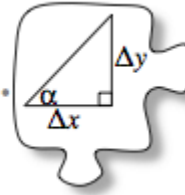
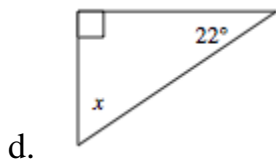
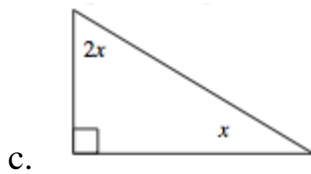
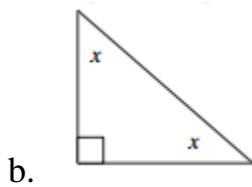
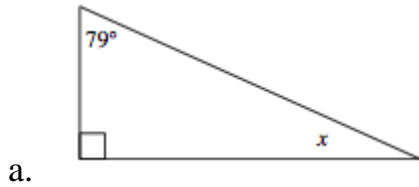


4.1.1 What patterns can I use?

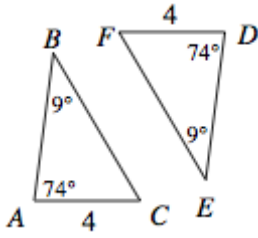
Constant Ratios in Right Triangles



4-6. Use what you know about the angles of a triangle to find the value of x and the angles in each triangle below.



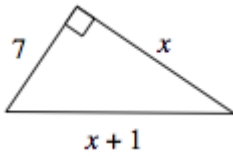
4-7. Use the triangles below to answer the following questions.



a. Are the triangles at right similar? How do you know? Show your reasoning in a flowchart.

b. Examine your work from part (a). Are the triangles also congruent? Explain why or why not.

4-8. As Randi started to solve for x in the diagram below, she wrote the equation $7^2 + x^2 = (x + 1)^2$.



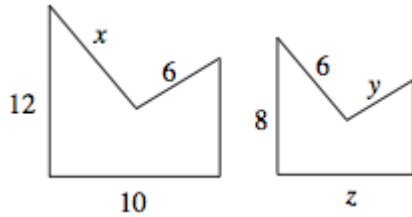
a. Is Randi's equation valid? Explain your thinking.

b. To solve her equation, first rewrite $(x + 1)^2$ by multiplying $(x + 1)(x + 1)$. You may want to review the Math Notes box for Lesson 2.2.2.

c. Now solve your equation for x .

d. What is the perimeter of Randi's triangle?

4-9. Assume that the shapes below are similar. Find the values of x , y , and z .



4-10. ROLL AND WIN

You begin the game *Roll and Win* by picking a number. Then you roll two regular dice, each numbered 1 through 6, and *add* the numbers that come up together. If the sum is the number you chose, you win a point. For example, if you choose “11,” and a 6 and a 5 are rolled, you win!

- a. What is the sample space, which can be thought of as the set of all the possible outcomes, when two dice are rolled and their numbers added?

- b. One way to analyze this situation is to make a model of all the possible outcomes like the one at right. Copy and complete this table of sums on your paper. Are each of the outcomes in this table equally likely?

		Dice #1					
		1	2	3	4	5	6
Dice #2	1				5		
	2		4				
	3						
	4			7			
	5						
	6						

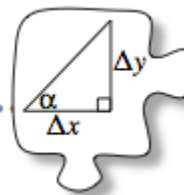
- c. What is $P(\text{even})$? $P(10)$? $P(15)$?

- d. Which sum is the most likely result? What is the probability of rolling that sum?

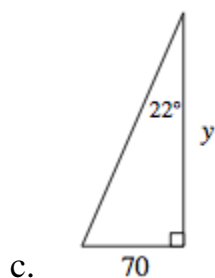
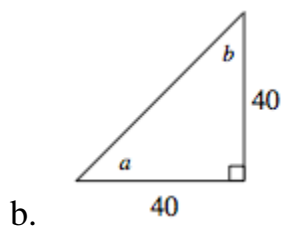
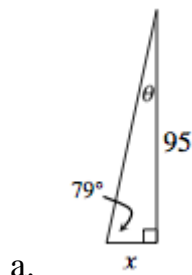
4-11. The temperature in San Antonio, Texas is currently 77°F and is increasing by 3° per hour. The current temperature in Bombay, India is 92°F and the temperature is dropping by 2° per hour. When will it be as hot in San Antonio as it is in Bombay? What will the temperature be?

