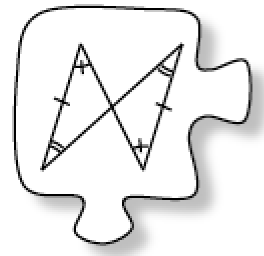
6.1.1 Are the triangles congruent?



Congruent Triangles

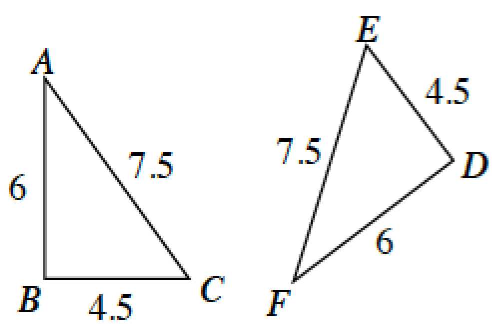
In Chapter 3, you learned how to identify similar triangles and used them to solve problems. But what can be learned when triangles are congruent? In today's lesson, you will practice identifying congruent triangles using what you know about similarity. As you search for congruent triangles in today's problems, focus on these questions:

What do I know about these triangles?

How can I show similarity?

What is the common ratio?

6-1. Examine the triangles below. Explore using the 6-1 Student eTool (CPM).



a. Make a flowchart showing that these triangles are similar.

b. Are these triangles also congruent? Explain how you know.

c. While the symbol for similar figures is "~", the symbol for congruent figures is "≅". How is the congruence symbol related to the similarity symbol? Why do you think mathematicians chose this symbol for congruence?

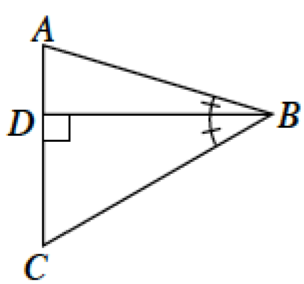
d. Luis wanted to write a statement to convey that these two triangles are congruent. He started with "ΔCAB …", but then got stuck because he did not know the symbol for congruence. Now that you know the symbol for congruence, complete Luis's statement for him.

6-2. The diagrams below are not drawn to scale. For each pair of triangles:

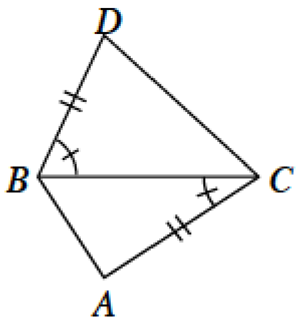
- Determine if the two triangles are congruent.
- If you find congruent triangles, write a congruence statement (such as $\Delta PQR \cong \Delta XYZ$).
- If the triangles are not congruent or if there is not enough information to determine congruence, write "cannot be determined."

a.

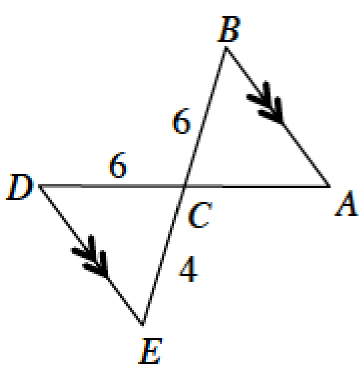
$\overline{AC}$ is a straight segment:



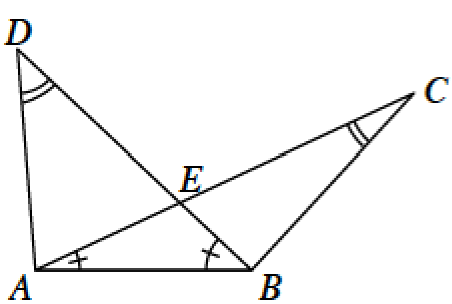
b.



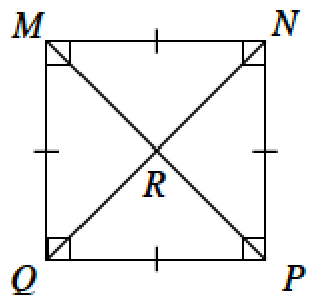
c.



d.



6-3. Consider square *MNPQ* with diagonals intersecting at *R*, as shown below.



a. How many triangles are there in this diagram? (Hint: There are more than 4!)

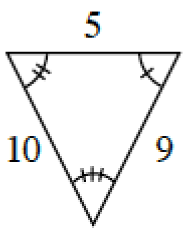
b. How many lines of symmetry does *MNPQ* have? On your paper, trace *MNPQ* and indicate the location of each line of symmetry with a dashed line.

c. Write as many triangle congruence statements as you can that involve triangles in this diagram. Be prepared to justify each congruence statement you write.

d. Write a similarity statement for two triangles in the diagram that are not congruent. Justify your similarity statement with a flowchart.

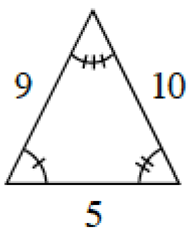
Methods and Meanings

Math Notes



Congruent Shapes

The information below is from Chapter 3 and is reprinted here for your convenience.



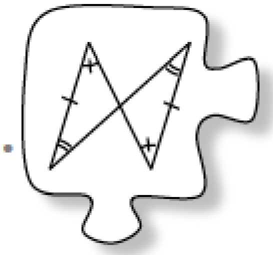
If two figures have the same shape and are the same size, they are **congruent** . Since the figures must have the same shape, they must be similar.

Two figures are congruent if they meet both the following conditions:

- The two figures are similar, and
- Their side lengths have a common ratio of 1.

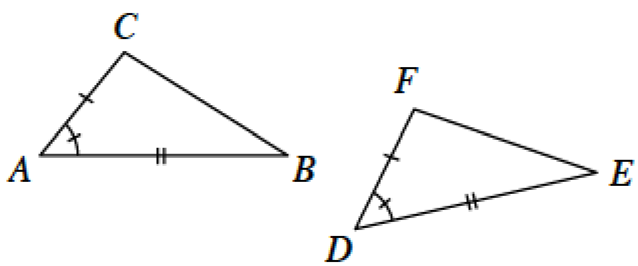
6.1.2 What information do I need?

Conditions for Triangle Congruence



In Lesson 6.1.1, you identified congruent triangles by looking for similarity and a common side length ratio of 1. Must you go through this two-step process every time you want to argue that triangles are congruent? Are there shortcuts for establishing triangle congruence? Today you will investigate multiple triangle congruence conditions in order to quickly determine if two triangles are congruent.

