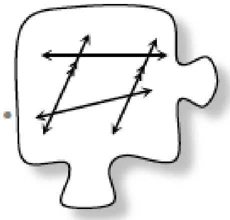
2.1.1 What is the relationship?

Complementary, Supplementary, and Vertical Angles

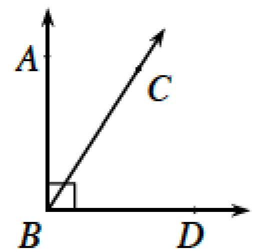


METHODS AND MEANINGS

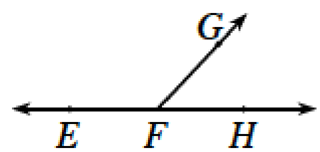
MATH NOTES

Angle Relationships

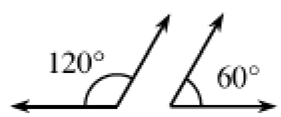
If two angles have measures that add up to 90°, they are called **complementary angles**. For example, in the diagram at right, $\angle ABC$ and $\angle CBD$ are complementary because together they form a right angle.



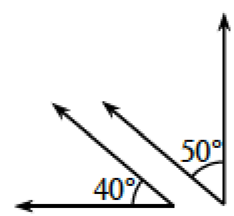
If two angles have measures that add up to 180°, they are called **supplementary angles**. For example, in the diagram at right, $\angle EFG$ and $\angle GFH$ are supplementary because together they form a straight angle.



Two angles do not have to share a vertex to be complementary or supplementary. The first pair of angles at right are supplementary; the second pair of angles are complementary.

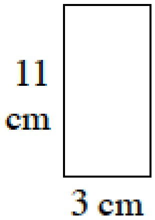


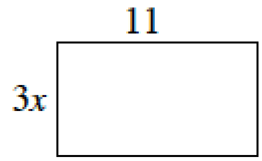
Supplementary

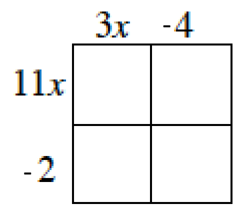


Complementary

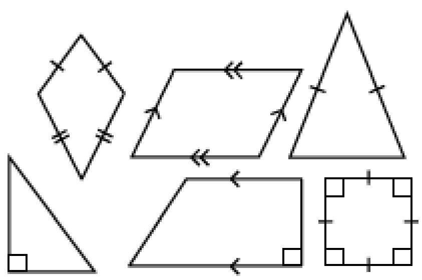
2-8. Find the area of each rectangle below.

a. 

b. 

c. 

2-9. Mei puts the shapes below into a bucket and asks Brian to pick one out.



a. What is the probability that he pulls out a quadrilateral with parallel sides?

b. What is the probability that he pulls out a shape with rotation symmetry?

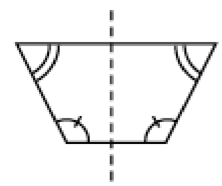
2-10. Camille loves guessing games. She is going to tell you a fact about her shape to see if you can guess what it is.

a. "My triangle has only one line of symmetry. What is it?"

b. "My triangle has three lines of symmetry. What is it?"

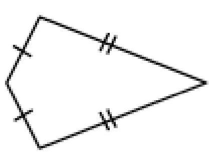
c. "My quadrilateral has no lines of symmetry but it does have rotation symmetry. What is it?"

2-11. Jerry has an idea. Since he knows that an isosceles trapezoid has reflection symmetry, he reasons, "That means that it must have two pairs of angles have equal measure." He marks this relationship on his diagram to the right.

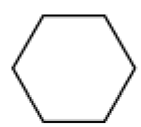


ISOSCELES TRAPEZOID

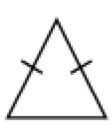
Similarly mark which angles must have equal measure due to reflection symmetry.

a. 

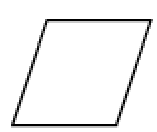
KITE

c. 

REGULAR HEXAGON

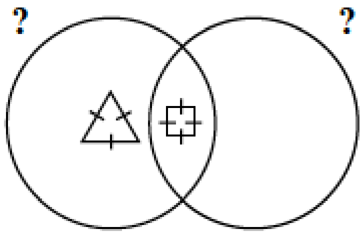
b. 

ISOSCELES TRIANGLE

d. 

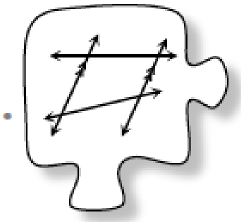
RHOMBUS

2-12. Larry saw Javon's incomplete Venn diagram below, and he wants to finish it. However, he does not know the condition that each circle represents. Find a possible label for each circle, and place two more shapes into the diagram.



2.1.2 What is the relationship?

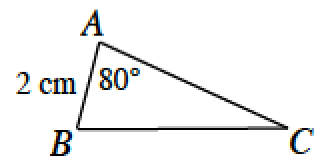
Angles Formed by Transversals



METHODS AND MEANINGS

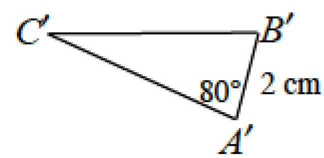
MATH NOTES

Naming Parts of Shapes



Part of geometry is the study of parts of shapes, such as points, line segments, and angles. To avoid confusion, standard notation is used to name these parts.

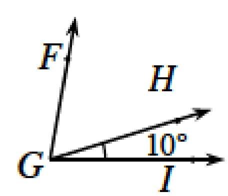
A **point** is named using a single capital letter. For example, the vertices (corners) of the triangle at right are named *A*, *B*, and *C*.



If a shape is transformed, the image shape is often named using **prime notation**. The image of point *A* is labeled *A*′ (read as "A prime"), the image of *B* is labeled *B*′ (read as "B prime"), etc. At right, *A*′*B*′*C*′ is the image of Δ*ABC*.

The side of a polygon is a line segment. A **line segment** is a portion of a line between two points and is named by naming its endpoints and placing a bar above them. For example, one side of the first triangle above is named $\overline{AB}$. When referring to the length of a segment, the bar is omitted. In Δ*ABC* above, *AB* = 2 cm.

A **line**, which differs from a segment in that it extends infinitely in either direction, is named by naming two points on the line and placing a bar with arrows above them. For example, the line below is named $\overleftrightarrow{DE}$. When naming a segment or line, the order of the letters is unimportant. The line below could also be named $\overleftrightarrow{ED}$.

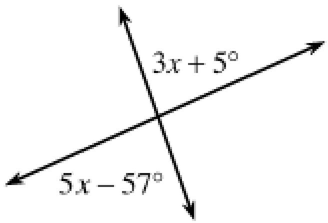


An **angle** can be named by putting an angle symbol in front of the name of the angle's vertex. For example, the angle measuring 80° in Δ*ABC* above is named ∠*A*. Sometimes using a single letter makes it unclear which angle is being referenced. For example, in the diagram at right, it is unclear which angle is referred to by ∠*G*. When this happens, the angle is named with three letters. For example, the angle measuring 10° is called ∠*HGI* or ∠*IGH*. Note that the name of the vertex must be the second letter in the name; the order of the other two letters is unimportant.

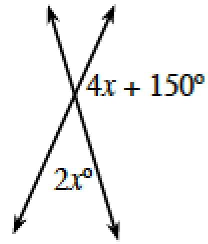
To refer to an angle's measure, an *m* is placed in front of the angle's name. For example, $m\angle HGI = 10°$ means "the measure of ∠*HGI* is 10°."

2-19. Examine the diagrams below. What is the geometric relationship between the labeled angles? What is the relationship of their measures? Then, use the relationship to write an equation and solve for *x*.

a.

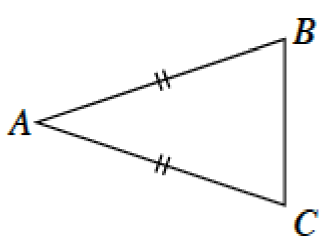


b.



