half-twists? What will happen when that strip is cut in half down the middle?*" can help start an investigation to find out what happens when the Möbius strip is slightly altered.

Once a question is asked, you can make an educated guess, called a **conjecture**. This is a mathematical statement that has not yet been proven.

Next, **exploration** begins. This part of the process may last awhile as you gather more information about the mathematical concept. For example, you may first have an idea about the diagonals of a rectangle, but as you draw and measure a rectangle on graph paper, you find out that your conjecture was incorrect.

When a conjecture seems to be true, the final step is to **prove** that the conjecture is always true. A proof is a convincing logical argument that uses definitions and previously proven conjectures in an organized sequence.

1.1.3 How can I predict the area?

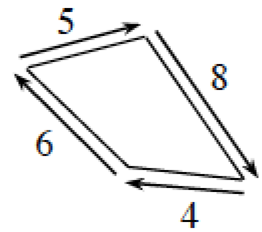
Perimeters and Areas of Enlarging Tile Patterns

Methods and Meanings

Math Notes

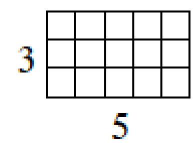
The Perimeter and Area of a Figure

The **perimeter** of a two-dimensional figure is the distance around its exterior (outside) on a flat surface. It is the total length of the boundary that encloses the interior (inside) region. See the example at right.



Perimeter = $5 + 8 + 4 + 6 =$ 23 units

The **area** indicates the number of square units needed to fill up a region on a flat surface. For a rectangle, the area is computed by multiplying its length and width. The rectangle at right has a length of 5 units and a width of 3 units, so the area of the rectangle is 15 square units.



Area = $5 \cdot 3 = 15$ square units

1.1.4 Are you convinced?

Logical Arguments

METHODS AND MEANINGS

MATH NOTES

Solving Linear Equations

In Algebra, you learned how to solve a linear equation. This course will help you apply your algebra skills to solve geometric problems. Review how to solve equations by reading the example below.

- **Simplify.** Combine like terms on each side of the equation whenever possible.

- **Keep equations balanced.** The equal sign in an equation tells you that the expressions on the left and right are balanced. Anything done to the equation must keep that balance.

$3x - 2 + 4 = x - 6$	Combine like terms
$3x + 2 = x - 6$	
$-x = -x$	Subtract x on both sides
$2x + 2 = -6$	
$-2 = -2$	Subtract 2 on both sides
$\frac{2x}{2} = \frac{-8}{2}$	Divide both sides by 2
$x = -4$	

- **Move your x-terms to one side of the equation.** Isolate all variables on one side of the equation and the constants on the other.

- **Undo operations.** Use the fact that addition is the opposite of subtraction and that multiplication is the opposite of division to solve for x. For example, in the equation $2x = -8$, since the 2 and the x are multiplied, then dividing both sides by 2 will get x alone.

1-32. One goal of this course will be to review and enhance your algebra skills. Read the Math Notes box for this lesson. Then solve for *x* in each equation below, show all steps leading to your solution, and check your answer.

a. $34x - 18 = 10x - 9$

b. $4x - 5 = 4x + 10$

c. $3(x - 5) + 2(3x + 1) = 45$

d. $-2(x + 4) + 6 = -3$

1-33. The day before Gerardo returned from a two-week trip, he wondered if he left his plants inside his apartment or outside on his deck. He knows these facts:

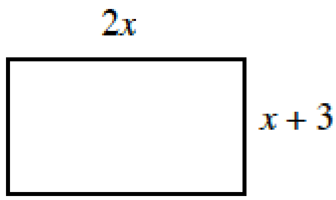
- If his plants are indoors, he must water them at least once a week or they will die.
- If he leaves his plants outdoors and it rains, then he does not have to water them. Otherwise, he must water them at least once a week or they will die.
- It has not rained in his town for 2 weeks. When Gerardo returns, will his plants be dead? Explain your reasoning.

1-34. For each of the equations below, solve for *y* in terms of *x*.

a. $2x - 3y = 12$

b.$5x + 2y = 7$

1-35. Examine the rectangle below.

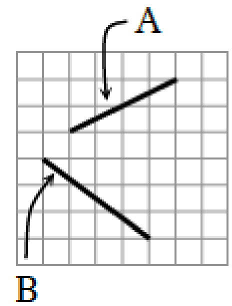


a. What is the perimeter in terms of *x*? In other words, find the perimeter.

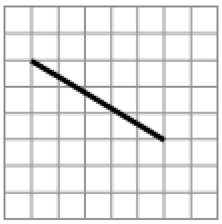
b. If the perimeter is 78 cm, find the dimensions of the rectangle. Show all your work.

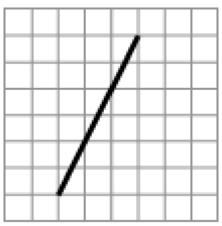
c. Verify that the area of this rectangle is 360 sq. cm. Explain how you know this.

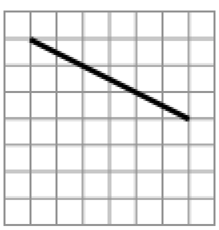
1-36. The **slope** of a line is a measure of its steepness and indicates whether it goes up or down from left to right. For example, the slope of the line segment A at right is $\frac{1}{2}$, while the slope of the line segment B is $-\frac{3}{4}$.

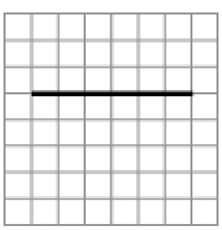


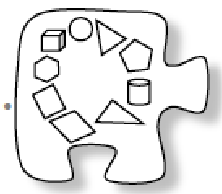
For each line segment below, find the slope. You may want to copy each line segment on graph paper in order to draw slope triangles.

a. 

b. 

c. 

d. 



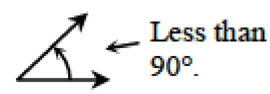
METHODS AND MEANINGS

MATH NOTES

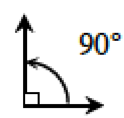
Types of Angles

When trying to describe shapes, it is convenient to classify types of angles. An angle is formed by two rays joined at a common endpoint. The measure of an angle represents the number of degrees of rotation from one ray to the other about the vertex. This course will use the following terms to refer to angles:

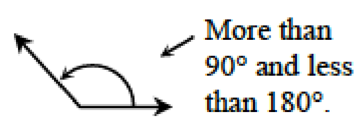
ACUTE: Any angle with measure *between* (but not including) 0°and 90°.



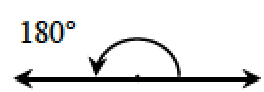
RIGHT: Any angle that measures 90˚.



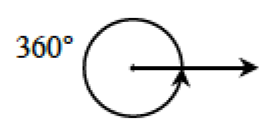
OBTUSE: Any angle with measure *between* (but not including) 90°and 180°.



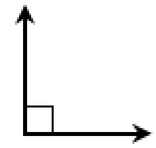
STRAIGHT: Straight angles have a measure of 180° and are formed when the sides of the angle form a straight line.

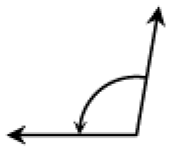


CIRCULAR: Any angle that measures 360°.