6-11. Review your work from problem 6-5, which required you to determine if two triangles are congruent. Then work with your team to answer the questions below.



a. Derek wants to find general conditions that can help determine if triangles are congruent. To help, he draws the diagram above to show the relationships between the triangles in problem 6-5.

If two triangles have the relationships shown in the diagram, do they have to be congruent? How do you know?

b. Write a theorem in the form of a conditional statement or arrow diagram based on this relationship. If you write a conditional statement, it should look like, "If two triangles …, then the triangles are congruent." What would be a good name (abbreviation) for this theorem?

6-12. TRIANGLE CONGRUENCE SHORTCUTS

Derek wonders, "*What other types of information can determine that two triangles are congruent?*"

Your Task: Examine the pairs of triangles below to decide what other types of information force triangles to be congruent. Notice that since no measurements are given in the diagrams, you are considering the general case of each type of pairing. For each pair of triangles below that you can prove must be congruent, enter the appropriate triangle congruence theorem on your Lesson 6.1.2 Resource Page with an explanation defending your decision. An entry for SAS ≅ (the theorem you looked at problem 6-11) is already created on the resource page as an example. Be prepared to share your results with the class**.** Test your ideas using the Similarity Toolkit (CPM).

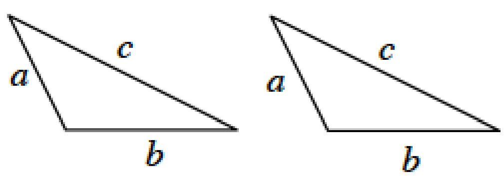
Discussion Points

In what ways can you show that triangles are similar?

What must be true in order for triangles to be congruent?

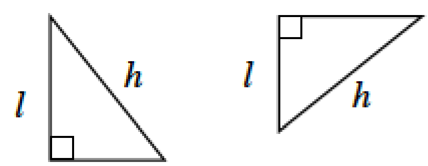
Which of the conditions below DO NOT assure congruence?

a.



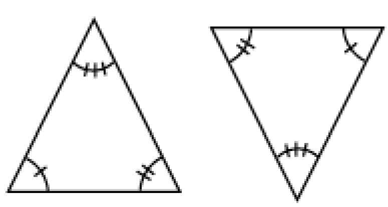
Side-Side-Side: SSS

b.



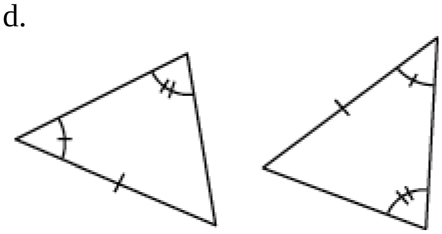
Hypotenuse-Leg: HL

c.



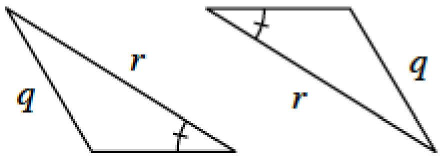
Angle-Angle-Angle: AA

d.



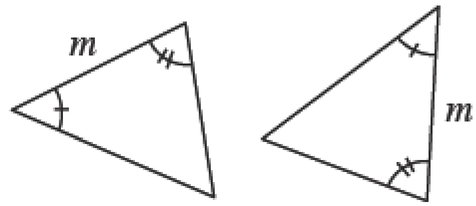
Angle-Angle-Side: AAS

e.



Side-Side-Angle: SSA

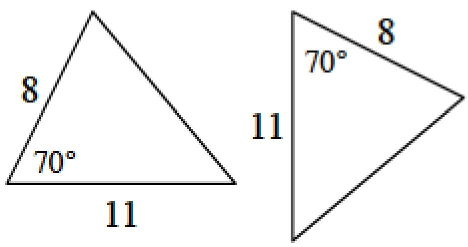
f.



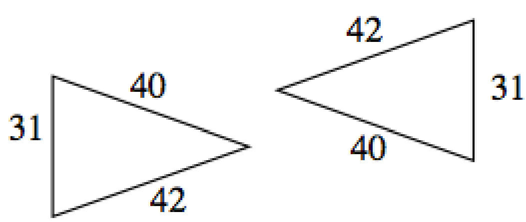
Angle-Side-Angle: ASA

6-13. Use your triangle congruence theorems to determine if the following pairs of triangles must be congruent. Note: The diagrams are not necessarily drawn to scale.

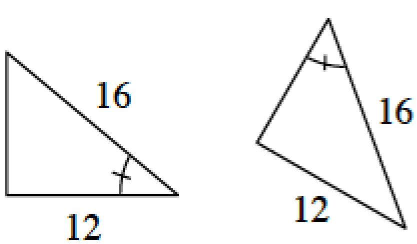
a.



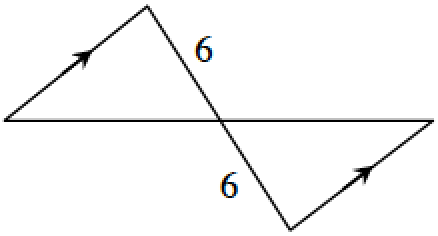
b.



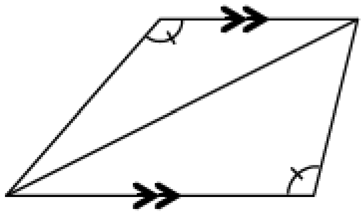
c.



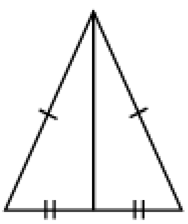
d.



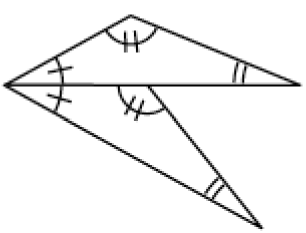
e.



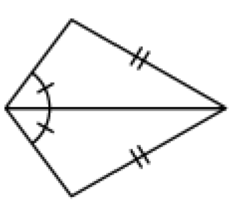
f.



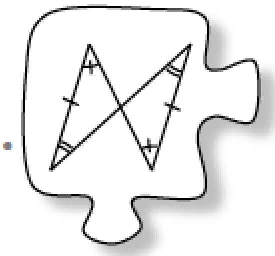
g.



h.