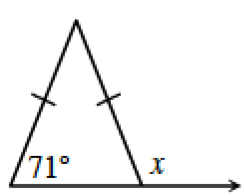
2-20. In problem 2-11, you determined that because an isosceles triangle has reflection symmetry, then it must have two angles that have equal measure.

a. How can you tell which angles have equal measure? For example, in the diagram below, which angles must have equal measure? Name the angles and explain how you know.



b. Examine the diagram for part (a). If you know that $m\measuredangle B + m\measuredangle C = 124°$, then what is the measure of $\measuredangle B$? Explain how you know.

c. Use this idea to find the value of *x* in the diagram below. Be sure to show all work.



2-21. On graph paper, draw the quadrilateral with vertices $(-1, 3)$, $(4, 3)$, $(-1, -2)$, and $(4, -2)$.

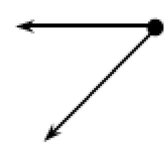
a. What kind of quadrilateral is this?

b. Translate the quadrilateral 3 units to the left and 2 units up. What are the new coordinates of the vertices?

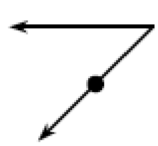
2-22. Find the equation for the line that passes through $(-1, -2)$ and $(4, 3)$. Is the point $(3, 1)$ on this line? Be sure to justify your answer.

2-23. Juan decided to test what would happen if he rotated an angle.

a. He copied the angle at right on tracing paper and rotated it 180° about its vertex. What type of angle pair did he create? What is the relationship of these angles?



b. Juan then rotated the same angle 180° through a different point (see the diagram below). On your paper, draw Juan's angle and the rotated image. Describe the overall shape formed by the two angles.



2.1.3 What is the relationship?

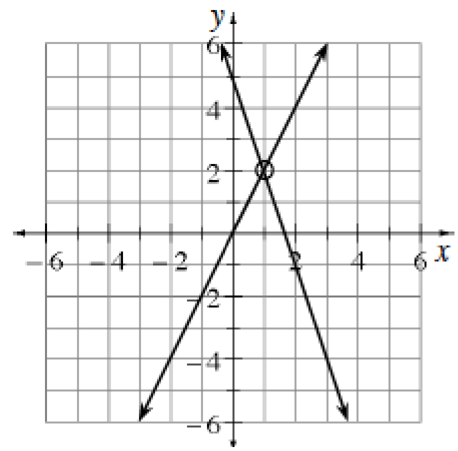
More Angles Formed by Transversals

Systems of Linear Equations

In a previous course, you learned that a **system of linear equations** is a set of two or more linear equations that are given together, such as the example at right. In a system, each variable represents the same quantity in both equations. For example, *y* represents the same quantity in *both* equations below.

$$y = 2x \ \& \ y = -3x + 5$$

To represent a system of equations graphically, you can simply graph each equation on the same set of axes. The graph may or may not have a **point of intersection**, as shown circled at right.



Sometimes two lines have *no* points of intersection. This happens when the two lines are parallel. It is also possible for two lines to have an *infinite* number of intersections. This happens when the graphs of two lines lie on top of each other. Such lines are said to **coincide**.

The **Substitution Method** is a way to change two equations with two variables into one equation with one variable. It is convenient to use when only one equation is solved for a variable. For example, to solve the system at right:

$$x = -3y + 1$$
$$4x - 3y = -11$$

- Use substitution to rewrite the two equations as one. In other words, replace *x*with (−3*y* + 1) to get 4(−3*y* + 1 − 3*y* = −11. This equation can then be solved to find *y*. In this case, *y* = 1.

$$x = -3y + 1$$
$$4(\quad) - 3y = -11$$
$$4(-3y + 1) - 3y = -11$$
$$-12y + 4 - 3y = -11$$
$$-15y + 4 = -11$$
$$-15y = -15$$
$$y = 1$$

- To find the point of intersection, substitute to find the other value.

- Substitute *y* = 1 into *x* = −3*y* + 1 and write the answer for *x* and *y* as an ordered pair.

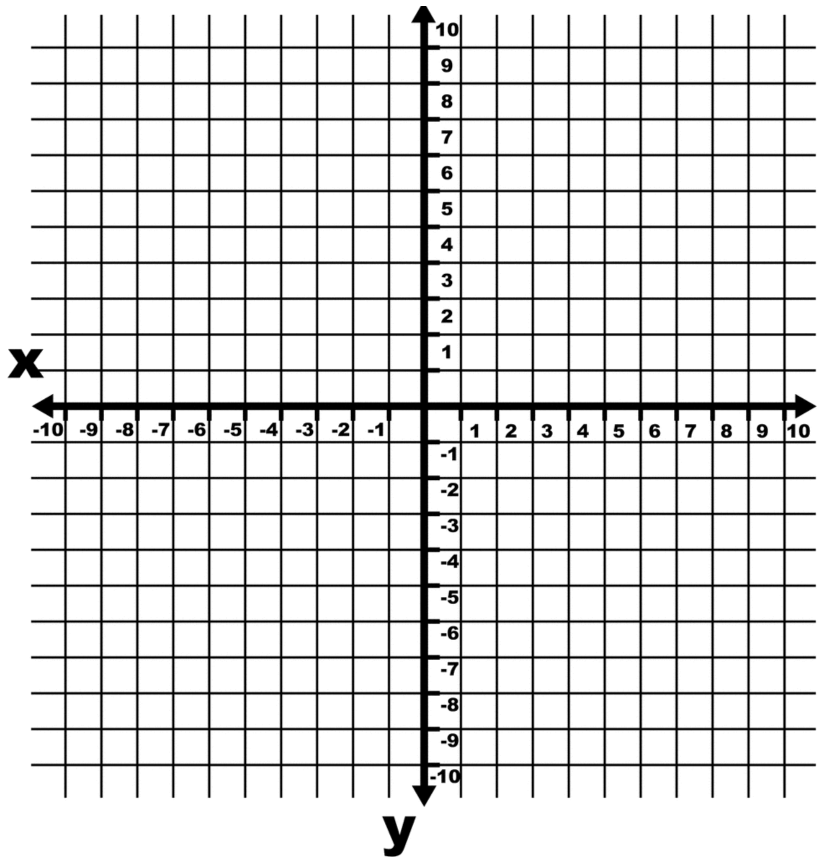
$$x = -3(1) + 1 = -2$$
$$(-2, 1)$$

- To test the solution, substitute *x* = −2 and *y* = 1 into 4*x* − 3*y* = −11 to verify that it makes the equation true. Since 4(−2) − 3(1) = −11, the solution (−2, 1) must be correct.

2-31. The set of equations below is an example of a **system of equations**. Read the Math Notes box for this lesson on how to solve systems of equations. Then answer the questions below.

$$y = -x + 1$$
$$y = 2x + 7$$

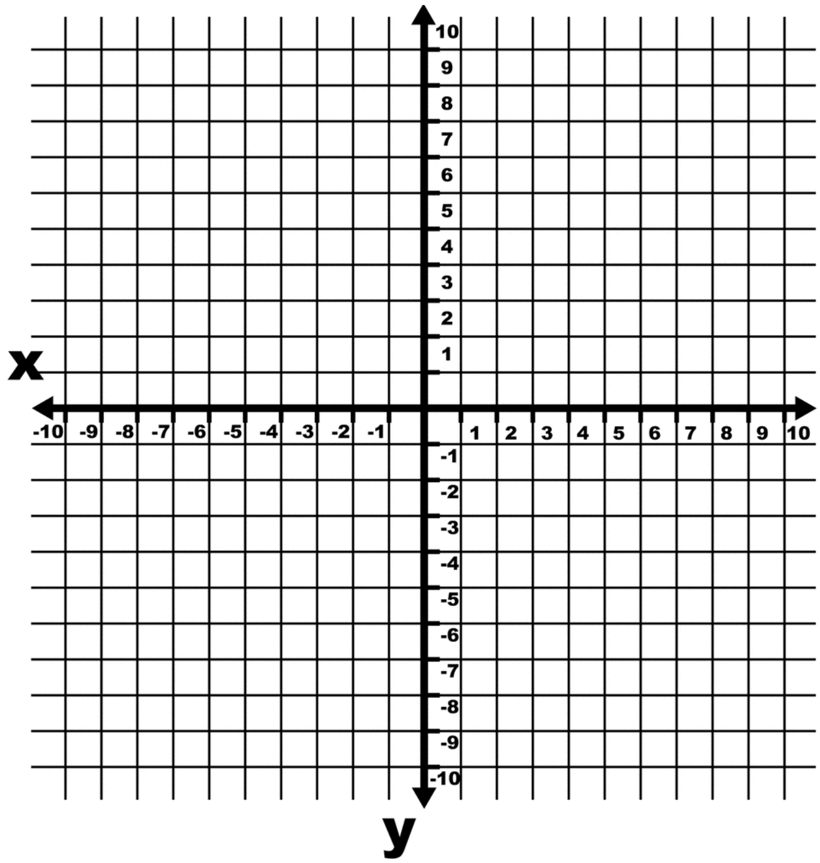
a. Graph the system on graph paper. Then write its solution (the point of intersection) in (*x*, *y*) form.