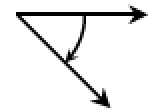


1-42. Estimate the size of each angle below to the nearest 10°. A right angle is shown for reference so you should not need a protractor.

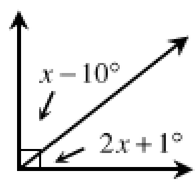


a. 

b. 

c. 

1-43. Rosalinda examined the angles at right and wrote the equation below. $(2x + 1^\circ) + (x - 10^\circ) = 90^\circ$

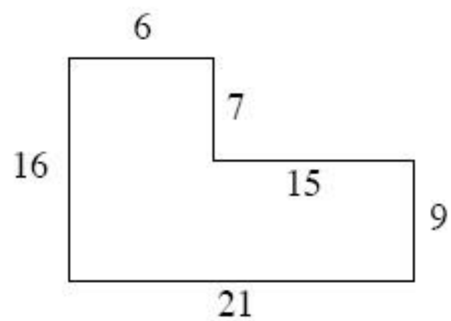


a. Does her equation make sense? If so, explain why her equation must be true. If it is not correct, determine what is incorrect and write the equation.

b. If you have not already done so, solve her equation, clearly showing all your steps. What are the measures of the two angles?

c. Verify that your answer is correct.

1-44. Angela had a rectangular piece of paper and then cut a rectangle out of a corner as shown below. Find the area and perimeter of the resulting shape.



1-45. For each equation below, solve for the given variable. If necessary, refer to the Math Notes box in Lesson 1.1.4 for guidance. Show the steps leading to your solution and check your answer.

a. $75 = 14y + 5$

b. $-7r + 13 = -71$

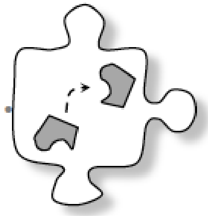
c. $3a + 11 = 7a - 13$

d. $2m + m - 8 = 7$

1-46. On graph paper, draw four different rectangles that each have an area of 24 square units. Then find the perimeter of each one.

1.2.1 How do you see it?

Spatial Visualization and Reflections



METHODS AND MEANINGS

MATH NOTES

Probability Vocabulary and Definitions

Event: Any outcome, or set of outcomes, from a probabilistic situation. A **successful event** is the set of all outcomes that are of interest in a given situation. For example, rolling a die is a probabilistic situation. Rolling a 5 is an event. If you win a prize for rolling an even number, you can consider the set of three outcomes {2, 4, 6} a successful event.

Sample space: All possible outcomes from a probabilistic situation. For example, the sample space for flipping a coin is heads and tails; rolling a die has a sample space of {1, 2, 3, 4, 5, 6}.

Probability: The likelihood that an event will occur. Probabilities may be written as ratios (fractions), decimals, or percents. An event that is certain to happen has a probability of 1, or 100%. An event that has no chance of happening has a probability of 0, or 0%. Events that "might happen" have probabilities between 0 and 1, or between 0% and 100%. The more likely an event is to happen, the greater its probability.

Experimental probability: The probability based on data collected in experiments.

$$\text{Experimental probability} = \frac{\text{number of successful outcomes in the experiment}}{\text{total number of outcomes in the experiment}}$$

Theoretical probability: Probability that is mathematically calculated. When each of the outcomes in the sample space has an *equally likely chance* of occurring, then

$$\text{Theoretical probability} = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$$

For example, to calculate the probability of rolling an even number on a die, first figure out how many possible (equally likely) outcomes there are. Since there are six faces on the number cube, the total number of possible outcomes is 6. Of the six faces, three of the faces are even numbers—there are three successful outcomes. Thus, to find the probability of rolling an even number, you would write:

$$P(\text{even}) = \frac{\text{number of ways to roll an even number}}{\text{number of faces on a number cube}} = \frac{3}{6} = 0.5 = 50\%$$

1-54. Graph each line below on the same set of axes.

a. $y = 3x - 3$

b. $y = -\frac{2}{3}x + 3$

c. $y = -4x + 5$

1-55. Probability is used to make predictions. See the Math Notes in this lesson for more details. Whenever the outcomes are *equally likely*, the probability in general is:

$$\text{P(success)} = \frac{\text{number of successes}}{\text{total number of possible outcomes}}$$

For example, if you were to reach into a bag with 16 total shapes, four of which have right angles, and randomly pull out a shape, you could use probability to predict the chances of the shape having a right angle.

$$\text{P(right angle)} = \frac{\text{number of successes}}{\text{total number of possible outcomes}} = \frac{\text{4 shapes with right angles}}{\text{16 total shapes}}$$

$$= \frac{4}{16} = \frac{1}{4} = 0.25 = 25\%$$

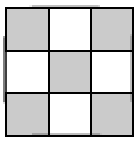
The example above shows all forms of writing probability: $\frac{4}{16}$ (read "4 out of 16") is the probability as a ratio, 0.25 is its decimal form, and 25% is its equivalent percent. What else can probability be used to predict? Analyze each of the situations below:

a. The historic carousel at the park has 4 giraffes, 4 lions, 2 elephants, 18 horses, 1 monkey, 6 unicorns, 3 ostriches, 3 zebras, 6 gazelles, and even 1 dinosaur. Eric's niece wants for Eric to randomly pick an animal to ride. What is the probability (expressed as a percent) that Eric picks a horse, a unicorn, or a zebra?

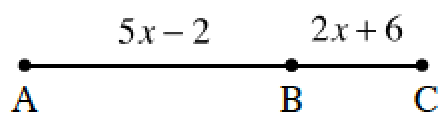
b. Eduardo has in his pocket \$1 in pennies, \$1 in nickels, and \$1 in dimes. If he randomly pulls out just one coin, what is the probability that he will pull out a dime?

c. P(rolling an 8) with one regular die if you roll the die just once.

d. P(dart hitting a shaded region) if the dart is randomly thrown and hits the target below.



1-56. The distance along a straight road is measured as shown in the diagram below. If the distance between towns A and C is 67 miles, find the distance between towns A and B.



1-57. For each equation below, solve for *x*. Show all work. The answers are provided so that you can check them. If you are having trouble with any solutions and cannot find your errors, you may need to see your teacher for extra help (you can ask your team as well).

a. $5x - 2x + x = 15$ Answer: $x = 3.75$

b. $3x - 2 - x = 7 - x$ Answer: $x = 3$

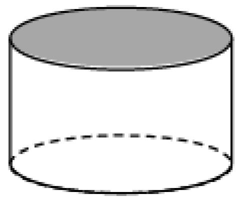
c. $3(x - l) = 2x - 3 + 3x$ Answer: $x = 0$

d. $3(2 - x) = 5(2x - 7) + 2$ Answer: $x = 3$

e. $\frac{26}{57} = \frac{849}{5x}$ Answer: $x \approx 372.25$

f. $\frac{4x+1}{3} = \frac{x-5}{2}$ Answer: $x = -3.4$

1-58. The three-dimensional shape below is called a **cylinder**. Its bottom and top bases are both circles, and its side is perpendicular to the bases. What would the shape of a flag need to be in order to generate a cylinder when it rotates about its pole? (You may want to refer to problem 1-49 to review how flags work.)