4.1.2 How important is the angle?

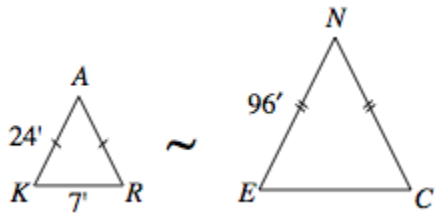
.....
Connecting Slope Ratios to Specific Angles



4-17. Use your Trig Table Toolkit from problem 4-16 to help you find the value of each variable below.



4-18. The triangles shown below are similar.

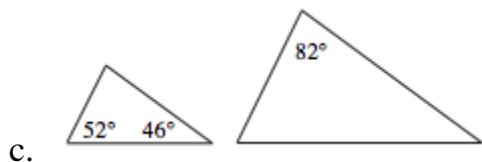
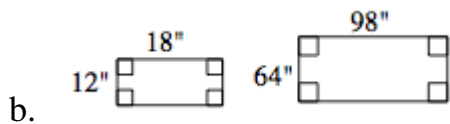
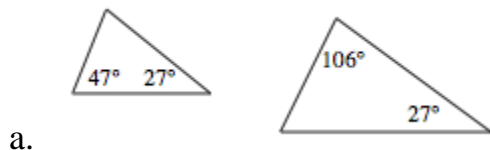


a. What is the ratio of side length NE to side length AK ?

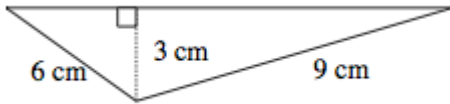
b. Use a ratio to compare the perimeters of $\triangle ENC$ and $\triangle KAR$. How is the perimeter ratio related to the side length ratio?

c. If you have not already done so, find the length of \overline{EC} .

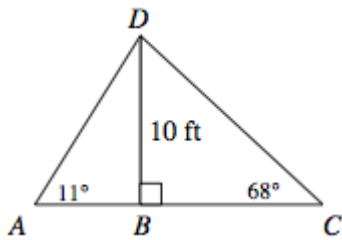
4-19. Examine each pair of figures below. Are they similar? Explain how you know.



4-20. Find the area and perimeter of the triangle below.



4-21. Examine the figure below, which is not drawn to scale. Which is longer, \overline{AB} or \overline{BC} ? Explain your answer.



4-22. Joan and Jim are planning a dinner menu including a main dish and dessert. They have 4 main dish choices (steak, vegetable-cheese casserole, turkey burgers, and vegetarian lasagna) and 3 dessert choices (chocolate brownies, strawberry ice cream, and chocolate chip cookies.)

a. Joan and Jim would like to know how many different dinner menus they have to choose from. One way to make sure you have considered the entire sample space – all the possible menu outcomes – is to make a table like the one at right. How many different menus are there?

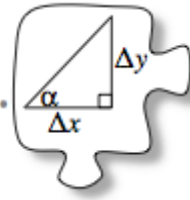
	steak	vegetable casserole	turkey burgers	vegetable lasagna
chocolate brownies				
strawberry ice cream				
chocolate chip cookies				

b. Assume the main dish choice and the dessert choices are both chosen randomly. Are all the menus equally likely?

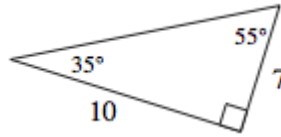
c. What is the probability they pick a menu without meat? What is the probability they pick a menu with chocolate?

4.1.3 What if the angle changes?

Expanding the Trig Table

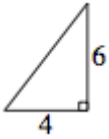


4-27. Ben thinks that the slope ratio for this triangle is $\frac{7}{10}$. Carlissa thinks the ratio is $\frac{10}{7}$. Who is correct? Explain your thinking fully.

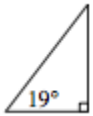


4-28. Use your observations from problem 4-26 to answer the following questions:

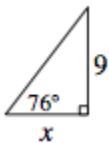
a. Thalia did not have a tool to help her find the slope angle in the triangle below. However, she claims that the slope angle has to be more than 45° . Do you agree with Thalia? Why?



b. Lyra was trying to find the slope ratio for the triangle at right, and she says the answer is $\frac{\Delta y}{\Delta x} = 2.675$. Isiah claims that cannot be correct. Who is right? How do you know?



c. Without finding the actual value, what information do you know about x in the diagram at right?



4-29. Examine each sequence below. State whether it is arithmetic, geometric, or neither. For the sequences that are arithmetic or geometric, find the equation for $t(n)$ or a_n . Refer to the Math Notes box in this lesson if you need additional help.

a. 1, 4, 7, 10, 13, ...

b. 0, 5, 12, 21, 32, ...

c. 2, 4, 8, 16, 32, ...

d. 5, 12, 19, 26, ...