6.1.3 How can I prove it?

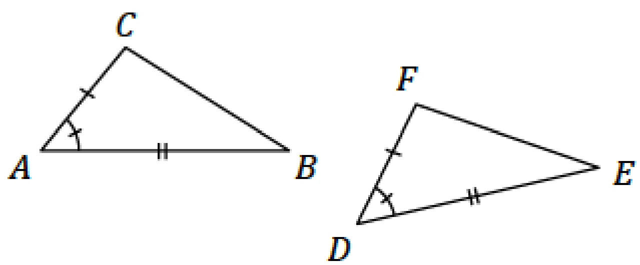


Congruence of Triangles Through Rigid Transformations

In Lesson 6.1.2, you identified five conditions that guarantee triangle congruence. They are true because they require the triangles to be similar and because you know the ratio of the side lengths equals 1. However, the definition of congruence uses rigid transformations. So it is reasonable to assume these conditions can be proved without similarity. In this lesson, you will revisit each condition and develop new logic for why each one is true.

6-20. PROVING SAS TRIANGLE CONGRUENCE

A team is working together to try to prove SAS ≅. Given the triangles shown below, they want to prove that $\Delta ABC \cong \Delta DEF$.



Jurgen said, "*We know that congruent means that the two triangles have the same size and shape, so we have to be able to move ΔABC right on top of ΔDEF using rigid motions, since they preserve lengths and angles. Does anyone see how we can do this?*"

Carlos suggested, "*Well, we can certainly move point A on top of point D with a translation to get $\Delta A'B'C'$ with points $A' = D$, but nothing else matches.*"

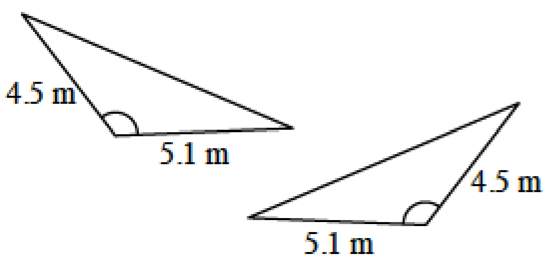
Then Mary Sue added, "*Oh, then we can rotate $\Delta A'B'C'$ about point D to get $\Delta A''B''C''$ with $\overrightarrow{A''B''}$ pointing in the same direction as $\overrightarrow{DE}$, and since point $A'' = D$, the two rays are the same. But does point B'' lie on top of point E then?*"

After a moment Emmy said, "*Sure. We know that rigid motions like translation and rotations preserve angles and lengths. So $AB = A'B' = A''B''$, right? And since we are assuming that $AB = DE$, then $A''B'' = DE$. Since point A'' is on point D, then point B'' must be at point E.*"

Jurgen added, "And look, since $\Delta A''B''C''$ lies on top of ΔDEF, they are congruent.

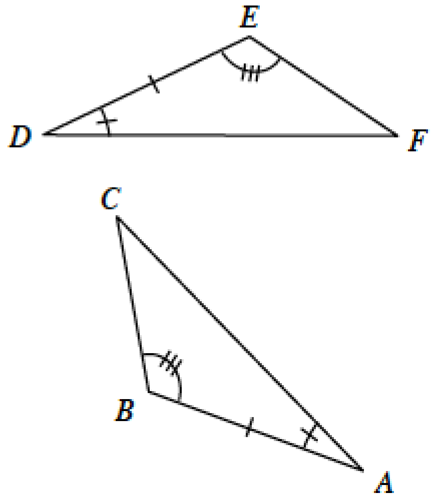
a. Use the SAS ~ tech tool, 6-20a Student eTool (CPM), and repeat their strategy to move ΔABC onto ΔDEF to prove these triangles are congruent.

b. Emmy asks, "*What about these triangles? How can we make them coincide?*" Discuss how their previous strategy needs to be changed to show that these triangles are congruent using translations. Explain why this proves that all pairs of triangles with SAS ≅ must be congruent. Explore using the 6-20b Student eTool (CPM).



6-21. PROVING ASA TRIANGLE CONGRUENCE

In problem 6-20 you proved SAS ≅ by finding a sequence of rigid transformations that would move one triangle onto another. A similar strategy can be used to prove the ASA ≅ condition. Suppose that ΔABC and ΔDEF are triangles with $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$, and $\angle B \cong \angle E$ as shown in the diagram below. Look at the work of a second team below as they work to prove that $\Delta ABC \cong \Delta DEF$ using rigid transformations. Explore with the 6-21 Student eTool (CPM).



a. Janet said, "*Like last time, we can translate and then rotate the triangle so that A″ lies on D and B″ lies on E. Except this time, we have to show that even though these points are on top of each other, the others (C and F) are also.*"

   She thinks for a bit and adds, "I think ∠E ≅ ∠B″." How can she be sure that ∠E ≅ ∠B″?

b. Patrick points out, "*This means that the* $\overrightarrow{B''C''}$ *is identical to* $\overrightarrow{EF}$. *Using the same reasoning, I know that* $\overrightarrow{A''C''}$ *is identical to* $\overrightarrow{DF}$." *Is Patrick correct? Explain why or why not.*

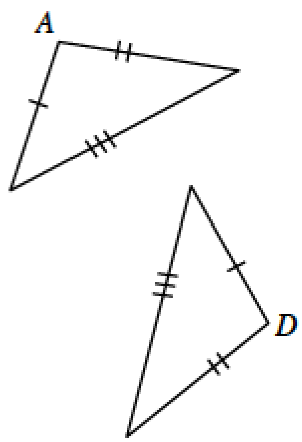
c. Aleesha then exclaimed, "*That's it! They must be congruent because this means that point C″ is the same as point F.*" Help her justify this conclusion.

d. Eddie asks, *"What if we need a reflection to have the triangles lie on top of each other?"* Does this affect the result? Explain why or why not.

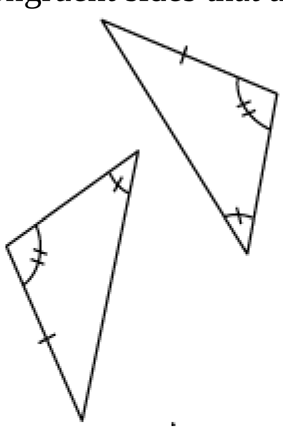
6-22. PROVING SSS, ASA, AND HL TRIANGLE CONGRUENCE

Interestingly, each of the other triangle congruence conditions can be shown to be true by either ASA ≅ or SAS ≅. Finish proving these three remaining conditions by answering the questions below.