1.2.2 What if it is reflected more than once?

Rigid Transformations: Rotations and Translations

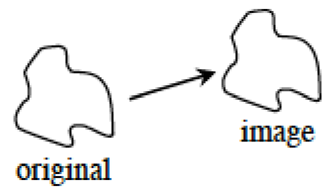
METHODS AND MEANINGS

MATH NOTES

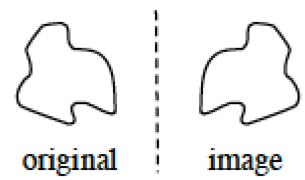
Rigid Transformations

A **rigid transformation** maps each point of a figure to a new point, so that the resulting image has the same size and shape of the original. There are three types of rigid transformations, described below.

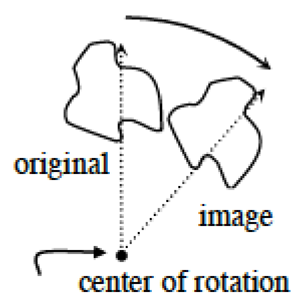
A transformation that preserves the size, shape, and orientation of a figure while *sliding* it to a new location is called a **translation**.



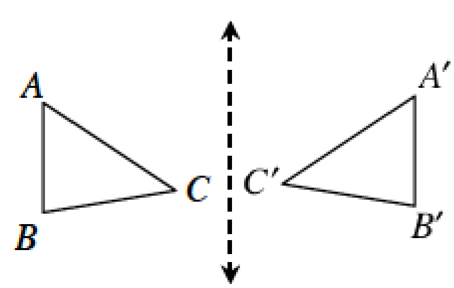
A transformation that preserves the size and shape of a figure across a line to form a mirror image is called a **reflection**. The mirror line is a **line of reflection**. One way to find a reflection is to *flip* a figure over the line of reflection.



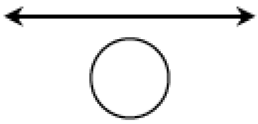
A transformation that preserves the size and shape while *turning* an entire figure about a fixed point is called a **rotation**. Figures can be rotated either clockwise (↻) or counterclockwise (↺).



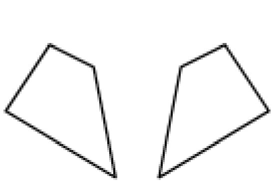
When labeling a transformation, the new figure (image) is often labeled with **prime notation.** For example, if ΔABC is reflected across the vertical dashed line, its image can be labeled $\Delta A'B'C'$ to show exactly how the new points correspond to the points in the original shape. We also say that ΔABC is **mapped** to $\Delta A'B'C'$.

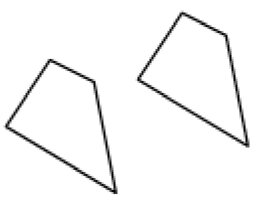


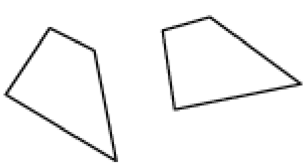
1-63. The diagram below shows a flat surface containing a line and a circle with no points in common. Can you visualize moving the line and/or circle so that they intersect at exactly one point? Two points? Three points? Explain each answer and illustrate each with an example when possible.

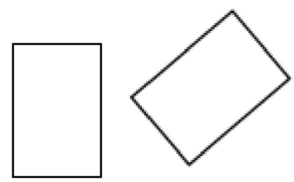


1-64. Decide which transformation was used on each pair of shapes below. Some may have undergone more than one transformation, but try to name a single transformation, if possible.

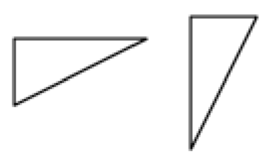
a. 

b. 

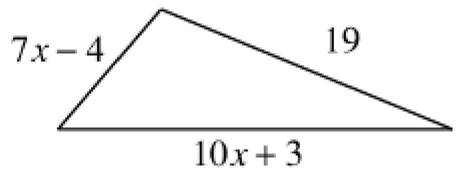
c. 

d. 

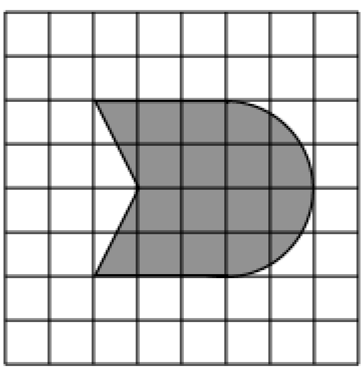
e. 

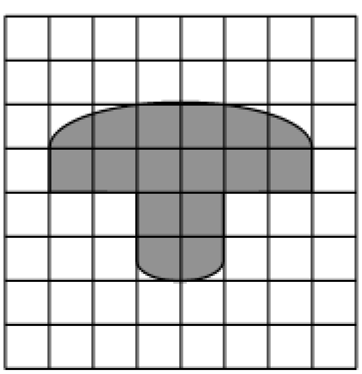
f. 

1-65. The perimeter of the triangle at right is 52 units. Write and solve an equation based on the information in the diagram. Use your solution for *x* to find the measures of each side of the triangle. Be sure to confirm that your answer is correct.



1-66. Bertie placed a transparent grid made up of unit squares over each of the shapes she was measuring below. Using her grid, approximate the area of each region.

a. 

b. 

1-67. For each equation below, find y if $x = -3$.

a. $y = -\frac{1}{3}x - 5$

b. $y = 2x^2 - 3x - 2$

c. $2x - 5y = 4$

1.2.3 What is the relationship?

Slopes of Parallel and Perpendicular Lines

1-76. TOP OF THE CHARTS

Renae's MP3 player can be programmed to randomly play songs from her playlist without repeating a single song. Currently, Renae's MP3 player has 5 songs loaded on it, which are listed below. As she walks between class, she only has time to listen to one song.

PLAYLIST

I Love My Mama (country)
by the Strings of Heaven

Don't Call Me Mama (country)
Duet by Sapphire and Hank Tumbleweed

Carefree and Blue (R & B)
by Sapphire and Prism Escape

Go Back To Mama (Rock)
Duet by Bjorn Free and Sapphire

Smashing Lollipops (Rock)
by Sapphire

a. Is each song equally likely to be chosen as the first song?

b. What is the probability that her MP3 player will select a country song?

c. What is the probability that Renae will listen to a song with "Mama" in the title?

d. What is the probability she listens to a duet with Hank Tumbleweed?

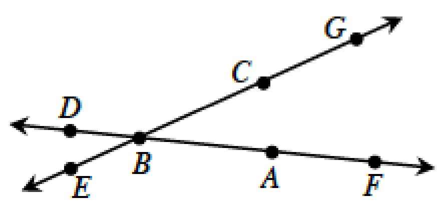
e. What is the probability she listens to a song that is not R & B?

1-77. Use what you learned about the slopes of parallel and perpendicular lines to find the equation of a line that would meet the criteria given below.

a. Find the equation of the line that goes through the point (0, −3) and is perpendicular to the line $y = -\frac{2}{5}x + 6$.

b. Find the equation of the line that is parallel to the line $-3x + 2y = 10$ and goes through the point (0, 7).