b. Now solve the system using an algebraic method of your choice. Did your solution match your result from part (a)? If not, check your work carefully and look for any mistakes in your algebraic process or on your graph.

2-32. On graph paper, graph the rectangle with vertices at (2, 1), (2, 5), (7, 1), and (7, 5).

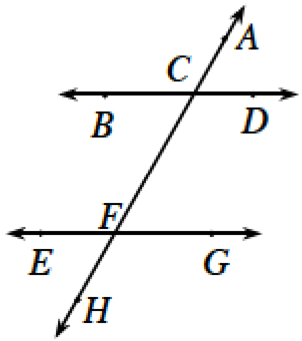
a. What is the area of this rectangle?

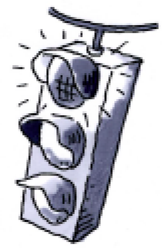


b. Shirley was given the following points and asked to find the area, but her graph paper is not big enough. Find the area of Shirley's rectangle, and explain to her how she can find the area without graphing the points.

Shirley's points: (352, 150), (352, 175), (456, 150), and (456, 175)

2-33. Looking at the diagram below, John says that $m\angle BCF = m\angle EFH$.





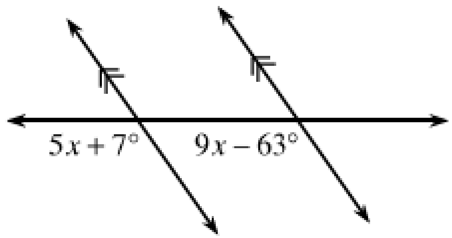
Note: This stoplight icon will appear periodically throughout the text. Problems with this icon display common errors that can be made. Be sure not to make the same mistakes yourself!

a. Do you agree with John? Why or why not?

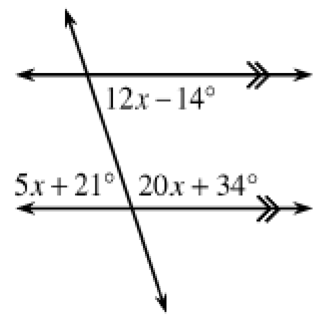
b. Jim says, "*You can't be sure those angles are equal. An important piece of information is missing from the diagram!*" What is Jim talking about?

2-34. Use your knowledge of angle relationships to solve for *x* in the diagrams below. Justify your solutions by naming the geometric relationship.

a.



b.



2-35. When Ms. Shreve randomly selects a student in her class, she has a $\frac{1}{3}$ probability of selecting a boy.

a. If her class has 36 students, how many boys are in Ms. Shreve's class?

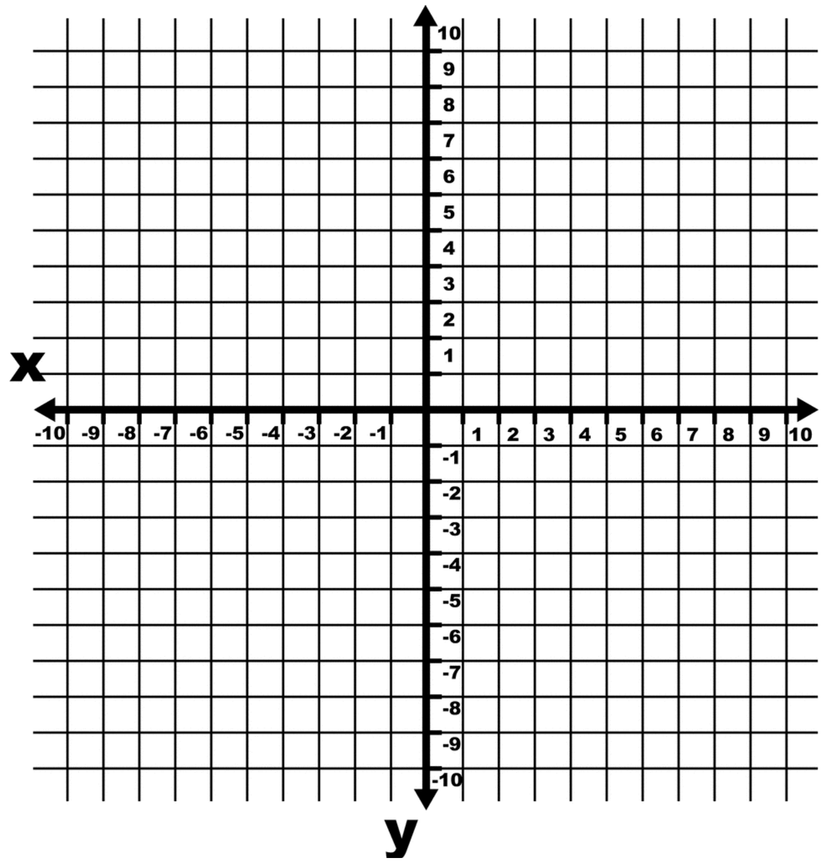
b. If there are 11 boys in her class, how many girls are in her class?

c. What is the probability that she will select a girl?

d. Assume that Ms. Shreve's class has a total of 24 students. She selected one student (who was a boy) to attend a fieldtrip and then was told she needed to select one more student to attend. What is the probability that the second randomly selected student will also be a boy?

2-36. On graph paper, draw line segment $\overline{AB}$ if *A* (6, 2) and *B* (3, 5).

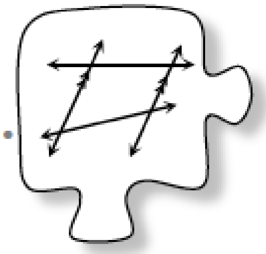
a. Reflect $\overline{AB}$ across the line *x* = 3 and connect points *A* and *A′*. What shape is created by this reflection? Be as specific as possible.



b. What polygon is created when $\overline{AB}$ is reflected across the line *y* = −*x* + 6 and all endpoints are connected to form a polygon?

2.1.4 How can I use it?

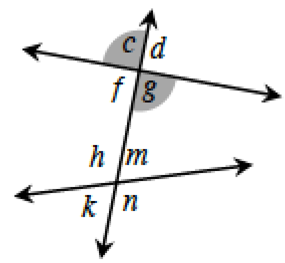
Angles in a Triangle



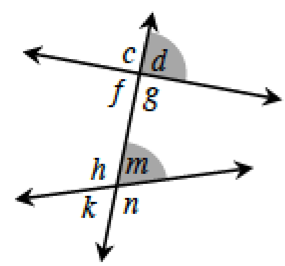
METHODS AND MEANINGS

MATH NOTES

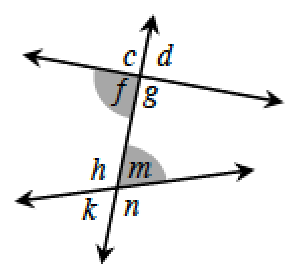
More Angle Pair Relationships



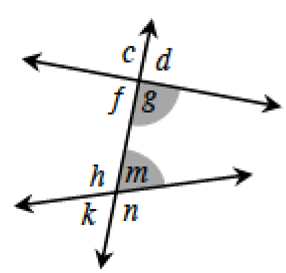
Vertical angles are the two opposite (that is, non-adjacent) angles formed by two intersecting lines, such as angles $\angle c$ and $\angle g$ in the diagram at right. $\angle c$ by itself is not a vertical angle, nor is $\angle g$, although $\angle c$and $\angle g$ together are a pair of vertical angles. Vertical angles always have equal measure.