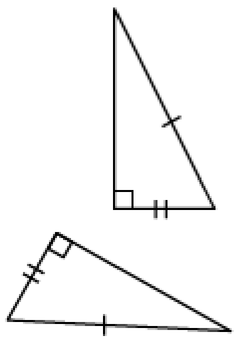
a. For the SSS ≅ condition, start with two triangles that have three pairs of congruent sides and explain why the triangles must be congruent. (Hint: Think about why the Law of Cosines guarantees that at least one of the pairs of angles has the same measure. To help, consider how you would calculate $m\angle A$ and $m\angle D$ below.)



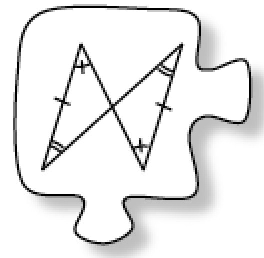
b. Explain why any pair of triangles with the AAS ≅ condition (two pairs of congruent angles and congruent sides that are not between them) can also be proved with the ASA ≅.



c. Finally, consider what you know about the side lengths of right triangles. How can you use SAS $\cong$ to prove the HL$\cong$ condition, that is, that right triangles with a pair of congruent legs and a pair of congruent hypotenuses must be congruent?



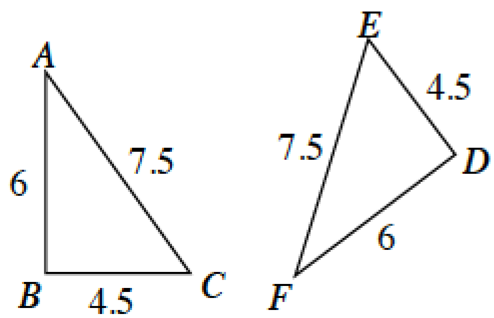
6.1.4 How can I organize my reasoning?



Flowcharts for Congruence

Now that you have shortcuts for establishing triangle congruence, how can you organize information in a flowchart to show that triangles are congruent? Consider this question as you work today with your team.

6-29. In problem 6-1, you determined that the triangles below are congruent.



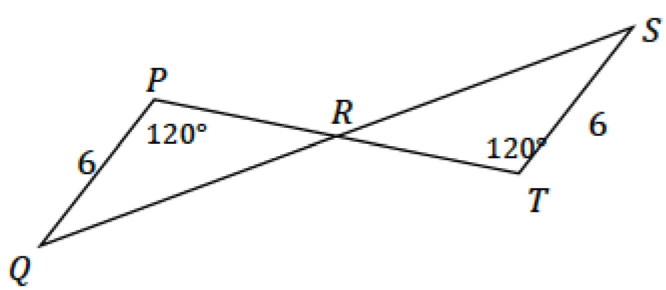
a. Which triangle congruence theorem shows that these triangles are congruent?

b. Make a flowchart showing your argument that these triangles are congruent.

c. How is a flowchart showing congruence different from a flowchart showing similarity? List every difference you can find.

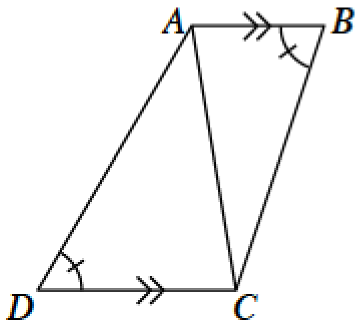
6-30. In Don's congruence flowchart for problem 6-29, oneof the ovals said "$\frac{AB}{FD}=1$". In Phil's flowchart, one of the ovals said, "$AB = FD$". Discuss with your team whether these ovals say the same thing. Can equality statements like Phil's always be used in congruence flowcharts?

6-31. Make a flowchart showing that the triangles below are congruent.

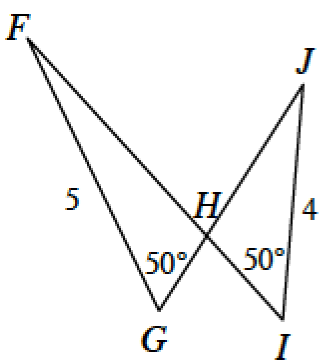


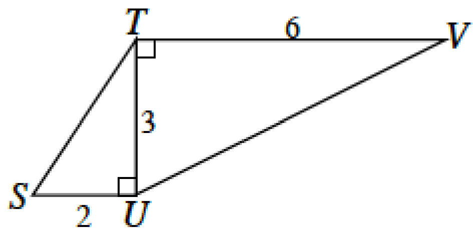
6-32. In each diagram below, determine whether the triangles are congruent, similar but not congruent, or not similar. If you claim the triangles are similar or congruent, make a flowchart justifying your answer.

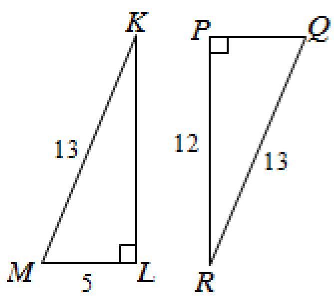
a.



b.



c. 

d. 

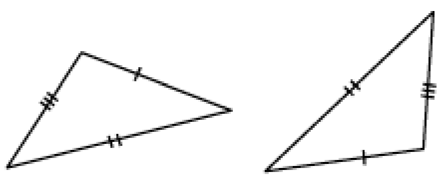
6-33. Suppose you are working on a problem involving the two triangles ΔUVW and ΔXYZ and you know that $\Delta UVW \cong \Delta XYZ$. What can you conclude about the sides and angles of ΔUVW and ΔXYZ? Write down every equation involving side lengths or angle measures that must be true.

METHODS AND MEANINGS

MATH NOTES

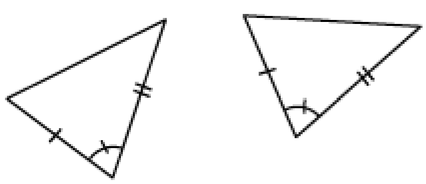
Triangle Congruence Conditions

To show that triangles are congruent, you can show that the triangles are similar and that the common ratio between side lengths is 1 or you can use rigid motions (transformations). However, you can also use certain combinations of congruent, corresponding parts as shortcuts to determine if triangles are congruent. These combinations, called **triangle congruence conditions** are:



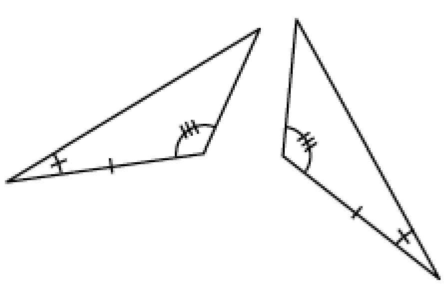
SSS ≅

(Pronounced "side–side–side") If all three pairs of corresponding sides have equal lengths, then the triangles are congruent.