1-78. Examine the diagram below. Which angle below is another name for ∠*ABC*? Note: More than one solution is possible.



a. ∠*ABE*

b. ∠*GBD*

c. ∠*FBG*

d. ∠*EBC*

e. None of these

1-79. Solve for the variable in each equation below. Show your steps and check your answer.

a. $4p + 8p + 12 = -48$

b. $9w + 33 = 12w - 27$

c. $-2x + 7 = -21$

d. $25 = 10y + 8$

1-80. Copy the table below, complete it, and write an equation relating *x* and *y*.

x	–3	–2	–1	0	1	2	3	4
y	–7			2	5			14

1.2.4 How can I move it?

Defining Transformations

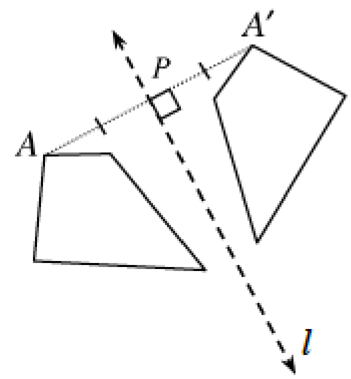
Methods and Meanings

Math Notes

Formal Definitions of Rigid Transformations

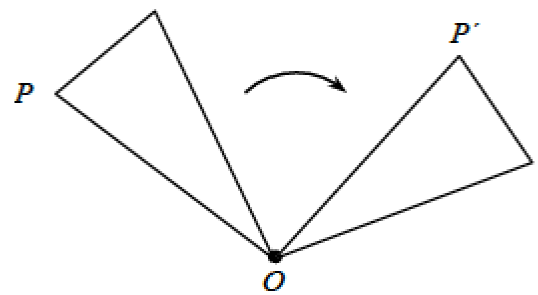
In algebra, you learned that a function is an equation that assigns each input a unique output. Most of these functions involved expressions and numbers such as $f(x) = 3x - 5$, so $f(2) = 1$.

In this course, you are now studying functions that assign each point in the plane to a unique point in the plane. These functions are called **rigid transformations (or motions)** because they move the entire plane with any figures you have drawn so that all of the figures remain unchanged. Therefore, angles and distances are preserved. There are three basic rigid motions that we will consider: reflections, translations, and rotations. All rigid motions can be seen as a combination of them.

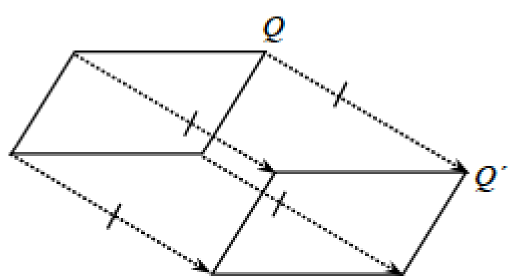


Reflections: When a figure is reflected across a line of reflection, such as the figure at right, it appears that the figure is "flipped" over the line. However, formally, a reflection across a line of reflection is defined as a function of each point (such as A) to a point (such as A') so that the line of reflection is the perpendicular bisector of the segments connecting the points and their images (such as $\overline{AA'}$). Therefore, $AP = A'P$.

Rotations: Formally, a rotation about a point O is a function that assigns each point (P) in the plane a unique point (P') so that all angles of rotation $\angle POP'$ have the same measure (which is the angle of rotation) and $OP = OP'$.



Translations: Formally, a translation is a function that assigns each point (Q) in the plane a unique point (Q') so that all line segments connecting an original point with its image have equal length and are parallel.



1-85. Plot the following points on another sheet of graph paper (insert a new page and use the graph paper option) and connect them in the order given. Then connect points *A* and *D*.

$A(-3,4)$, $B(1, 6)$, $C(5, -2)$, and $D(1, -4)$

a. A rectangle is a four-sided polygon with four right angles. Does the shape you graphed appear to be a rectangle? Use slope to justify your answer.

b. If *ABCD* is rotated 90° clockwise (↻) about the origin to form *A′B′C′D′*, what are the coordinates of the vertices of *A′B′C′D′*?

1-86. Solve for the variable in each equation below. Show the steps leading to your answer.

a. $8x - 22 = -60$

b. $\frac{1}{2}x - 37 = -84$

c. $\frac{3x}{4} = \frac{6}{7}$

d. $9a + 15 = 10a - 7$

1-87. While waiting for a bus after school, Renae programmed her MP3 player to randomly play two songs from her playlist, below. Assume that the MP3 player will not play the same song twice.

PLAYLIST

a. **I Love My Mama** (country)
by the Strings of Heaven

b. **Don't Call Me Mama** (country)
Duet by Sapphire and Hank Tumbleweed

c. **Carefree and Blue** (R & B)
by Sapphire and Prism Escape

d. **Go Back To Mama** (Rock)
Duet by Bjorn Free and Sapphire

e. **Smashing Lollipops** (Rock)
by Sapphire

a. A **sample space** is a list of all possible outcomes for a probabilistic situation. List the sample space for all the combinations of two songs that Renae could select. The order that she hears the songs does not matter for your list. How can you be sure that you listed all of the song combinations?

b. Are each of the combinations of two songs equally likely? Why is that important?

c. Find the probability that Renae will listen to two songs with the name "Mama" in the title.

PLAYLIST

a. **I Love My Mama** (country)
by the Strings of Heaven

b. **Don't Call Me Mama** (country)
Duet by Sapphire and Hank Tumbleweed

c. **Carefree and Blue** (R & B)
by Sapphire and Prism Escape

d. **Go Back To Mama** (Rock)
Duet by Bjorn Free and Sapphire

e. **Smashing Lollipops** (Rock)
by Sapphire

d. What is the probability that at least one of the songs will have the name “Mama” in the title?

e. Why does it make sense that the probability in part (d) is higher than the probability in part (c)?

1-88. On graph paper, graph the line through the point (0, −2) with slope $\frac{4}{3}$.

a. Write the equation of the line.

b. Translate the graph of the line up 4 and to the right 3 units. What is the result? Write the equation for the resulting line.

c. Now translate the original graph down 5 units. What is the result? Write the equation for the resulting line.

d. How are the three lines you graphed related to each other? Justify your conclusion.

e. Write the equation of a line that is perpendicular to these lines and passes through point (12, 7).

1-89. Evaluate the expression $\frac{1}{4}k^5 - 3k^3 + k^2 - k$ for $k = 2$.

1.2.5 What shapes can I create with triangles?

Using Transformations to Create Shapes

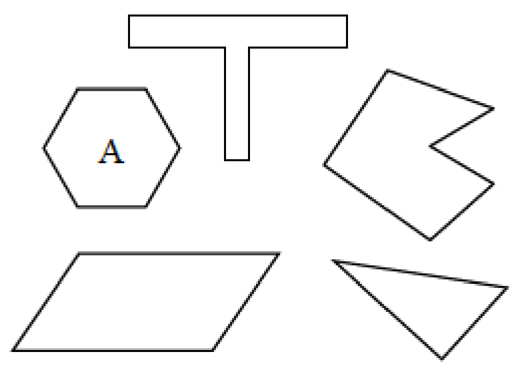
METHODS AND MEANINGS

MATH NOTES

Polygons

A **polygon** is defined as a two-dimensional closed figure made up of straight line segments connected end-to-end. These segments may not cross (intersect) at any other points.