4-30. Edwina has created her own Shape Bucket and has provided the clues below about her shapes. List one possible group of shapes that could be in her bucket.

$$P(\text{equilateral}) = 1$$

$$P(\text{triangle}) = \frac{1}{3}$$

4-31. Renae has programmed her music player to play all five songs in her playlist in a random order without repeating songs.

PLAYLIST	
a.	I Love My Mama (country) by the Strings of Heaven
b.	Don't Call Me Mama (country) Duet by Sapphire and Hank Tumbleweed
c.	Carefree and Blue (R & B) by Sapphire and Prism Escape
d.	Go Back To Mama (Rock) Duet by Bjorn Free and Sapphire
e.	Smashing Lollipops (Rock) by Sapphire

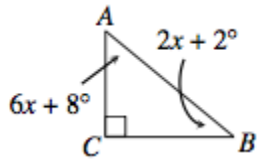
- a. What is the probability that the first song is a country song?

- b. If the first song is a country song, does that affect the probability that the second song is a country song? Explain your thinking.

- c. As songs are playing, the number of songs left to play decreases. Therefore, the probability of playing each of the remaining songs depends on which songs that have played before it. This is an example of events that are **not independent**. If Renae has already listened to “Don’t Call Me Mama,” “Carefree and Blue,” and “Smashing Lollipops,” what is the probability that one of the singers of the fourth song will be Sapphire? Explain your reasoning.

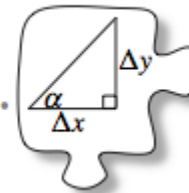
- d. To get home, Renae can take one of four buses: #41, #28, #55, or #81. Once she is on a bus, she will randomly select one of the following equally likely activities: listening to her music player, writing a letter, or reading a book. Her choice of bus and choice of entertainment are **independent events**, because the bus that Renae took did not affect which activity she chose. For example, what is the probability that Renae writes a letter if she takes the #41 bus? What is the probability that Renae writes a letter if she takes the #55 bus?

4-32. Use what you know about the sum of the angles of a triangle to find $m\angle ABC$ and $m\angle BAC$. Are these angles acute or obtuse? Find the sum of these two angles. How can you describe the relationship of these two angles?

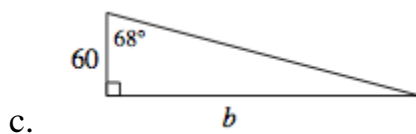
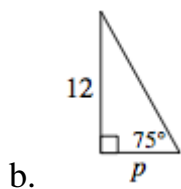
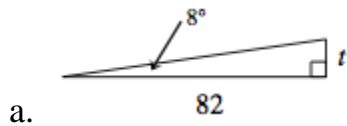


4.1.4 What about other right triangles?

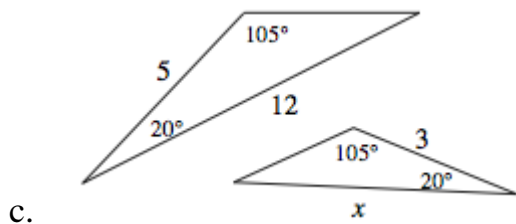
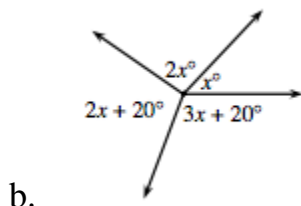
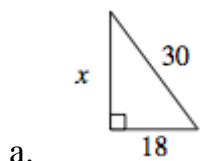
The Tangent Ratio



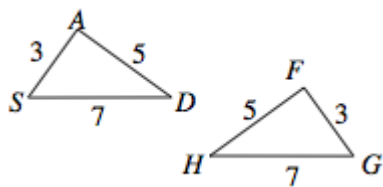
4-39. Find the missing side length for each triangle. Use the tangent button on your calculator to help.



4-40. Use the relationships in the diagrams below to write an equation and solve for x .



4-41. What is the relationship of the triangles below? Justify your conclusion using rigid transformations.



4-42. Alexis, Bart, Chuck, and Dariah all called in to a radio show to get free tickets to a concert. List all the possible orders in which their calls could have been received.

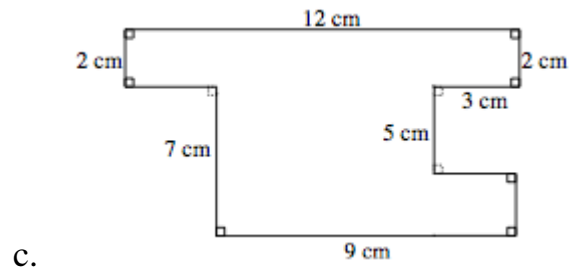
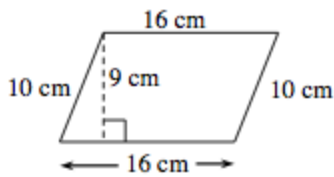
4-43. When she was younger, Mary had to look up at a 68° angle to see into her father's eyes whenever she was standing 15 inches away. How high above the flat ground were her father's eyes if Mary's eyes were 32 inches above the ground?