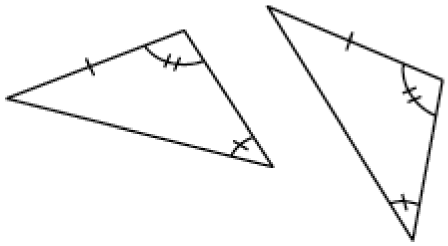
SAS ≅

(Pronounced "side–angle–side") If two pairs of corresponding sides have equal lengths *and* the angles between them (the included angle) are equal, then the triangles are congruent.



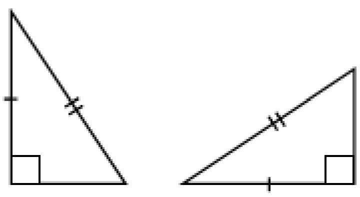
ASA ≅

(Pronounced "angle–side–angle") If two angles and the side between them in a triangle are congruent to the corresponding angles and side in another triangle, then the triangles are congruent.



AAS ≅

(Pronounced "angle–angle–side") If two pairs of corresponding angles *and* a pair of corresponding sides that are not between them have equal measures, then the triangles are congruent.

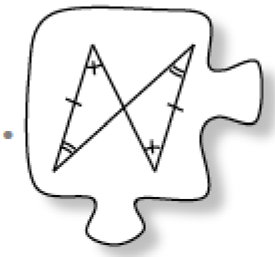


HL ≅

(Pronounced "hypotenuse–leg") If the hypotenuse and a leg of one right triangle have the same lengths as the hypotenuse and a leg of another right triangle, the triangles are congruent.

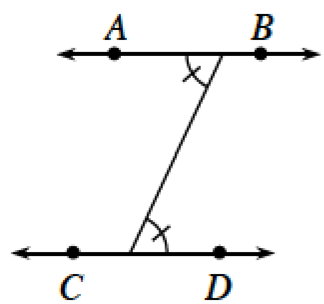
6.1.5 What is the relationship?

Converses



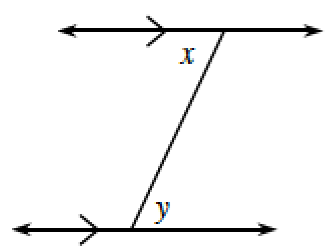
So far in this chapter, you have completed several problems in which you were given certain information and had to determine whether triangles were congruent. But what if you already know triangles are congruent? What information can you conclude then? Thinking this way requires you to reverse your triangle congruence theorems. Today you will look more generally at what happens when you reverse a theorem.

6-41. Jorge is working with the diagram below, and concludes that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. He writes the following conditional statement to justify his reasoning:



If alternate interior angles are equal, then lines are parallel.

a. Margaret is working with a different diagram, shown below. She concludes that $x = y$. Write a conditional statement or arrow diagram that justifies her reasoning.



b. How are Jorge's and Margaret's statements related? How are they different?

c. Conditional statements that have this relationship are called **converses**. Write the converse of the conditional statement below.

If lines are parallel, then corresponding angles are equal.

6-42. In problem 6-41, you learned that each conditional statement has a converse. Do you think that all converses of true statements are also true? Consider the arrow diagram of a familiar theorem below.

Triangles congruent $\rightarrow$ *corresponding sides are congruent*.

a. Is this arrow diagram true?

b. Write the converse of this arrow diagram as an arrow diagram or as a conditional statement. Is this converse true? Justify your answer.

c. Now consider another true congruence conjecture below. Write its converse and decide if it is true. If it is true, prove it. If it is not always true, explain why not.

Triangles congruent $\rightarrow$ *corresponding angles are congruent*.

d. Write the converse of the arrow diagram below. Is this converse true? Justify your answer.

$$\textit{A shape is a rectangle} \quad \rightarrow \quad \textit{the area of the shape is } b \cdot h.$$

6-43. CRAZY CONVERSES

For each of these problems below, make up a conditional statement or arrow diagram that meets the stated conditions. You must use a different example each time, and none of your examples can be about math!

a. A true statement whose converse is true.

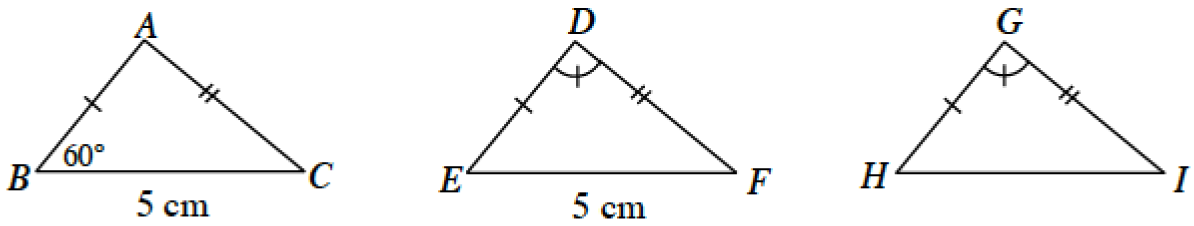
b. A true statement whose converse is false.

c. A false statement whose converse is true.

d. A false statement whose converse is false.

6-44. INFORMATION OVERLOAD

Raj is solving a problem about three triangles. He is trying to find the measure of $\angle H$ and the length of $\overline{HI}$. Raj summarizes the relationships he has found so far in the diagrams below:



a. Help Raj out! Assuming everything marked in the diagram is true, find m∠H and the length of $\overline{HI}$. Make sure to justify all your claims – do not make assumptions based on how the diagram looks!

b. Raj is still confused. Write a careful explanation of the reasoning you used to find the values in part (a). Whenever possible, use arrow diagrams or conditional statements in explaining your reasoning.

6-45. LEARNING LOG

Write an entry in your Learning Log about the converse relationship. Explain what a converse is, and give an example of a conditional statement and its converse. Also discuss the relationship between the truth of a statement and its converse. Title this entry "Converses" and label it with today's date.

METHODS AND MEANINGS

MATH NOTES

Converse

When conditional statements (also called "If …, then …" statements) are written backwards so that the condition (the "if" part) is switched with the conclusion (the "then" part), the new statement is called a**converse**. For example, examine the theorem and its converse below:

Theorem: If two parallel lines are cut by a transversal, then pairs of corresponding angles are equal.