Below are some examples of polygons.



Shape A above is an example of a **regular polygon** because its sides are all the same length and its angles have equal measure.

Polygons are named according to the number of sides that they have. Polygons that have 3 sides are **triangles**, those with 4 sides are **quadrilaterals**, polygons with 5 sides are **pentagons**, polygons with 6 sides are **hexagons**, polygons with 8 sides are **octagons**, polygons with 10 sides are **decagons**. For most other polygons, people simply name the number of sides, such as "11-gon" to indicate a polygon with 11 sides.

1-94. Augustin is in line to choose a new locker at school. The locker coordinator has each student reach into a bin and pull out a locker number. There is one locker at the school that all the kids dread! This locker, # 831, is supposed to be haunted, and anyone who has used it has had strange things happen to him or her! When it is Augustin's turn to reach into the bin and select a locker number, he is very nervous. He knows that there are 535 lockers left and locker # 831 is one of them. What is the probability that Augustin reaches in and pulls out the dreaded locker # 831? Should he be worried? Explain.

1-95. Lourdes has created the following challenge for you: She has given you three of the four points necessary to determine a rectangle on a graph. She wants you to find the points that "complete" each of the rectangles below. Create a new page and use the graph paper option.

a. (−1, 3), (−1, 2), (9, 2)

b. (3, 7), (5, 7), (5, −3)

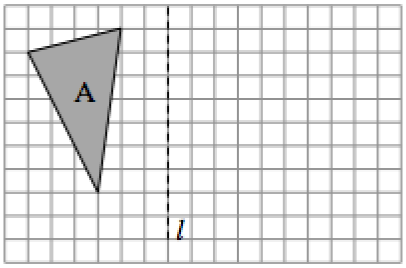
c. (−5, −5), (1, 4), (4, 2)

d. (−52, 73), (96, 73), (96, 1483)

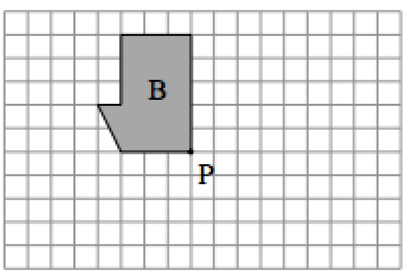
1-96. Find the area of the rectangles formed in parts (a), (b), and (d) of problem 1-95.

1-97. Copy the diagrams below on graph paper. Then find the result when each indicated transformation is performed.

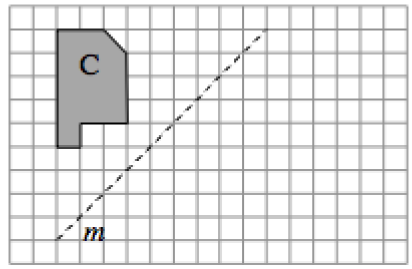
a. Reflect Figure A across line *l*.



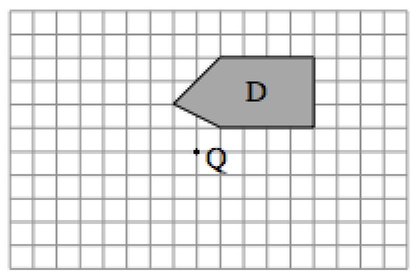
b. Rotate Figure B 90° clockwise (↻) about point P.



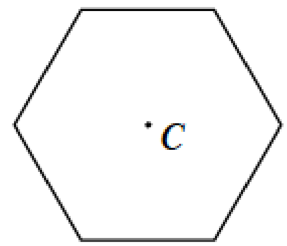
c. Reflect Figure C across line *m*.



d. Rotate Figure D 180° about point Q.

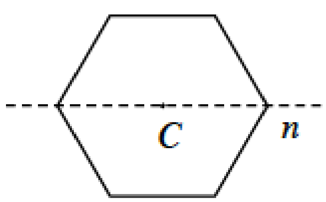


1-98. Below is a diagram of a **regular hexagon** with center *C*. A polygon is regular if all sides are equal and all angles are equal. Copy this figure on your paper, then answer the questions below.



a. Draw the result of rotating the hexagon about its center 180°. Explain what happened. When this happens, the shape has **rotation symmetry**.

b. What is the result when the original hexagon is reflected across line *n*, as shown below? A shape with this quality is said to have **reflection symmetry** and line *n* is a **line of symmetry** of the hexagon (not of the reflection).



c. Does a regular hexagon have any other lines of symmetry? That is, are there any other lines you could fold across so that both halves of the hexagon will match up? Find as many as you can.

1.2.6 What shapes have symmetry?

Symmetry

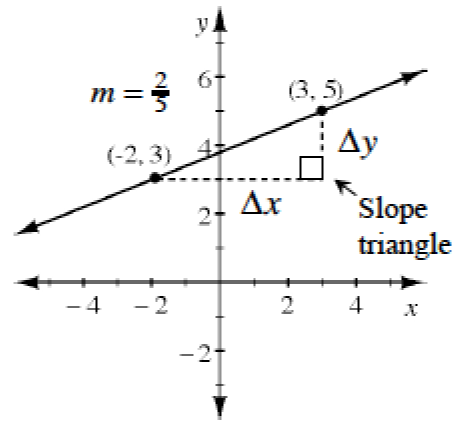
Methods and Meanings

Math Notes

Slope of a Line and Parallel and Perpendicular Slopes

During this course, you will use your algebra tools to learn more about shapes. One of your algebraic tools that can be used to learn about the relationship of lines is slope. Review what you know about slope below.

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\Delta y}{\Delta x}$$

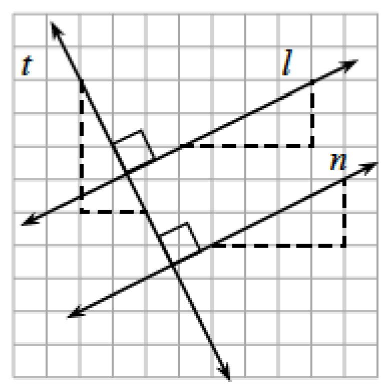


The **slope** of a line is the ratio of the change in y (Δy) to the change in x (Δx) between any two points on the line. It indicates both how steep the line is and its direction, upward or downward, left to right.

Lines that point upward from left to right have positive slope, while lines that point downward from left to right have negative slope. A horizontal line has zero slope, while a vertical line has undefined slope. The slope of a line is denoted by the letter m when using the $y = mx + b$ equation of a line.

One way to calculate the slope of a line is to pick two points on the line, draw a slope triangle (as shown in the example above), determine Δy and Δx, and then write the slope ratio. Be sure to verify that your slope correctly resulted in a negative or positive value based on its direction.

Parallel lines lie in the same plane (a flat surface) and never intersect. They have the same steepness, and therefore they grow at the same rate. Lines l and n below are examples of parallel lines.



On the other hand, **perpendicular** lines are lines that intersect at a right angle. For example, lines m and n above are perpendicular, as are lines m and l. Note that the small square drawn at the point of intersection indicates a right angle.