Corresponding angles lie in the same position but at different points of intersection of the transversal. For example, in the diagram at right, $\angle m$ and $\angle d$ form a pair of corresponding angles, since both of them are to the right of the transversal and above the intersecting line. Corresponding angles are congruent when the lines intersected by the transversal are parallel.



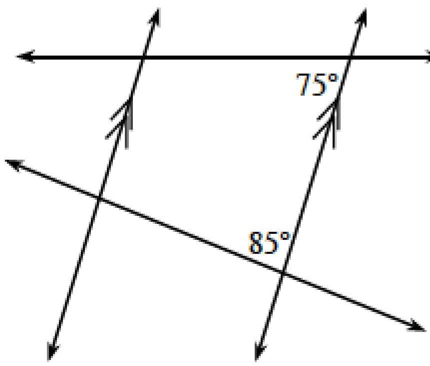
$\angle f$ and $\angle m$ are **alternate interior angles** because one is to the left of the transversal, one is to the right, and both are between (inside) the pair of lines. Alternate interior angles are congruent when the lines intersected by the transversal are parallel.



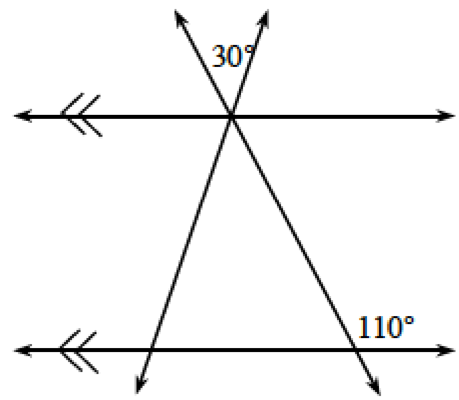
$\angle g$ and $\angle m$ are **same-side interior angles** because both are on the same side of the transversal and both are between the pair of lines. Same-side interior angles are supplementary when the lines intersected by the transversal are parallel.

2-41. Find all missing angles in the diagrams below.

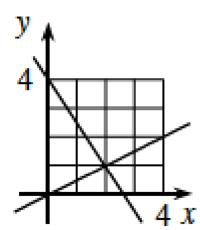
a.



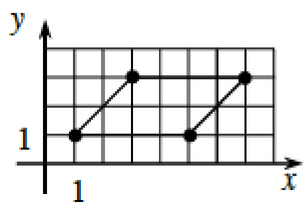
b.



2-42. Robert believes the lines graphed below are perpendicular, but Mario is not convinced. Find the slope of each line, and explain how you know whether or not the lines are perpendicular.



2-43. The diagram below represents only half of a shape that has the graph of $y = 1$ as a line of symmetry. Draw the completed shape on your paper, and label the coordinates of the missing vertices.



2-44. Janine measured the sides of a rectangle and found that the sides were 12 inches and 24 inches. Howard measured the same rectangle and found that the sides were 1 foot and 2 feet. When their math teacher asked them for the area, Janine said 288, and Howard said 2. Why did they get two different numbers for the area of the same rectangle?

2-45. Solve the system of equations at right using the method of your choice. Then state the solution to the system. If there is not a solution, explain why.

$$y = -\tfrac{2}{5}x + 1$$
$$y = -\tfrac{2}{5}x - 2$$

2.1.5 What is the relationship?

Applying Angle Relationships

METHODS AND MEANINGS

MATH NOTES

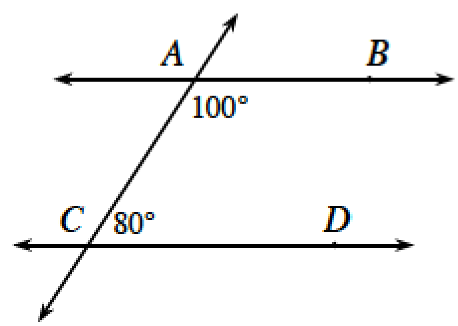
Proof by Contradiction

The kind of argument you used in Lesson 2.1.5 to justify “If same-side interior angles are supplementary, then lines are parallel” is sometimes called a **proof by contradiction**. In a proof by contradiction, you prove a claim by thinking about what the consequences would be if it were false. If the claim’s being false would lead to an impossibility, this shows that the claim must be true.

For example, suppose you know Mary’s brother is seven years younger than Mary. Can you argue that Mary is at least five years old? A proof by contradiction of this claim would go:

> *Suppose* Mary is less than five years old.
>
> *Then* her brother’s age is negative!
>
> *But* this is impossible, so Mary must be at least five years old.

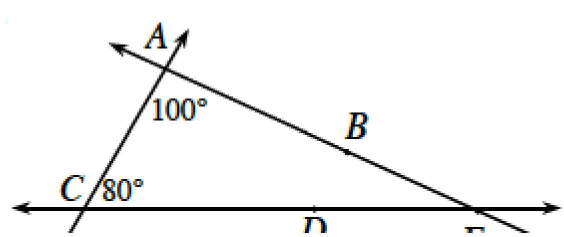
To show that lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ must be parallel in the diagram to the right, you used a proof by contradiction. You argued:



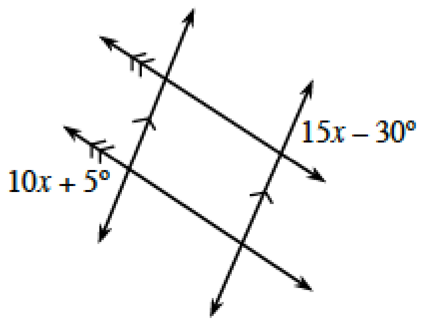
Suppose $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at some point *E*.
Then the angles in ΔAEC add up to more than 180°.

But this is impossible, so $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ must be parallel.

This is true no matter on which side of $\overleftrightarrow{AC}$ point *E* is assumed to be.

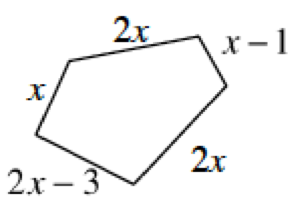


2-55. Solve for *x* in the diagram below.

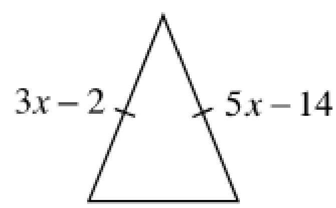


2-56. For each diagram below, set up an equation and solve for *x*.

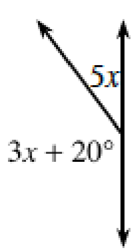
a. Perimeter = 76 units



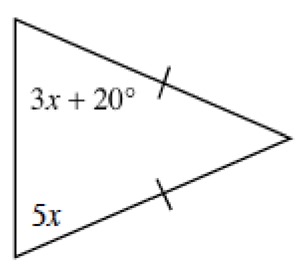
b.



c.



d.



2-57. Solve each system of equation below. Then verify that your solution makes each equation true. You may want to refer to the Math Notes box in Lesson 2.1.3.

a. $y = 5x - 2$

$y = 2x + 10$

b. $x = -2y - 1$

$2x + y = -20$

2-58. Graph the line $y = \frac{3}{4}x$ on graph paper.

a. Draw a slope triangle.

b. Rotate your slope triangle 90° around the origin to get a new slope triangle. What is the new slope?

c. Find the equation of a line perpendicular to $y = \frac{4}{3}x$.

2-59. Mario has 6 shapes in a bucket. He tells you that the probability of pulling an isosceles triangle out of the bucket is $\frac{1}{3}$. How many isosceles triangles are in his bucket?