Converse: If two corresponding angles formed when two lines are cut by a transversal are equal, then the lines cut by the transversal are parallel.

Since the second statement is a reversal of the first, it is called its converse. Note that just because a theorem is true does not mean that its converse must be true. For example, if the conditional statement, "If the dog has a meaty bone, then the dog is happy," is true, then its converse, "If the dog is happy, then the dog has a meaty bone," is not necessarily true. The dog could be happy for other reasons, such as going for a walk.

6.2.1 How can I use it? What is the connection?

Angles on a Pool Table

The activities in this section review several big topics you have studied so far. Work with your team to decide which combination of tools you will need for each problem. As you work together, think about which skills and tools you are comfortable using and which ones you need more practice with.

As you work on this activity, keep in mind the following questions:

What mathematical concepts did you use to solve this problem?

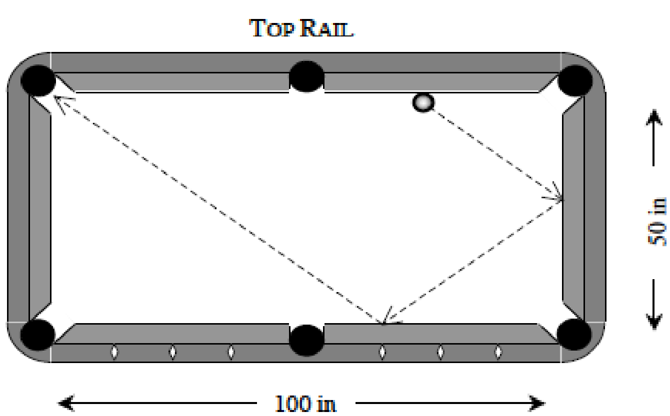
What do you still want to know more about?

What connections did you find?

6-53. TAKE IT TO THE BANK

Ricky just watched his favorite pool player, Montana Mike, make a double bank shot in a trick-shot competition. Montana bounced a ball off two rails (sides) of the table and sank it in the corner pocket. "*That doesn't look too hard,*" Ricky says, "*I just need to know where to put the ball and in which direction to hit it.*"

A diagram of Montana's shot is shown below. The playing area of a tournament pool table is 50 inches by 100 inches. Along its rails, a pool table is marked with a diamond every 12.5 inches. Montana started the shot with the ball against the top rail and the ball hit the bottom rail three diamonds from the right rail.



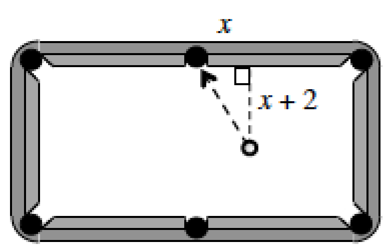
Your Task: Figure out where on the top rail Ricky needs to place his ball and where he needs to aim to repeat Montana Mike's bank shot. Write instructions that tell Ricky how to use the diamonds on the table to place his ball correctly, and at what angle from the rail to hit the ball. Explore using the Lesson 6.2.1 Resource Page.

6-54. EXTENSIONS

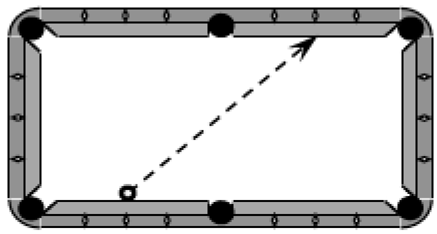
The algebraic and geometric tools you have developed so far will enable you to answer many questions about the path of a ball on a pool table. Work with your team to analyze the situations below.

a. Ricky decided he wants to alter Montana's shot so that it hits the right rail exactly at its midpoint. Where would Ricky need to place the ball along the top rail so that his shot bounces off the right rail, then the bottom rail, and enters the upper left pocket? At what angle with the top rail would he need to hit the ball?

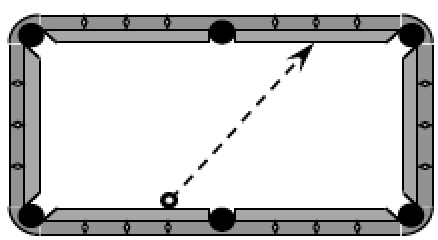
b. During another shot, Ricky noticed that Montana hit the ball as shown in the diagram below. He estimated that the ball traveled 18 inches before it entered the pocket. Before the shot, the announcers noted that the distance of the ball to the top rail was 2 inches more than the distance along the top rail, as shown in the diagram. Where was the ball located before Montana hit it?



c. Ricky wants to predict how Montana's next shot will end. The ball is placed at the second diamond from the left along the bottom rail, as shown below. Montana is aiming to hit the ball toward the second diamond from the right along the top rail. Assuming he hits the ball very hard so that the ball continues traveling indefinitely, will the ball ever reach a pocket? If so, show the path of the ball. If not, explain how you know.



d. After analyzing the path in part (c), Montana decided to start his ball from the third diamond from the left along the bottom rail, as shown below. He is planning to aim at the same diamond as he did in part (c). If he hits the ball sufficiently hard, will the ball eventually reach a pocket? If so, show the path of the ball. If not, explain how you know.



6.2.2 How can I use it? What is the connection?

Investigating a Triangle

The activities in this section review several big topics you have studied so far. Work with your team to decide which combination of tools you will need for each problem. As you work together, think about which skills and tools you are comfortable using and which ones you need more practice with.

As you work on this activity, keep in mind the following questions:

What mathematical concepts did you use to solve this problem?

What do you still want to know more about?

What connections did you find?

6-61. GETTING TO KNOW YOUR TRIANGLE

a. If you were asked to give every possible measurement of a triangle, what measurements could you include?

b. Consider a triangle on a coordinate grid with the following vertices:

$$A(2, 3),\ B(32, 15),\ C(12, 27)$$

Your Task: On graph paper, graph ΔABC and find all of its measurements. Be sure to find every measurement you listed in part (a) of this problem and show all of your calculations. With your team, be prepared to present your method for finding the area. Explore with the 6-61 Student eTool (Desmos).

6.2.3 How can I use it? What is the connection?

Creating a Mathematical Model

The activities in this section review several big topics you have studied so far. Work with your team to decide which combination of tools you will need for each problem. As you work together, think about which skills and tools you are comfortable using and which ones you need more practice with.