The **slopes of parallel lines** are the same. In general, the slope of a line parallel to a line with slope m is m.

The **slopes of perpendicular lines** are opposite reciprocals. For example, if one line has slope $\frac{4}{5}$, then any line perpendicular to it has slope $-\frac{5}{4}$. If a line has slope -3, then any line perpendicular to it has slope $\frac{1}{3}$. In general, the slope of a line perpendicular to a line with slope m is $-\frac{1}{m}$.

1-105. On graph paper, graph each of the lines below on the same set of axes. What is the relationship between lines (a) and (b)? What about between (b) and (c)?

a. $y = \frac{1}{3}x + 4$

b. $y = -3x + 4$

c. $y = -3x - 2$

1-106. The length of a side of a square is $5x + 2$ units. If the perimeter is 48 units, complete the following.

a. Write an equation to represent this information.

b. Solve for *x.*

c. What is the area of the square?

1-107. What is the probability of drawing each of the following cards from a standard playing deck? See the entry "playing cards" in the glossary to learn what playing cards are included a deck.

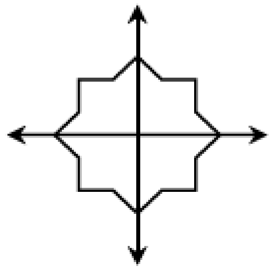
a. P(Jack)

b. P(spade)

c. P(Jack of spades)

d. P(not spade)

1-108. Examine the figure graphed on the axes below.



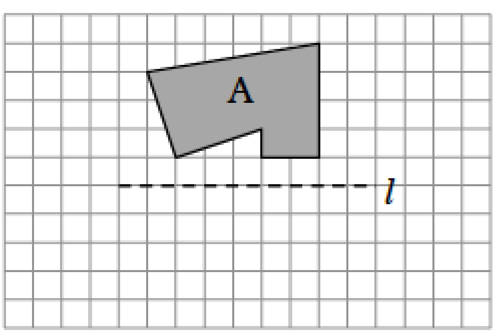
a. What happens when you rotate this figure about the origin 90°? 45°? 180°?

a. What other angle could the figure above be rotated so the shape does not appear to change?

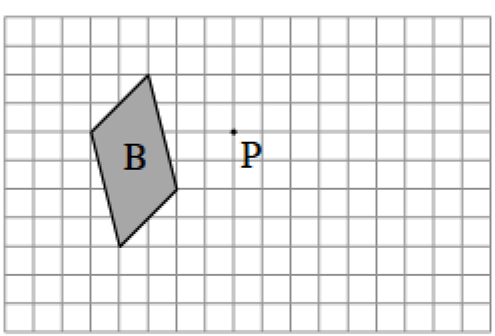
b. What shape will stay the same no matter how many degrees it is rotated?

1-109. Copy the diagrams below on graph paper. Then find each result when each indicated transformation is performed.

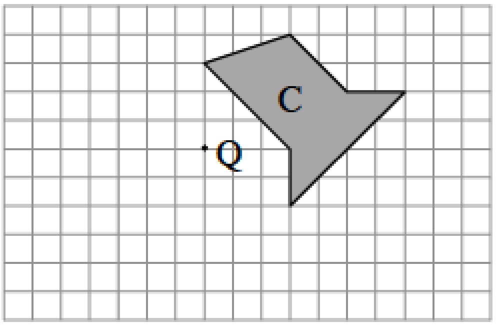
a. Reflect A across line *l*.



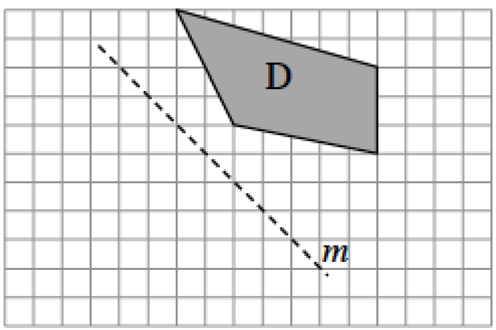
b. Rotate B 90° counterclockwise (↺) about point P.



c. Rotate C 180° about point Q.



d. Reflect D across line *m*.



1.3.1 How can I classify this shape?

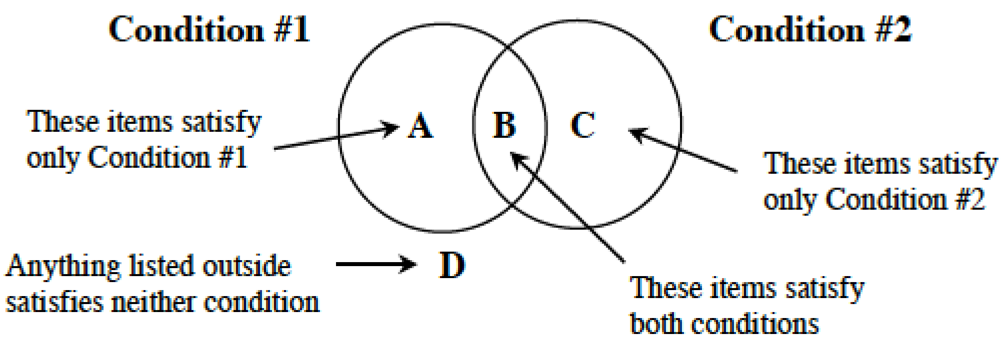
Attributes and Characteristics of Shapes

METHODS AND MEANINGS

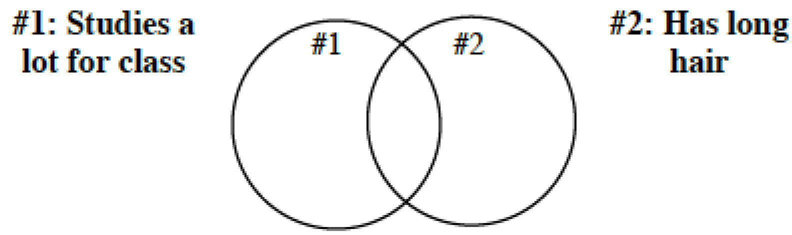
MATH NOTES

Venn Diagrams

A Venn diagram is a tool used to classify objects. It is usually composed of two or more circles that represent different conditions. An item is placed or represented in the Venn diagram in the appropriate position based on the conditions it meets. See the example below:



1-112. Copy the Venn diagram below on your paper. Then show where each person described should be represented in the diagram. If a portion of the Venn diagram remains empty, describe the qualities a person would need to belong there.



a. Carol: *"I rarely study and enjoy braiding my long hair."*

b. Bob: *"I never do homework and have a crew cut."*

c. Pedro: *"I love joining after school study teams to prepare for tests and I like being bald!"*

1-113. Sandy has a square, equilateral triangle, rhombus, and regular hexagon in her Shape Bucket, while Robert has a scalene triangle, kite, isosceles trapezoid, non-special quadrilateral, and obtuse isosceles triangle in his. Sandy will randomly select a shape from her Shape Bucket, while Robert will randomly select a shape from his.

a. Who has a greater probability of selecting a quadrilateral? Justify your conclusion.

b. Who has a greater probability of selecting an equilateral shape? Justify your conclusion.

c. What is more likely to happen: Sandy selecting a shape with at least two sides that are parallel or Robert selecting a shape with at least two sides that are equal?