2.2.1 How can I measure an object?

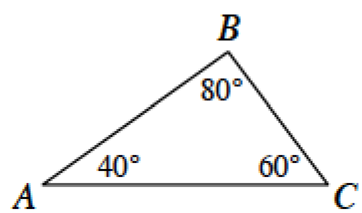
Units of Measure

METHODS AND MEANINGS

MATH NOTES

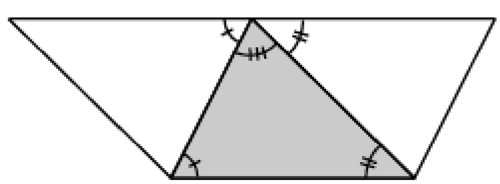
Triangle Angle Sum Theorem

The **Triangle Angle Sum Theorem** states that the measures of the angles in a triangle add up to 180°. For example, in ΔABC below:



$$m\angle A + m\angle B + m\angle C = 180°$$

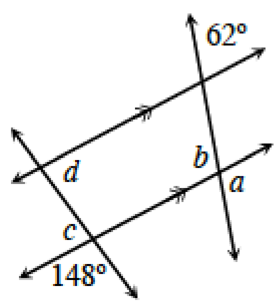
The Triangle Angle Sum Theorem can be verified by using a tiling of the given triangle (shaded below). Because the tiling produces parallel lines, the alternate interior angles must be congruent. As seen in the diagram below, the three angles of a triangle form a straight angle. Therefore, the sum of the angles of a triangle must be 180°.



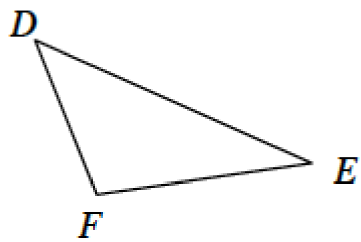
2-65. Examine the shapes in your Shape Toolkit. Then name all of the shapes in the Shape Toolkit that share both of the following qualities.

- They have only one line of symmetry.
- They have fewer than four sides.

2-66. Examine the diagram below. Then use the information provided in the diagram to find the measures of angles *a*, *b*, *c*, and *d*. For each angle, name the relationship from your Angle Relationships Toolkit that helped justify your conclusion. For example, did you use vertical angles? If not, what type of angle did you use?



2-67. Examine the triangle below.



a. If $m\angle D = 48°$ and $m\angle F = 117°$, then what is $m\angle E$?

b. Solve for x if $m\angle D = 4x + 2°$, $m\angle F = 7x - 8°$, and $m\angle E = 4x + 6°$. Then find $m\angle D$.

c. If $m\angle D = m\angle F = m\angle E$, what type of triangle is ΔFED?

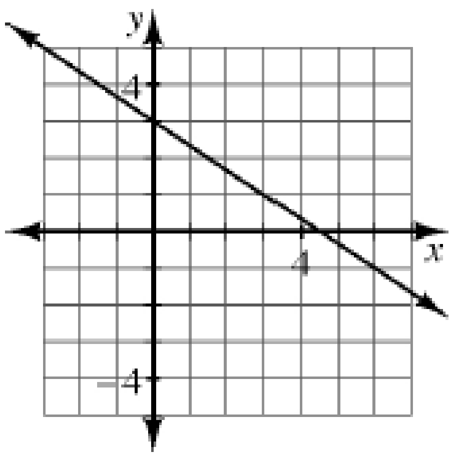
2-68. Plot ΔABC on graph paper if A (6, 3), B (2, 1), and C (5, 7).

a. ΔABC is rotated about the origin 180° to become $\Delta A'B'C'$. Name the coordinates of A', B', and C'.

b. This time ΔABC is rotated 180° about point C to form $\Delta A''B''C''$. Name the coordinates of B''.

c. If ΔABC is rotated 90° clockwise (↻) about the origin to form $\Delta A'''B'''C'''$, what are the coordinates of point A'''?

2-69. Examine the graph below.



a. Find the equation of the line.

b. Is the line $y = \frac{3}{2}x + 1$ perpendicular ($\perp$) to this line? How do you know?

c. On graph paper, graph $\overleftrightarrow{AB}$ if $A\ (-2, 4)$ and $B\ (4, 7)$. Then find the equation of $\overleftrightarrow{AB}$.

d. Find an equation of $\overleftrightarrow{AC}$ if $\overleftrightarrow{AC} \perp \overleftrightarrow{AB}$ from part (c).

2.2.2 How can I find the area?

Areas of Triangles and Composite Shapes

METHODS AND MEANINGS

MATH NOTES

Multiplying Binomials

One method for multiplying binomials is to use a generic rectangle. That is, use each factor of the product as a dimension of a rectangle and find its area. If $(2x + 5)$ is the base of a rectangle and $(3x - 1)$ is the height, then the expression $(2x + 5)(3x - 1)$ is the area of the rectangle. See the example below.

	$2x$	$+5$	
-1	$-2x$	-5	-1
$3x$	$6x^2$	$15x$	$3x$
	$2x$	$+5$	

Multiply: $(2x + 5)(3x - 1) = 6x^2 - 2x + 15x - 5$

$= 6x^2 + 13x - 5$

2-74. Review how to multiply binomials by reading the Math Notes box for this lesson. Then rewrite each of the expressions below by multiplying binomials and simplifying the resulting expression.

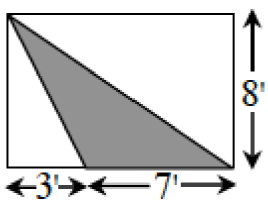
a. $(4x + 1)(2x - 7)$

b. $(5x - 2)(2x + 7)$

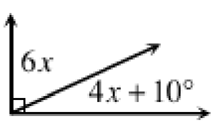
c. $(4x - 3)(x - 11)$

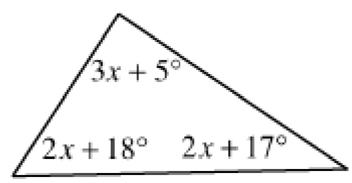
d. $(-3x + 1)(2x - 5)$

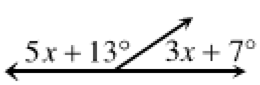
2-75. The shaded triangle below is surrounded by a rectangle. Find the area of the triangle.

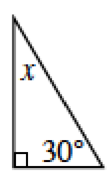


2-76. For each diagram below, solve for *x*. Explain what relationship(s) from your Angle Relationships Toolkit you used for each problem.

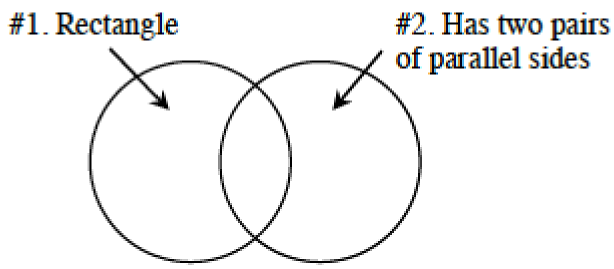
a. 

c. 

b. 

d. 

2-77. Daniel and Mike were having an argument about where to place a square in the Venn diagram below. Daniel wants to put the square in the intersection (the region where the two circles overlap).



Mike doesn't think that's right. "*I think it should go in the right region because it is a square, not a rectangle.*"

"*But a square IS a rectangle!*" protests Daniel. Who is right? Explain your thinking.

2-78. Flo thinks she may be able to increase sales if she makes a big show of flipping pancakes at her diner. But flipping pancakes high in the air takes a lot of practice!

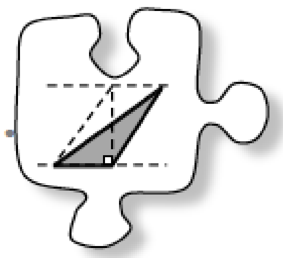
a. After flipping 35 pancakes, only 22 landed correctly on the grill. What is the probability (expressed in percent) of Flo correctly flipping a pancake?

b. Flo needs 42 pancakes for a large hungry group that just arrived. How many pancakes should she attempt to flip so that 42 flip correctly?

c. A customer orders a side of "Flo's grab bag of flapjacks" in which a customer gets one randomly chosen pancake. Flo has prepared a pan of 12 sourdough pancakes and 15 buttermilk pancakes. How many banana pancakes should Flo add to the pan if she wants the probability of randomly grabbing one banana pancake to be $\frac{1}{10}$?