4-44. This problem is a checkpoint for finding areas and perimeters of complex shapes. It will be referred to as Checkpoint 4.

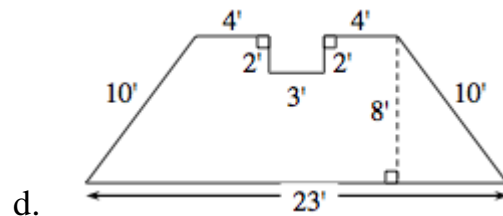
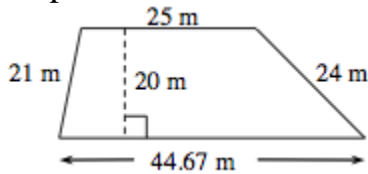


For each figure below, find the area and the perimeter.

a. Parallelogram

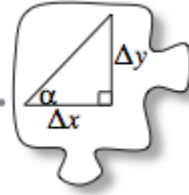


b. Trapezoid

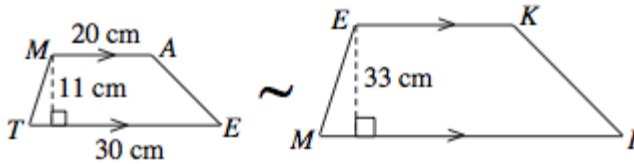


4.1.5 What if I can't measure it?

Applying the Tangent Ratio

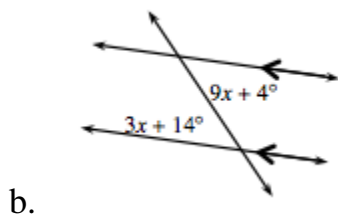
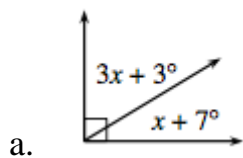


4-47. The trapezoids below are similar.



- What is the ratio of the heights?
- Compare the areas. What is the ratio of the areas?

4-48. For each diagram below, write an equation and solve for x , if possible.



4-49. Which of the following events are independent? Refer to the Math Notes in this lesson.

- a. Flipping a head, after flipping 5 heads in a row.
- b. Drawing an Ace from a deck of playing cards, after two Aces were just drawn (and not returned to the deck).
- c. Having blue eyes, if you have blonde hair.
- d. The probability of rain this weekend, if the debate team from North City High School wins the state championship.
- e. Randomly selecting a diet soda from a cooler filled with both diet and regular soda, after the person before you just selected a diet soda and drank it.

4-50. Leon is standing 60 feet from a telephone pole. As he looks up, a red-tailed hawk lands on the top of the pole. Leon's angle of sight up to the bird is 22° and his eyes are 5.2 feet above the ground.

a. Draw a detailed picture of this situation. Label it with all of the given information.

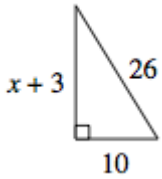
b. How tall is the pole? Show all of your work.

4-51. Examine each sequence below. State whether it is arithmetic, geometric, or neither. For the sequences that are arithmetic or geometric, find the equation for a_n .

a. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$

b. $-7.5, -9.5, -11.5, \dots$

4-52. Find the value of x in the triangle below. Refer to problem 4-8 for help. Show all work.



4.2.1 How can I represent it?

Using an Area Model



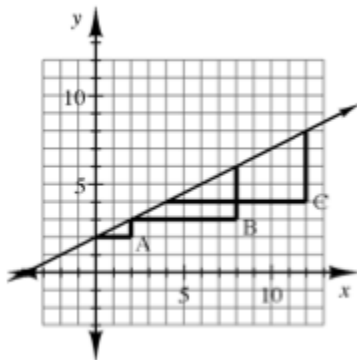
4-58. Out of the 20 contestants in the state math championships, 10 are girls. For this round, each contestant gets asked one question. The first question goes to a randomly chosen contestant.

- What is the probability the first contestant is a girl?
- If the first contestant is a girl, what is the probability that the second contestant is a girl?
- Is the probability that the second contestant a girl independent of the first contestant being a girl? Refer to the Math Notes box at the end of Lesson 4.1.5.

4-59. On graph paper, graph the parabola $y = 2x^2 - 5x - 3$.