1-114. Solve the equations below for *x*, if possible. Be sure to check your solution.

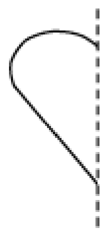
a. $\frac{3x-1}{4} = -\frac{5}{11}$

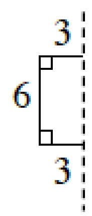
b. $(5 - x)(2x + 3) = 0$

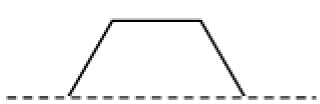
c. $6 - 5(2x - 3) = 4x + 7$

d. $\frac{3x}{4} + 2 = 4x - 1$

1-115. When the shapes below are reflected across the given line of reflection, the original shape and the image (reflection) create a new shape. For each reflection below, name the new shape that is created.

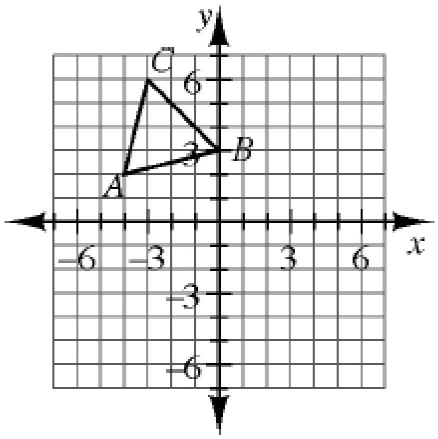
a. 

b. 

c. 

d. Use this method to create your own shape that has reflection symmetry. Add additional lines of symmetry. Note that the dashed lines of reflection in the figures above become lines of symmetry in the new shape.

1-116. Copy Δ*ABC* below on graph paper.



a. Rotate Δ*ABC* 90° counter-clockwise (↺) about the origin to create Δ*A′B′C′*. Name the coordinates of *C′* .

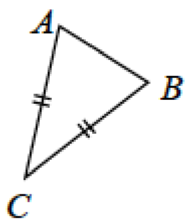
b. Reflect Δ*ABC* across the vertical line $x = 1$ to create *A″B″C″* . Name the coordinates of the vertices.

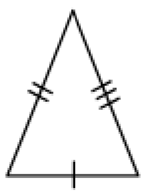
c. Translate Δ*ABC* so that *A‴* is at (4, −5). Name the coordinates of *B‴* .

1.3.2 How can I describe it?

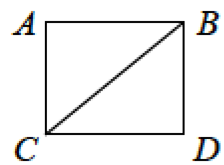
More Characteristics of Shapes

1-123. If no sides of a triangle have the same length, the triangle is called **scalene** (pronounced SCALE-een). And, as you might remember, if the triangle has two sides that are the same length, the triangle is called isosceles. Use the markings in each diagram below to decide if Δ*ABC* is isosceles or scalene. Assume the diagrams are not drawn to scale.

a. 

b. 

c. *ABDC* is a square



1-124. Find the probabilities of randomly selecting the following shapes from a Shape Bucket that contains all 16 basic shapes.

a. P(quadrilateral)

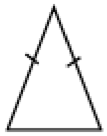
b. P(shape with an obtuse angle)

c. P(equilateral triangle)

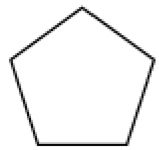
d. P(shape with parallel sides)

1-125. Without referring to your Shapes Toolkit, see if you can recall the names of each of the shapes below. Then check your answers with definitions from your Shapes Toolkit. How did you do?

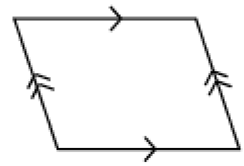
a.



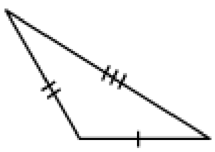
b.



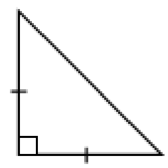
c.



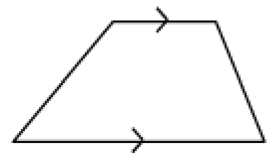
d.



e.

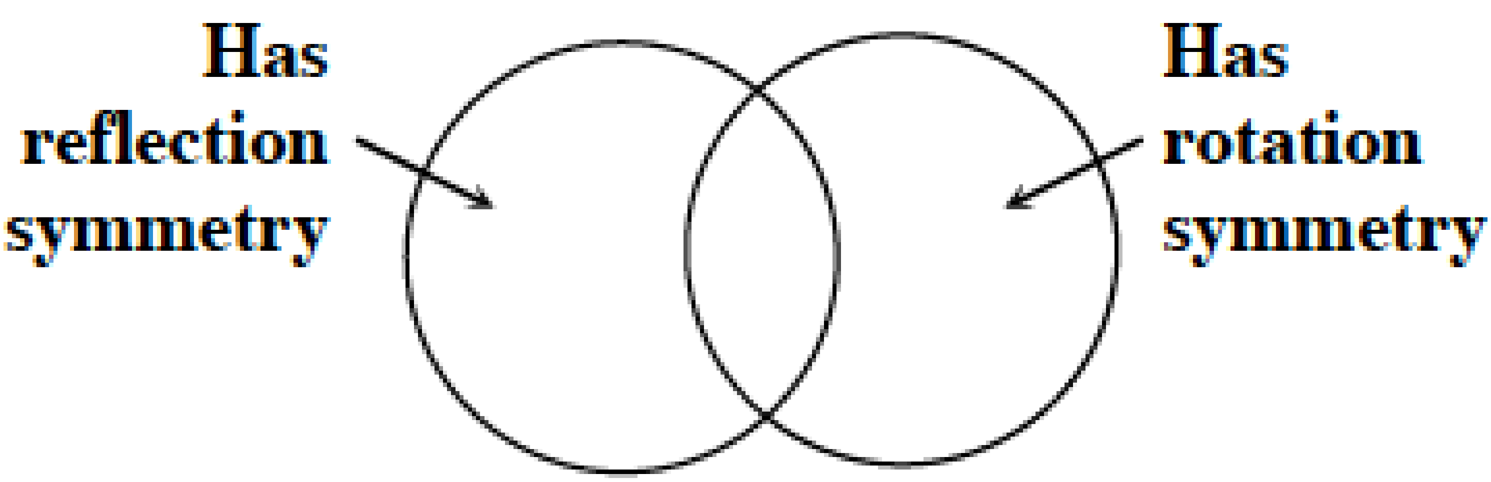


f.



1-126. Copy the Venn diagram below onto your paper. Then carefully place each capitalized letter of the alphabet below into your Venn diagram based on its type of symmetry.

A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z



1-127. Throughout this book, key problems have been selected as "checkpoints." Each checkpoint problem is marked with an icon like the one at left. These checkpoint problems are provided so that you can check to be sure you are building skills at the expected level. When you have trouble with checkpoint problems, refer to the review materials and practice problems that are available in the "Checkpoint Materials" section at the back of your book.