2.2.3 What is the area?

Areas of Parallelograms and Trapezoids



METHODS AND MEANINGS

MATH NOTES

Conditional Statements

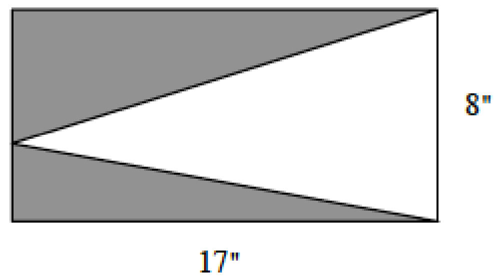
A **conditional statement** is written in the form "**If …, then …**." Here are some examples of conditional statements:

> *If a shape is a rhombus, then it has four sides of equal length.*
>
> *If it is February 14th, then it is Valentine's Day.*
>
> *If a shape is a parallelogram, then its area is $A = bh$.*

2-85. Berti is the Shape Factory's top employee. She has received awards every month for having the top sales figures so far for the year. If she stays on top, she will receive a $5000 bonus for excellence. She currently has sold 16,250 shapes and continues to sell 340 per month.

Since there are eight months left in the sales year, Sarita is working hard to catch up. While she has only sold 8,830 shapes, she is working overtime and on weekends so that she can sell 1,082 per month. Will Sarita catch up with Berti before the end of the sales year? If so, when?

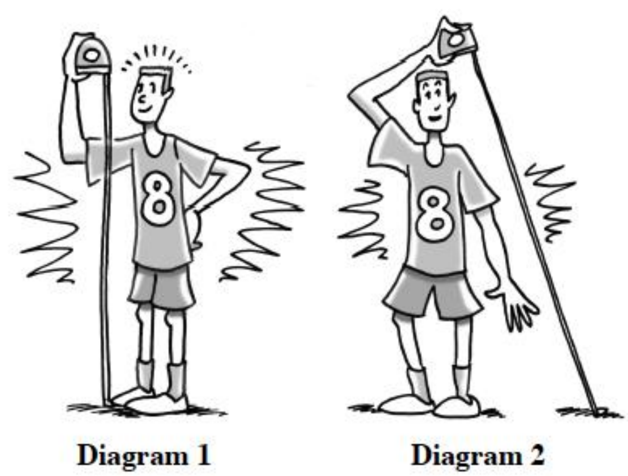
2-86. Calculate the area of the shaded region below. Use the appropriate units.



2-87. How tall are you? How do you measure your height? Consider these questions as you answer the questions below.

a. Why do you stand up straight to measure your height?

b. Which diagram (to the right) best represents how you would measure your height? Why?



c. When you measure your height, do you measure up to your chin? Down to your knees? Explain.

2-88. Read the Math Notes box for this lesson. Then rewrite each of the following statements as a conditional statement.

a. Mr. Spelling is always unhappy when it rains.

b. The sum of two even numbers is always even.

c. Marla has a piano lesson every Tuesday.

2-89. Simplify the following expressions.

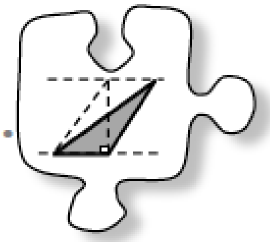
a. $2x + 8 + 6x + 5$

b. $15 + 3(2x - 4) - 4x$

c. $(x - 3)(3x + 4)$

d. $5x(2x + 7) + x(3x - 5)$

2.2.4 How can I find the height?



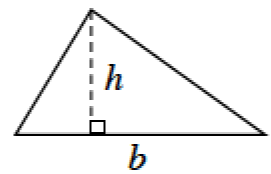
Heights and Areas

METHODS AND MEANINGS

MATH NOTES

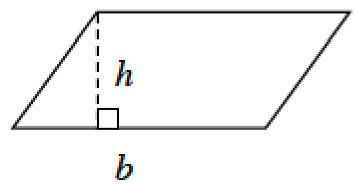
Areas of a Triangle, Parallelogram, and Trapezoid

The area of a triangle is half the area of a rectangle with the same base and height. If the base of the triangle is length *b* and the height length *h*, then the area of the triangle is:



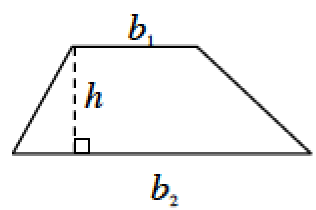
$$A = \frac{1}{2}bh.$$

The area of a parallelogram is equal to a rectangle with the same base and height. If the base of the parallelogram is length *b* and the height length *h*, then the area of the parallelogram is:



$$A = bh.$$

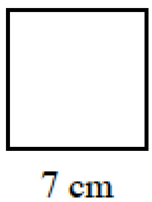
Finally, the area of a trapezoid is found by averaging the two bases and multiplying by the height. If the trapezoid has bases b_1 and b_2 and height *h*, then the area is:



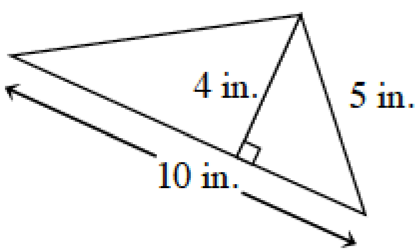
$$A = \frac{1}{2}(b_1 + b_2)h.$$

2-94. Find the area of each figure below. Show all work. Remember to include units in your answer.

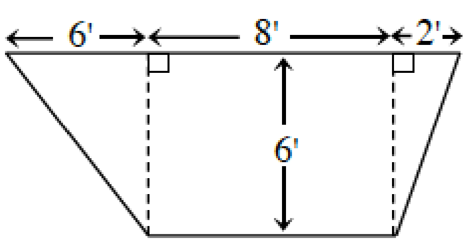
a. a square



b.



c.



2-95. Multiply the expressions below. Then simplify the result, if possible.