- What are the roots (x -intercepts) of the parabola?
- Read the Math Notes box for this lesson. Then solve the equation $2x^2 - 5x - 3 = 0$ algebraically. Did your solutions match your roots from part (a)?

4-60. Examine the graph below with slope triangles A, B, and C.



a. Find the slope of the line using slope triangle A, slope triangle B, and then slope triangle C.

b. Hernisha's slope triangle has a slope of $\frac{1}{2}$. What do you know about her line?

4-61. Francis and John are racing. Francis is 2 meters in front of the starting line at time $t = 0$ and he runs at a constant rate of 1 meter per second. John is 5 meters in front of the starting line and he runs at a constant rate of 0.75 meters per second. After how long will Francis catch up to John?

4-62. Solve each equation to find the value of x . Leave your answers in decimal form accurate to the thousandths place.

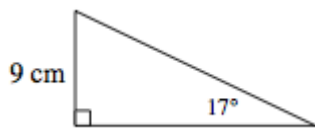
a. $\frac{3.2}{x} = \frac{7.5}{x^2}$

b. $4(x - 2) + 3(-x + 4) = -2(x - 3)$

c. $2x^2 + 7x - 15 = 0$

d. $3x^2 - 2x = -1$

4-63. Find the perimeter of the shape below. Clearly show all your steps.



4.2.2 How can I represent it?

Using a Tree Diagram

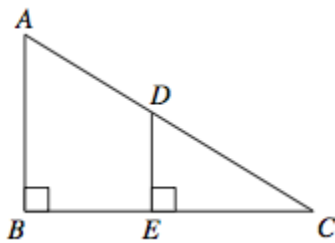


4-69. Eddie is arguing with Tana about the probability of flipping three coins. They decided to flip a penny, nickel, and a dime.

- a. Which would be better for determining the sample space, a tree diagram or an area model? Justify your answer.
- b. Make a sample space that shows all the possible outcomes. How many outcomes are there?
- c. Find the probability of each of the following events occurring. Be sure to show your thinking clearly:
 - i.* Three heads
 - ii.* One head and two tails
 - iii.* At least one tail
 - iv.* Exactly two tails
- d. Which is more likely, flipping at least 2 heads or at least 2 tails? Explain.

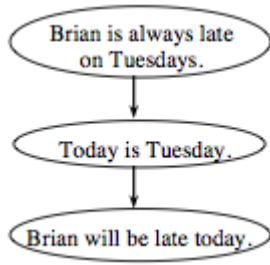
- e. How would the probabilities change if Tana found out that Eddie was using weighted coins (coins that were not fair) so that the probability of getting heads for each coin was $\frac{4}{5}$ instead of $\frac{1}{2}$? Would this change the sample space? Recalculate the probabilities in part (c) based on the new information.

4-70. Are the triangles at right similar? If so, write a flowchart that justifies your conclusion. If not, explain how you know.



4-71. You roll a die and it comes up a “6” three times in a row. What is the probability of rolling a “6” on the next toss?

4-72. Mr. Singer made the flowchart below about a student named Brian.



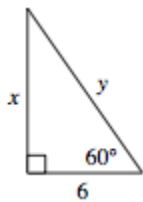
- a. What is wrong with Mr. Singer's flowchart?
- b. Rearrange the ovals so the flowchart makes more sense.

4-73. Write the first four terms of each of the following sequences.

a. $a_n = 3 \cdot 5^{n-1}$

b. $a_1 = 10, a_{n+1} = -5a_n$

4-74. Find x and y in the diagram below. Show all of the steps leading to your answer.



4.2.3 What model should I use?

Probability Models



4-81. Eddie told Alfred, “I’ll bet if I flip three coins I can get exactly two heads.” Alfred replied, “I’ll bet I can get exactly two heads if I flip four coins!” Eddie scoffed, “Well, so what? That’s easier.” Alfred argued, “No, it’s not. It’s harder.” Who is correct? Show all of your work and be prepared to defend your conclusion.

4-82. Find the equation of the line with a slope of $\frac{1}{3}$ that goes through the point (0, 9).

4-83. An airplane takes off and climbs at an angle of 11° . If the plane must fly over a 120-foot tower with at least 50 feet of clearance, what is the minimum distance between the point where the plane leaves the ground and the base of the tower?

a. Draw and label a diagram for this situation.

b. What is the minimum distance between the point where the plane leaves the ground and the tower? Explain completely.