This problem is a checkpoint for solving linear equations. It will be referred to as Checkpoint 1

a. $3x + 7 = -x - 1$

b. $1 - 2x + 5 = 4x - 3$

c. $-2x - 6 = 2 - 4x - (x - 1)$

d. $3x - 4 + 1 = -2x - 5 + 5x$

Check your answers by referring to the Checkpoint 1 materials located at the back of your book.

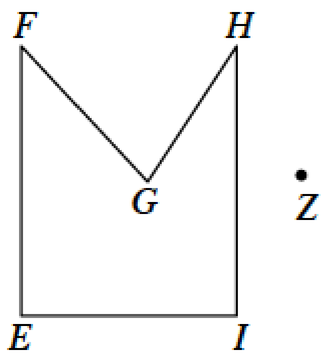
If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 1 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

Chapter 1 Closure What have I learned?

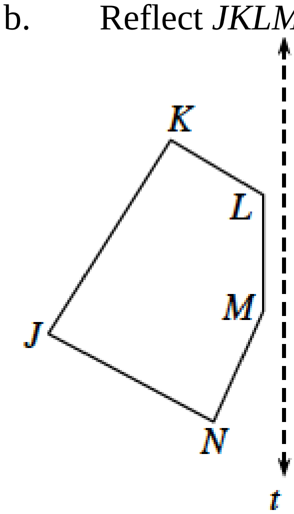
Reflection and Synthesis

CL 1-128. Trace the figures in parts (a) and (b) onto your paper and perform the indicated transformations. Copy the figure from part (c) onto graph paper and perform the indicated transformation. Label each image with prime notation ($A \rightarrow A'$).

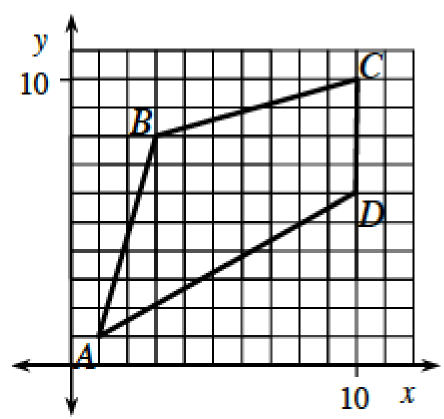
a. Rotate *EFGHI* 90° clockwise ↻ about point *Z*



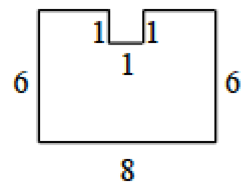
b. Reflect *JKLMN* over line *t*



c. Translate *ABCD* down 5 units and right 3 units

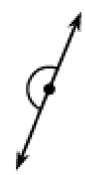


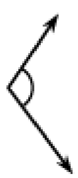
CL 1-129. Assume that all angles in the diagram below are right angles and that all the measurements are in centimeters. Find the perimeter of the figure.

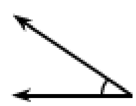


CL 1-130. Estimate the measures of the angles below. Are there any that you know for sure?

a.


c.


b.


d.


CL 1-131. Examine the angles in problem CL 1-130. If these four angles are placed in a bag, what is the probability of randomly selecting:

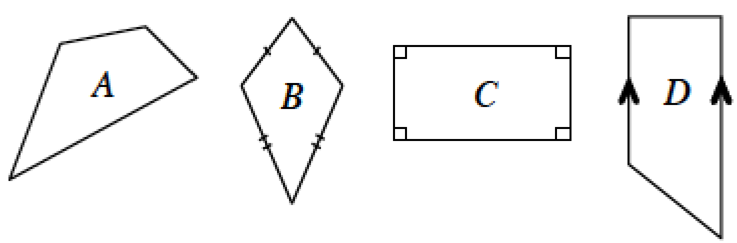
a. An acute angle

b. An angle greater than 60°

c. A 90° angle

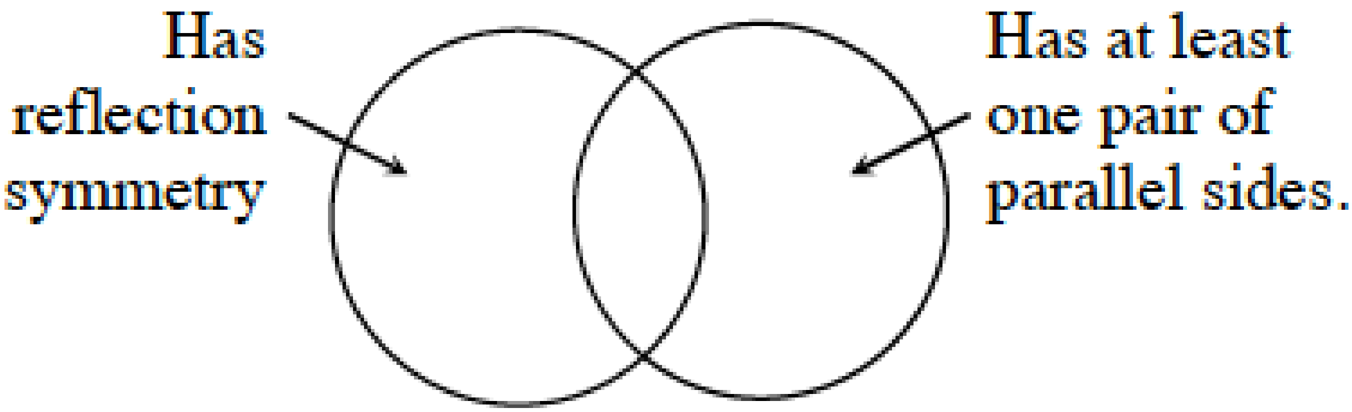
d. An angle less than or equal to 180°

CL 1-132. Examine the shapes below.



a. Describe what you know about each shape based on the information provided in the diagram. Then name the shape.

b. Decide where each shape would be placed in the Venn diagram below.



CL 1-133. Solve each equation below. Check your solution.

a. $3x - 12 + 10 = 8 - 2x$

b. $\frac{x}{7} = \frac{3}{2}$

c. $5 - (x + 7) + 4x = 7(x - 1)$

d. $x^2 + 11 = 36$

CL 1-134. Find the value of y for each equation twice: first for $x = 8$, then for $.x = -3$.

a. $y = x^2 + 13x + 2$

b. $y = 6x - 2$

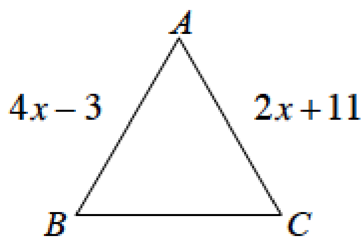
CL 1-135. Graph and connect the points in the table below. Then graph the equation in part (b) on the same set of axes. Also, find the equation for the data in the table.

a.

x	−4	−3	−2	−1	0	1	2	3	4	5	6
y	−5	−3	−1	1	3	5	7	9	11	13	15

b. $y = x^2 + x - 2$

CL 1-136. ΔABC below is equilateral. Use what you know about an equilateral triangle to write and solve an equation for x. Then find the perimeter of ΔABC.



CL 1-137. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in math classes you have taken before? Use the table to make a list of topics you need help on and a list of topics you need to practice more.