a. $3x\,(5x + 7)$

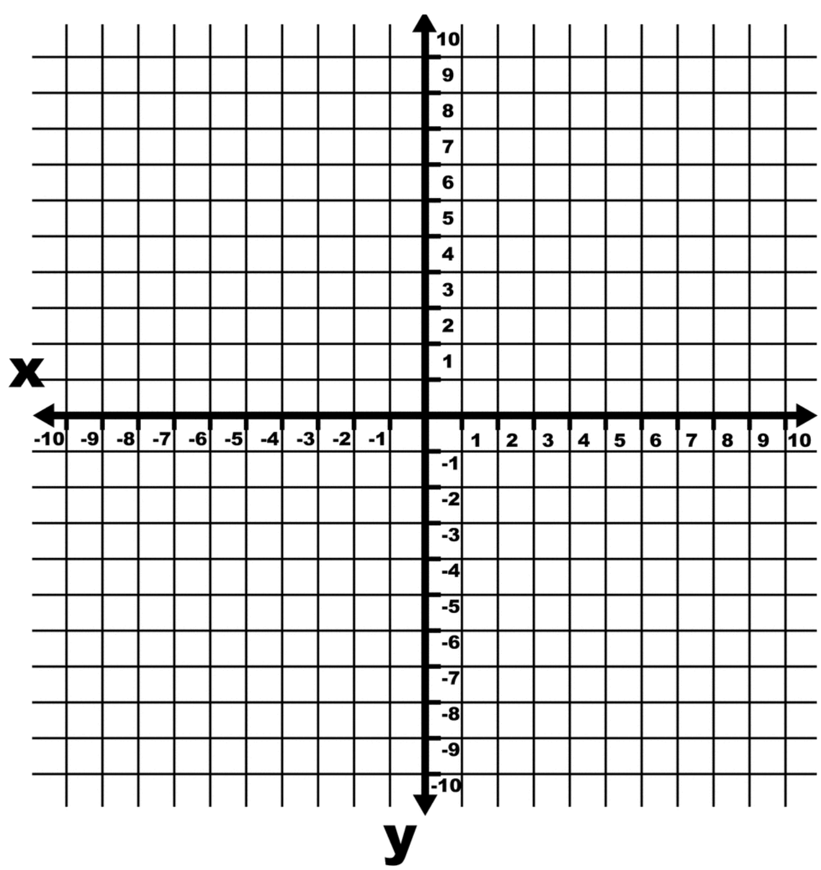
b. $(x + 2)(x + 3)$

c. $(3x + 5)(x - 2)$

d. $(2x + 1)(5x - 4)$

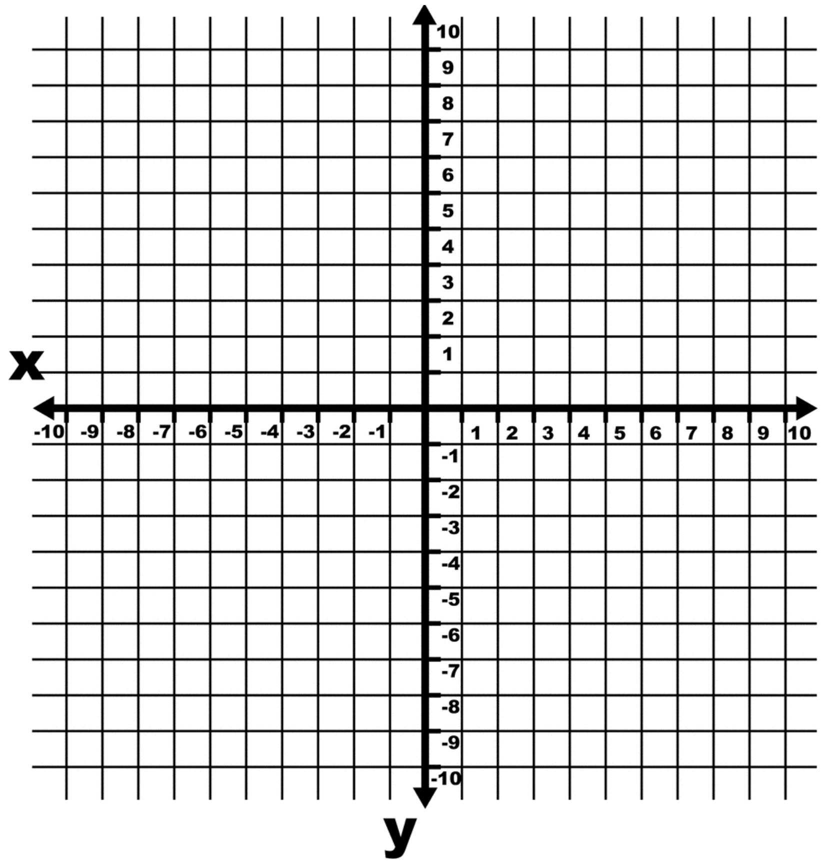
2-96. Graph the following equations on the same set of axes. Label each line or curve with its equation. Where do the two curves intersect?

$y = -x - 3$
$y = x^2 + 2x - 3$



2-97. On graph paper, plot quadrilateral *ABCD*.

A (2, 7), *B* (4, 8), *C* (4, 2), and *D* (2, 3).



a. What is the best name for this shape? Justify your conclusion.

b. Quadrilateral *A′B′C′D′* is formed by rotating *ABCD* 90° clockwise about the origin. Name the coordinates of the vertices.

c. Find the area of *ABCD*. Show all work.

2-98. What is the probability of drawing each of the following cards from a standard playing deck? Refer to the glossary entry "playing cards" if you need information about a deck of cards.

a. P(face card)

b. P(card printed with an even number)

c. P(red ace)

d. P(purple card)

2.3.1 Is the answer reasonable?

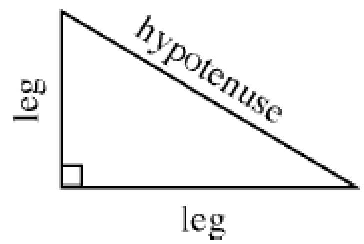
Triangle Inequality

METHODS AND MEANINGS

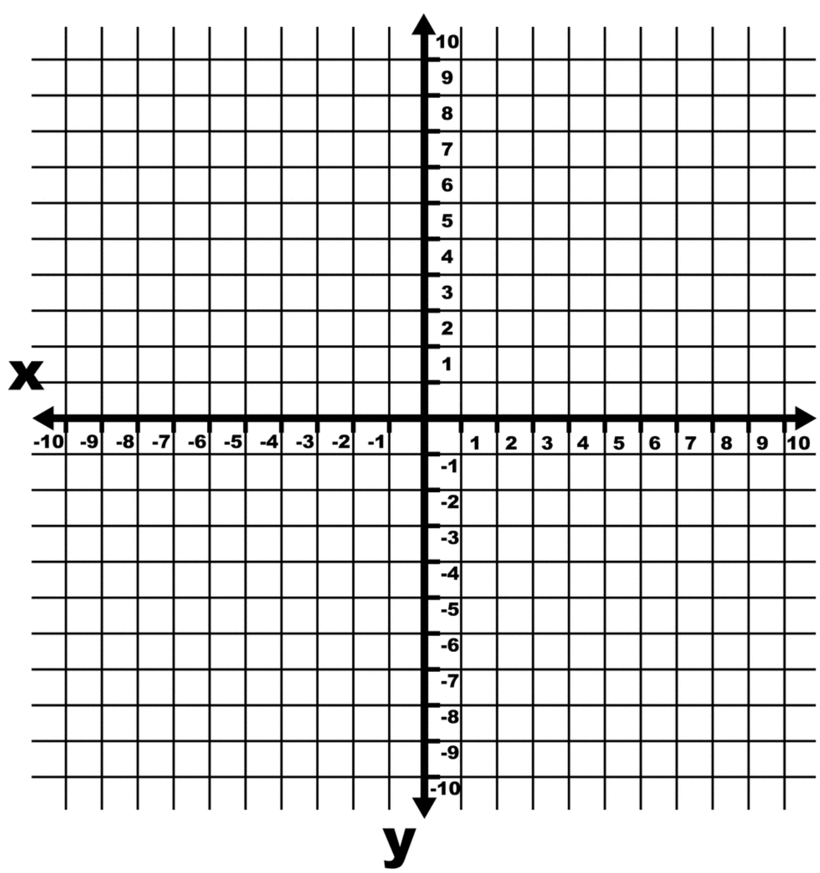
MATH NOTES

Right Triangle Vocabulary

Several of the triangles that you have been working with in this section are right triangles, that is, triangles that contain a 90° angle. The two shortest sides of the right triangle (the sides that meet at the right angle) are called the **legs** of the triangle and the longest side (the side opposite the right angle) is called the **hypotenuse** of the triangle.



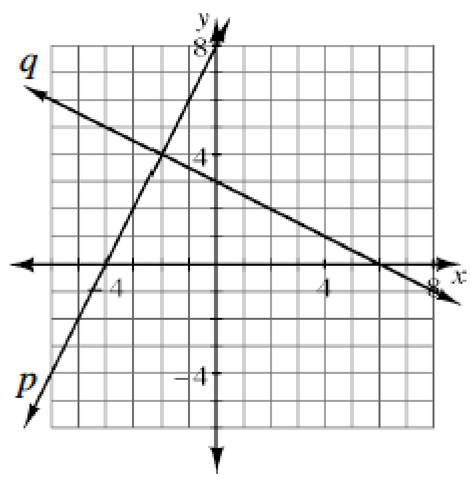
2-104. Draw a right triangle with legs of length 6 and 8 units, respectively, onto graph paper. Construct a square on the hypotenuse and use the square's area to find the length of the hypotenuse.



2-105. One of the algebra topics you have reviewed during this chapter is solving systems of equations. Assess what you know about solving systems as you answer the questions below.

a. Find the points of intersection of the lines below using any method. Write your solutions as a point (x, y).