4-84. Solve each equation below for the given variable. Show all work and check your answer.

a. $\sqrt{x} - 5 = 2$

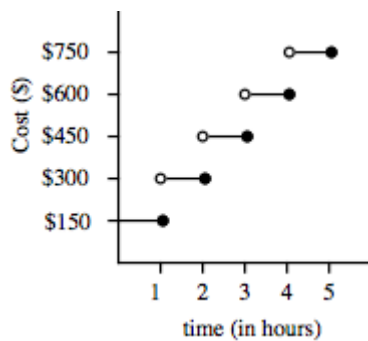
b. $-4(-2 - x) = 5x + 6$

c. $\frac{5}{x-2} = \frac{3}{2}$

d. $x^2 + 4x - 5 = 0$

4-85. Can a triangle be made with sides of length 7, 10, and 20 units? Justify your answer.

4-86. According to the graph below, how much money would it cost to speak to an attorney for 2 hours and 25 minutes?



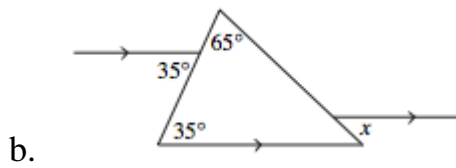
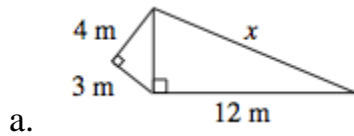
4.2.4 What if both events happen?

Unions, Intersections, and Complements

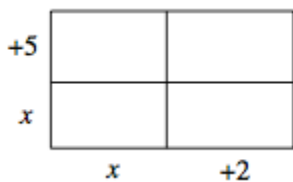


- 4-95.** Use an area model, a tree diagram, or refer to the table you created in problem 4-10 that represents the sample space for the sum of the numbers when rolling two standard six-sided dice.
- In a standard casino dice game the roller wins on the first roll if he rolls a sum of 7 or 11. What is the probability of winning on the first roll?
 - The player loses on the first roll if he rolls a sum of 2, 3, or 12. What is the probability of losing on the first roll?
 - If the player rolls any other sum, he continues to roll the dice until the first sum he rolled comes up again or until he rolls a 7, whichever happens sooner. What is the probability that the game continues after the first roll?
- 4-96.** A player in the casino dice game described in problem 4-10 rolled a sum of 6 on his first roll. He will win if he rolls a sum of six on the second roll but lose if he rolls a sum of seven. If anything else happens they ignore the result and he gets to roll again.
- How many ways are there to get a sum of six?
 - How many ways are there to get a sum of seven?
 - How many possible outcomes are important in this problem?
 - What is the probability of getting a sum of six before a sum of seven?

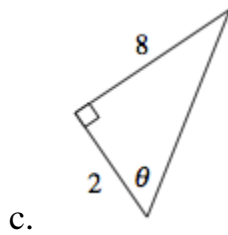
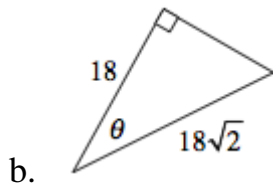
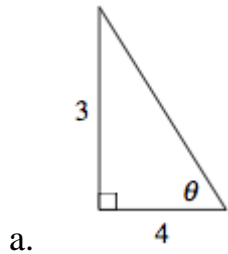
4-97. For each diagram below, solve for x . Name the relationship(s) you used. Show all work.



4-98. The area of the rectangle shown below is 40 square units. Write and solve an equation to find x . Then find the dimensions of the rectangle.



4-99. Based on the measurements provided for each triangle below, decide if the angle must be more than, less than, or equal to 45° . Assume the diagram is not drawn to scale. Show how you know.



4-100. Find the slope of the line through the points $(-5, 86)$ and $(95, 16)$. Then find at least one more point on the line.

4.2.5 How much can I expect to win?

.....
Expected Value



4-110. When he was in first grade, Harvey played games with spinners. One game he especially liked had two spinners and several markers that you moved around a board. You were only allowed to move if your color came up on *both* spinners.

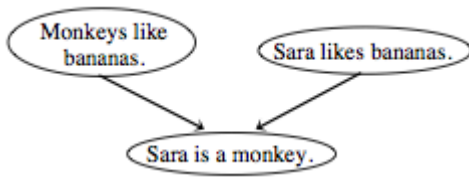


- Harvey always chose purple because that was his favorite color. What was the probability that Harvey could move his marker?
- Is the event that Harvey wins a union or an intersection of events?
- Was purple the best color choice? Explain.
- If both spinners are spun, what is the probability that no one gets to move because the two colors are not the same?
- There are at least two ways to figure out part (d). Discuss your solution method with your team and show a second way to solve part (d).

4-111. Consider the sequence $2, 8, 3y + 5, \dots$

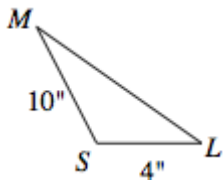
- Find the value of y if the sequence is arithmetic.
- Find the value of y if the sequence is geometric.