As you work on this activity, keep in mind the following questions:

What mathematical concepts did you use to solve this problem?

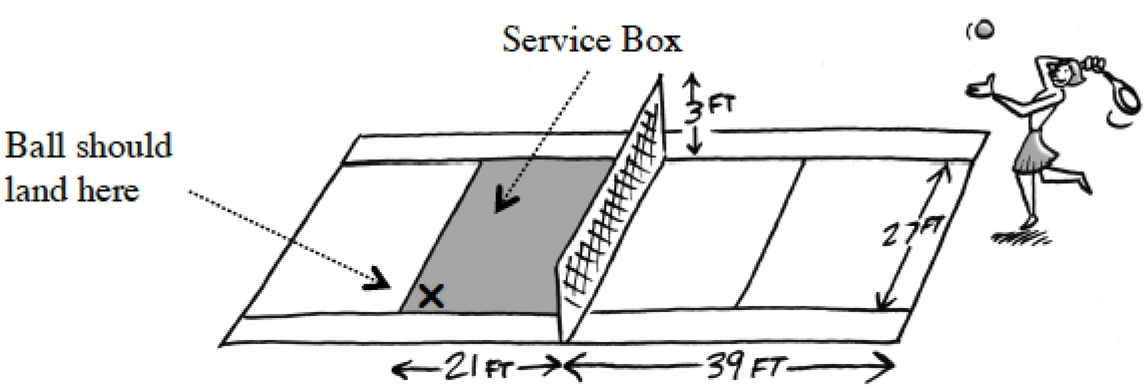
What do you still want to know more about?

What connections did you find?

6-68. AT YOUR SERVICE

Carina, a tennis player, wants to make her serve a truly powerful part of her game. She wants to hit the ball so hard that it appears to travel in a straight path. Unfortunately, the ball always lands beyond the service box. After a few practice serves, she realizes that the height at which you hit the ball determines where the ball lands. Before she gets tired from serving, she sits down to figure out how high the ball must be when she hits it so that her serve is legal.

In the game of tennis, every point begins with one player serving the ball. For a serve to be legal, the player must stand outside the court and hit the ball so that it crosses over the net and lands within the service box (shown shaded below). It can be difficult to make the ball land in the service box because the ball is often hit too low and touches the net or is hit too high and lands beyond the service box.



A tennis court is 78 feet long with the net located at the center. The distance from the net to the back of the service box is 21 feet, and the net is 3 feet tall.

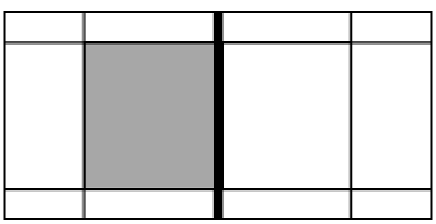
Your Task: Assuming Carina can hit the ball so hard that its path is linear, from what height must she hit the ball to have the serve just clear the net and land in the service box? Decide whether or not it is reasonable for Carina to reach this height if she is 5'7" tall. Also, at what angle does the ball hit the ground?
Your solution should include:

- A labeled diagram that shows a birds' eye view of the path of the ball.
- A labeled diagram that shows the side view of Carina, the ideal height of the tennis racket, the ideal path of the tennis ball, and the measurements that are needed from the birds' eye view diagram.

Discussion Points

What would you see if you were a bird looking down on the court as Carina served?

Which distances do you know and which do you need to find?



6-72. You've been hired as a consultant for the National Tennis Association. They are considering raising the net to make the serve even more challenging. They want players to have to jump to make a successful serve (assuming that the powerful serves will be hit so hard that the ball will travel in a straight line). Determine how high the net should be so that the players must strike the ball from at least 10 feet.

6.2.4 How can I use it? What's the connection?

Analyzing a Game

The activities in this section review several big topics you have studied so far. Work with your team to decide which combination of tools you will need for each problem. As you work together, think about which skills and tools you are comfortable using and which ones you need more practice with.

As you work on this activity, keep in mind the following questions:

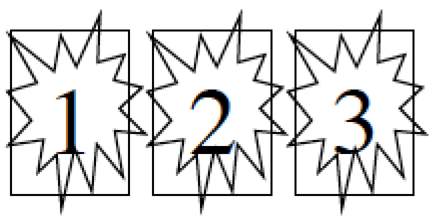
What mathematical concepts did you use to solve this problem?

What do you still want to know more about?

What connections did you find?

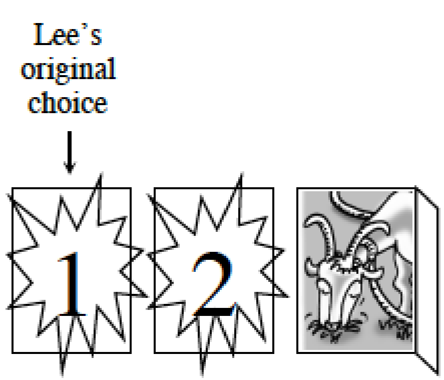
6-79. THE MONTY HALL PROBLEM

Wow! Your best friend, Lee, has been selected as a contestant in the popular "Pick-A-Door" game show. The game show host, Monty, has shown Lee three doors and, because he knows what is behind each door, has assured her that behind one of the doors lies a new car! However, behind each of the other two doors is a goat.



"*Which door do you pick?*" Monty asks.

"*I pick Door #1,*" Lee replies confidently.



"*Okay. Now, before I show you what is behind Door #1, let me show you what is behind Door #3. It is a goat! Now, would you like to change your mind and choose Door #2 instead?*" Monty asks.

What should Lee do? Should she stay with Door #1 or should she switch to Door #2? Does she have a better chance of winning if she switches, or does it not matter? Discuss this situation with the class and make sure you provide reasons for your statements.

6-80. Now test your prediction from problem 6-79 by simulating this game with a partner using either the dynamic tool, *The Monty Hall Problem*, or a programmable calculator. If no technology is available, collect data by playing the game with a partner as described below.

Choose one person to be the contestant and one person to be the game show host. As you play, carefully record information about whether the contestant switches doors and whether the contestant wins. Play as many times as you can in the time allotted, but be sure to record at least 10 results from switching and 10 results from not switching. Be ready to report your findings with the class.

- Secretly choose the winning door. Make sure that the contestant has no way of knowing which door has been selected.
- Ask the contestant to choose a door.
- "Open" one of the remaining two doors that does not have the winning prize.
- Ask the contestant if he or she wants to change his or her door.
- Show if the contestant has won the car and record the results.