(1) $y = -x + 8$
$y = x - 2$
(2) $2x - y = 10$
$y = -4x + 2$



b. Find the equation for each line on the graph below. Remember, the general form of any line in the **slope-intercept form** is $y = mx + b$.

c. What is the relationship between the two lines in part (b)? How do you know?

d. Solve the system of equations you found in part (b) algebraically. Verify that your solution matches the one shown in the graph above.

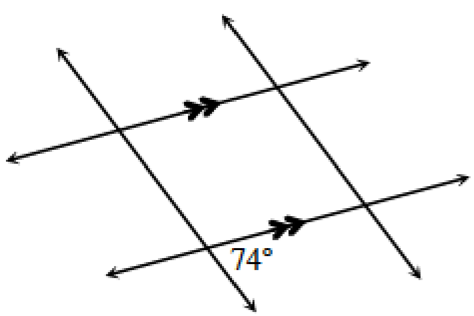
2-106. Lines *p* and *q* graphed in problem 2-105 form a triangle with the *x*-axis.

a. How can you describe this triangle? In other words, what is the most appropriate name for this triangle? How do you know?

b. Find the area of the triangle.

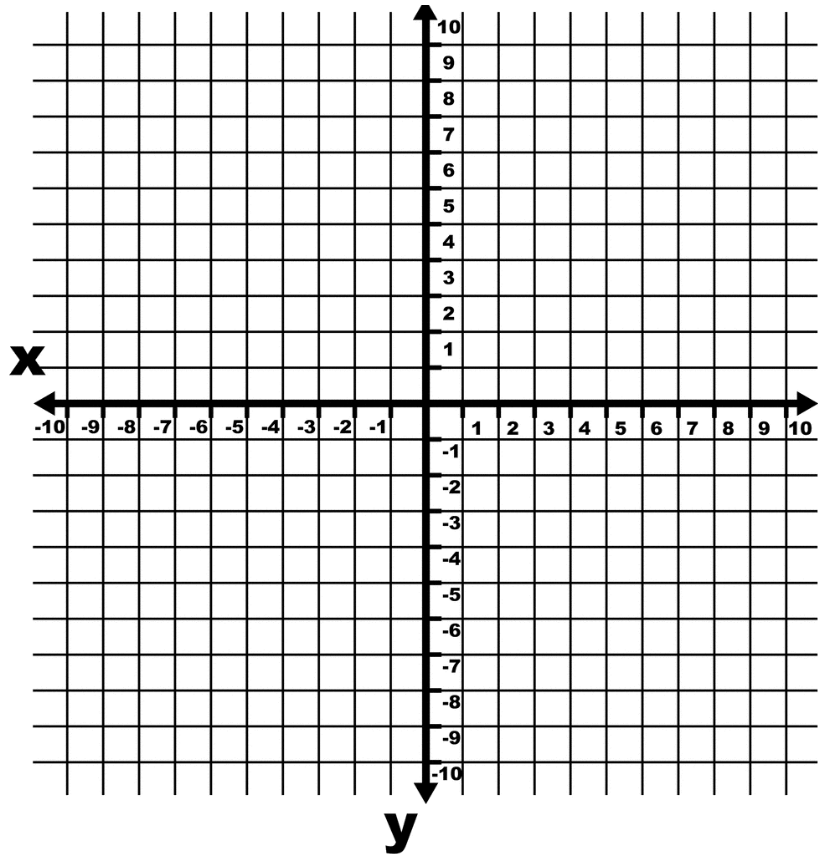
c. What is the perimeter?

2-107. Examine the diagram below. Based on the information in the diagram, which angles can you determine? Copy the diagram on your paper and find *only* those angles that you can justify.



2-108. On graph paper, plot *ABCD* if *A*(–1, 2), *B*(0, 5), *C*(2, 5), and *D*(6, 2).

a. What type of shape is *ABCD*? Justify your answer.



b. If *ABCD* is rotated 90° counterclockwise (↺) about the origin, name the coordinates of the image *A′B′C′D′*.

c. On your graph, reflect *ABCD* across the *y*-axis to find *A″B″C″D″*. Name the coordinates of *A″* and *C″*.

d. Find the area of *ABCD*. Show all work.

2.3.2 Is there a shortcut?

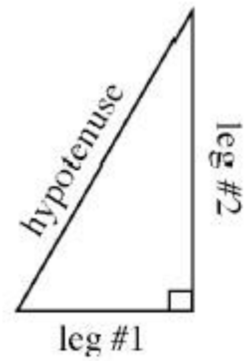
The Pythagorean Theorem

METHODS AND MEANINGS
MATH NOTES

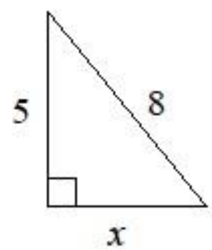
The Pythagorean Theorem

The **Pythagorean Theorem** states that in a right triangle,

(Length of leg #1)^2 + (Length of leg #2)^2 = (Length of hypotenuse)^2



The Pythagorean Theorem can be used to find a missing side length in a right triangle. See the example below.



$$5^2 + x^2 = 8^2$$

$$25 + x^2 = 64$$

$$x^2 = 39$$

$$x = \sqrt{39} \approx 6.24$$

In the example above, $\sqrt{39}$ is an example of an **exact** answer, while 6.24 is an **approximate** answer.

2-113. This problem is a checkpoint for solving linear systems of equations. It will be referred to as Checkpoint 2.

Solve each system of equations.

a. $y = 3x + 11$
$x + y = 3$

b. $y = 2x + 3$
$x - y = -4$

c. $x + 2y = 16$
$x + y = 2$

d. $2x + 3y = 10$
$3x - 4y = -2$

Check your answers by referring to the Checkpoint 2 materials located at the back of your book. If you needed help solving these problems correctly, then you need more practice. Review the Checkpoint 2 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily

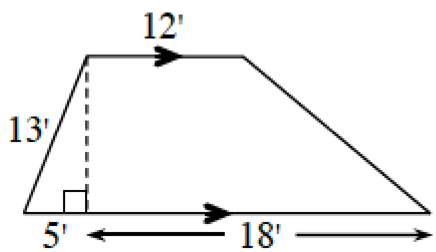
2-114. Hannah's shape bucket contains an equilateral triangle, an isosceles right triangle, a regular hexagon, a non-isosceles trapezoid, a rhombus, a kite, a parallelogram and a rectangle. If she reaches in and selects a shape at random, what is the probability that the shape will meet the criterion described below?

a. At least two sides congruent.

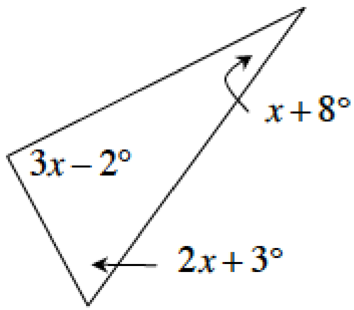
b. Two pairs of parallel sides.

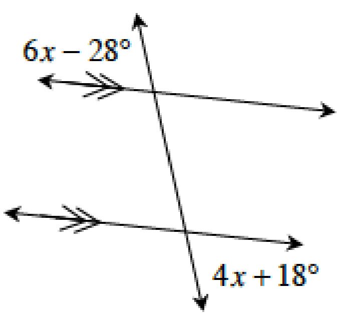
c. At least one pair of parallel sides.

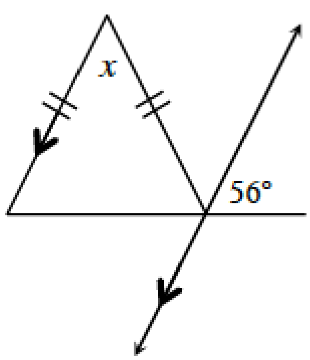
2-115. Find the area of the trapezoid below. What strategies did you use?



2-116. Use the relationships in the diagrams below to solve for *x*, if possible. If it is not possible, state how you know. If it is possible, justify your solution by stating which geometric relationships you use.

a. 

b. 

c. 

2-117. Find the minimum and maximum limits for the length of a third side of a triangle if the other two sides are 8" and 13".