4-112. What is wrong with the argument shown in the flow chart below? What assumption does the argument make?

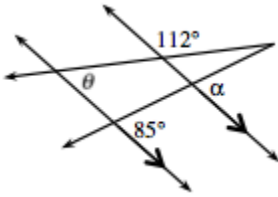


4-113. Kamillah decided to find the height of the Empire State Building. She walked 1 mile away (5280 feet) from the tower and found that she had to look up 15.5° to see the top. Assuming Manhattan is flat, if Kamillah's eyes are 5 feet above the ground how tall is the Empire State Building?

4-114. What are the possible lengths for side \overline{ML} in the triangle below? Show how you know.



4-115. Find the values of θ and α in the diagram below. State the relationships you used.

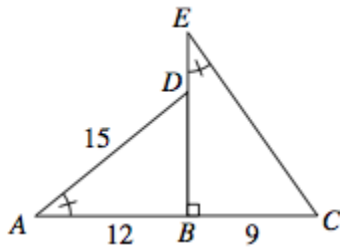


4-116. Avery has been learning to play some new card games and is curious about the probabilities of being dealt different cards from a standard 52-card deck. Help him figure out the probabilities listed below.

- What are $P(\text{king})$, $P(\text{queen})$, and $P(\text{club})$?
- What is $P(\text{king or club})$? How does your answer relate to the probabilities you calculated in part (a)?
- What is $P(\text{king or queen})$? Again, how does your answer relate to the probabilities you calculated in part (a)?
- What is the probability of not getting a face card? Jacks, queens, and kings are face cards.

4-117. Woottonville currently has a population of 1532 people and is growing at a rate of approximately 15 people per year. Nearby, Coynertown has a population of 2740 people but is decreasing at a rate of approximately 32 people per year. In how many years will the towns have the same population?

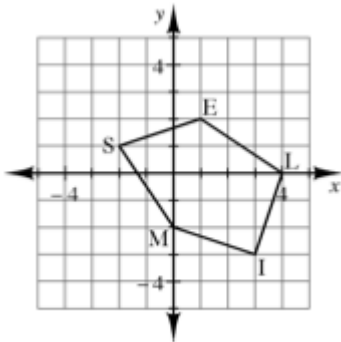
4-118. Examine the diagram below. If \overline{AC} passes through point B , then answer the questions below.



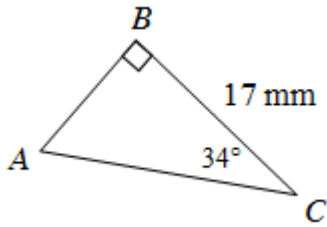
a. Are the triangles similar? If so, make a flowchart justifying your answer.

b. Are the triangles congruent? Explain how you know.

4-119. Examine pentagon $SMILE$ below. Do any of its sides have equal length? How do you know? Be sure to provide convincing evidence. You might want to copy the figure onto graph paper.



4-120. Find the area of the triangle below. Show all work.



4-121. On graph paper, plot $\triangle ABC$ if $A(-1, -1)$, $B(3, -1)$, and $C(-1, -2)$.

a. Enlarge (dilate) $\triangle ABC$ from the origin so that the ratio of the side lengths is 3. Name this new triangle $\triangle A'B'C'$. List the coordinates of $\triangle A'B'C'$.

b. Rotate $\triangle A'B'C'$ 90° clockwise (↻) about the origin to find $\triangle A''B''C''$. List the coordinates of $\triangle A''B''C''$.

c. If $\triangle ABC$ is translated so that the image of A is located at $(5, 3)$, where would the image of B lie?

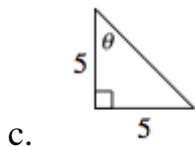
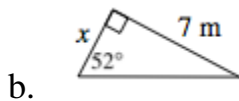
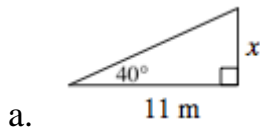
WHAT HAVE I LEARNED?



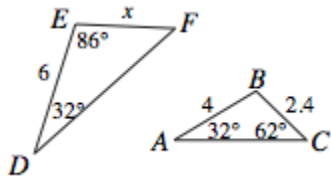
Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

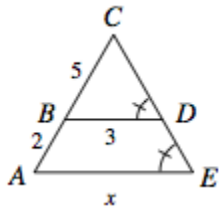
CL 4-122. Solve for the missing side length or angle below.



CL 4-123. Use a flowchart to show how you know the triangles are similar. Then find the value of each variable.