6-81. Examine the data the class collected in problem 6-80.

a. What does this data tell you? What should Lee do in problem 6-79 to maximize her chance of winning?

b. Your teammate, Kaye, is confused. "*Why does it matter? At the end, there are only two doors left. Isn't there a 50-50 chance that I will select the winning door?*" Explain to Kaye why switching is better.

c. Gerald asks, "*What if there are 4 doors? If Monty now reveals two doors with a goat, is it still better to switch?*" What do you think? Analyze this problem and answer Gerald's question.

6-82. LEARNING LOG

One of the topics you studied during Chapters 1 through 6 was probability. You investigated what made a game fair and how to predict if you would win or lose. Reflect on today's activity and write a Learning Log entry about the mathematics you used today to analyze the "Monty Hall" game. Title this entry "Game Analysis" and label it with today's date.

6.2.5 How can I use it? What is the connection?

Using Transformations and Symmetry to Design Snowflakes

The activities in this section review several big topics you have studied so far. Work with your team to decide which combination of tools you will need for each problem. As you work together, think about which skills and tools you are comfortable using and which ones you need more practice with. As you work on this activity, keep in mind the following questions:

What mathematical concepts did you use to solve this problem?

What do you still want to know more about?

What connections did you find?

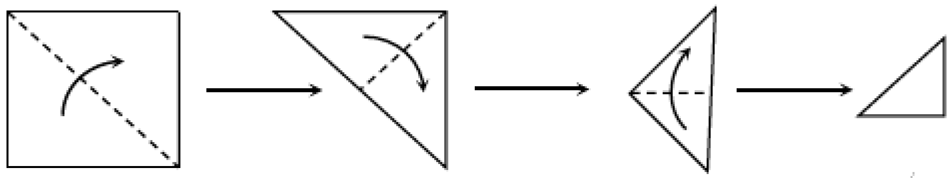
What tools would be useful to complete this task?

6-89. THE PAPER SNOWFLAKE

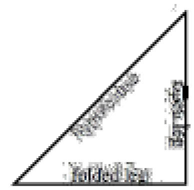
You have volunteered to help the decorating committee make paper snowflakes for the upcoming winter school dance. A paper snowflake is made by folding and cutting a square piece of paper in such a way that when the paper is unfolded, the result is a beautiful design with symmetric patterns similar to those of a real snowflake.

Looking through your drawer of craft projects, you find the directions for how to fold the paper snowflake (see below). However you cannot find any directions for how to cut the folded paper to make specific designs in the final snowflake.

DESIGNING A PAPER SNOWFLAKE: Cut out a 20 unit by 20 unit square from a piece of graph paper. Making sure that the gridlines are visible on the outside of the shape, fold the square three times as shown in the diagram below. Your final shape should be a 45°- 45°- 90° triangle that has folds along the hypotenuse.



Once you have folded your paper, correctly label the sides of your folded triangle "hypotenuse," "folded leg," and "open leg" (the leg comprised of the edges of your original square). Orient your triangle as shown in the diagram at right.



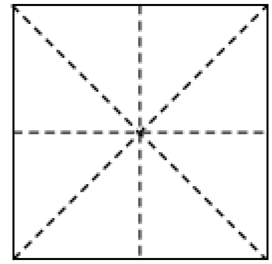
Your Task: You want to be fully prepared to help the decorating committee for the school dance. Explore and be ready to explain the relationships between the shapes that are cut out and the design that appears after unfolding the paper. For each possible location of a cutout, use what you know about symmetry and transformations to describe the shapes that result when you unfold the paper.

Discussion Points

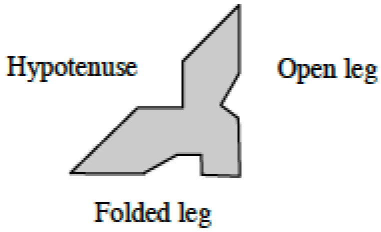
What are your goals for this task?

Visualize the result when a shape is cut along the hypotenuse.
What qualities will the result have?

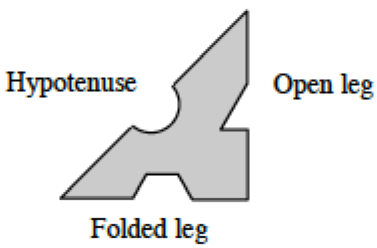
6-91. Some possible folded triangles with cutouts are shown below. What would each of these snowflakes look like when unfolded? Draw the resulting designs on the squares provided on the Lesson 6.2.5 Resource Page (or draw the result on your paper.)



a.



b.



6-92. What shape must you cut out along your folded triangle hypotenuse to get the following shapes on your snowflake when your paper is unfolded? Sketch and label diagrams to show that you can accomplish each result.

a. Rectangle with a length that is twice its width

b. Kite

c. Rhombus

d. Pentagon

6-93. EXTENSION

Get a final piece of grid paper from your teacher. Make one more snowflake that includes at least four of the following shapes. Make sure you sketch all of your planned cuts before cutting the paper with scissors. After you cut out your shapes and unfold your snowflake, answer questions (a) through (d) below.

- Draw a shape at the vertex where the hypotenuse and folded leg intersect that results in a shape with 8 lines of symmetry. Note that you will actually have to cut the vertex off to do this.
- Many letters, such as E, H, and I have reflection symmetry. Pick a letter (maybe one of your initials) and draw a shape along the folded triangle's hypotenuse or folded leg that results in a box letter when cut out and the snowflake is unfolded.
- Draw a shape along the folded triangle's hypotenuse or folded leg that results in a regular hexagon when cut out and the snowflake is unfolded.
- Draw a shape along the folded triangle's hypotenuse or folded leg that results in a star when cut out and the snowflake is unfolded.
- Draw curved shapes along the open leg so that there is at least one line of symmetry that is perpendicular to the open leg.

a. What has to be true about the shape you cut out in order for your unfolded shape to have 8 lines of symmetry?

b. There are two different shapes you could have drawn that would result in a regular hexagon. Sketch both shapes. Why do both shapes work?

c. Some shapes are impossible to create by cutting along the hypotenuse of the folded paper triangle. Sketch several different examples of these shapes. What is the common characteristic of these impossible shapes?

d. How is cutting along the open leg different from cutting along the folded leg or hypotenuse? Try to describe the difference in terms of symmetry. Use a diagram to help make your description clear.