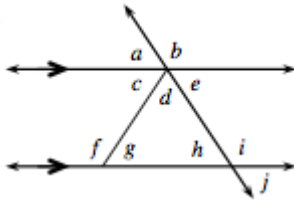
a.



b.

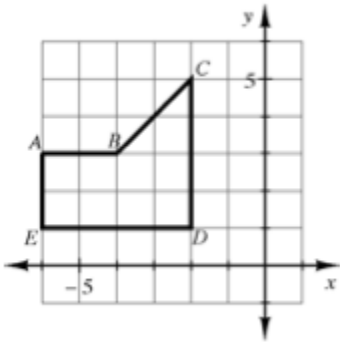
CL 4-124. Salvador has a hot dog stand 58 meters from the base of the Space Needle in Seattle. He prefers to work in the shade and knows that he can calculate when his hotdog stand will be in the shade if he knows the height of the Space Needle. To measure its height, Salvador stands at the hotdog stand, gets out his clinometer, and measures the angle to the top of the Space Needle to be 80° . Salvador's eyes are 1.5 meters above the ground. Assuming that the ground is level between the hotdog stand and the Space Needle, how tall is the Space Needle?

CL 4-125 Use the diagram below to answer the questions below.



- a. State the name of the geometric relationship between the angles below. Also describe the relationship between the angle measures, if one exists.
- $\angle a$ and $\angle h$
 - $\angle b$ and $\angle e$
 - $\angle c$ and $\angle g$
 - $\angle g$, $\angle d$, and $\angle h$
- b. Find the measure of each angle listed below and justify your answer. Let $m\angle c = 32^\circ$ and $m\angle e = 55^\circ$ in the figure above.
- $m\angle j$
 - $m\angle d$
 - $m\angle a$
 - $m\angle g$

CL 4-126. Draw a pair of axes in the center of a half sheet of graph paper. Then draw the figure below and perform the indicated transformations. For each transformation, label the resulting image $A'B'C'D'E'$.



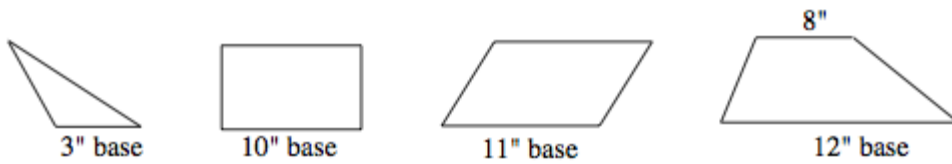
- Rotate $ABCDE$ 180° \curvearrowright around the origin.
- Rotate $ABCDE$ 90° \curvearrowright around the origin.
- Reflect $ABCDE$ across the y -axis.
- Translate $ABCDE$ up 5, left 7.

CL 4-127. Kiyomi has 4 pairs of pants (black, peach, gray, and cream), and she has 5 shirts (white, red, teal, black, and lavender).

- If any shirt can be worn with any pair of pants, represent the sample space of all possible outfits with both a probability area model and a tree diagram. How many outfits does she own?
- The closet light is burned out, so Kiyomi must randomly select a pair of pants and a shirt. What is the probability that she will wear something black?

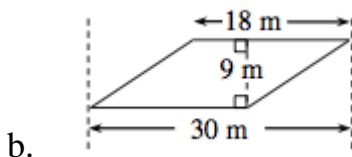
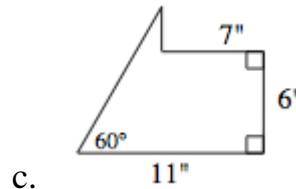
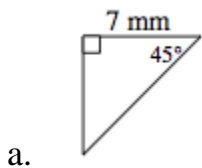
4-128. In a certain town, 45% of the population has dimples and 70% has a widow's peak (a condition where the hairline above the forehead makes a "V" shape). Assuming that these physical traits are independently distributed, what is the probability that a randomly selected person has both dimples and a widow's peak? What is the probability that he or she will have neither? Use a probability area model or a tree diagram to represent this situation.

CL 4-129. Trace each figure onto your paper and label the sides with the given measurements.



- On your paper, draw a height that corresponds to the labeled base for each figure.
- Assume that the height for each figure above is 7 inches. Add this information to your diagrams and find the area of each figure.

CL 4-130. Find the perimeter of each shape below. Assume the diagram in part (b) is a parallelogram.



CL 4-131. For each equation below, solve for x :

a. $\frac{x}{23} = \frac{15}{7}$

b. $(x + 2)(x - 5) = 6x + x^2 - 5$

c. $x^2 + 2x - 15 = 0$

d. $2x^2 - 11x = -3$

CL 4-132. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in math classes you have taken before? Use the table to make a list of topics you need help on and a list of topics you need to practice more.